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A note on the aggregated production function and the accounting identity

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Abstract
In this note, we give a more general, and drastically simpler, proof of the "Shaikh's relation" between the accounting identity and the Cobb-Douglas function, when the shares of the factors of production are constant. We also show that it is possible, with our simpler approach, to give a first order approximation of the error made when the shares of the factors vary slightly.
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In a recent review of Jesus Felipe and Mc Combie's important and excellent book, *The Aggregate Production Function and the Measurement of Technical Change : "Not Even wrong"* (Felipe and McCombie, 2013), Bernard Guerrien and Ozgur Gun remind us how decisive was Shaikh's criticism of Solow's famous paper "Technical Change and the Aggregated Production Function"(Guerrien and Gun, 2015), Shaikh proves that Solow's, and others, "good results", closely fitting data, can be explained by an accounting identity, completed by a "stylised fact" – the shares of the factors of production are almost constant in the data examined (Shaikh , 1974).

The aim of our note is to give a more general, and drastically simpler, proof of the relation between the accounting identity and the Cobb-Douglas function, when the shares of the factors of production are constant. We also show that it is possible, with our simpler approach, to give a first order approximation of the error made when the shares of the factors vary slightly.

A purely algebraic prove of Shaikh result

Shaikh's differential and integral reasoning is unquestionable as far as Solow's model is concerned. In a different setting it is somewhat strange to introduce a continuous parameter and to differentiate with respect to it, only to integrate back after a small rearrangement of terms.

It is much simpler to proceed directly, "algebraically", in the following way.

Let the six real numbers V, L, J, w, r and a satisfy the two relations:

(1) $V \equiv wL + rJ$ (accounting identity)

(2) $wL = aV$ (stylized fact).

The output, in value, V is equal to the payroll wL and the interests served rJ, where J is the value of capital. The number *a* gives the part of "labour" in total revenue ($a = wL/V$). It follows from (1) and (2) that the part of "capital" rJ/V is $1 - a$.

Now, the term $(wL)^b\ (rJ)^{1-b}$, with any b, can be developed in two different ways:

$$(wL)^b\ (rJ)^{1-b} = w^b L^b r^{1-b} J^{1-b} \quad \text{(evident)}$$

and

$$(wL)^b\ (rJ)^{1-b} = (aV)^b [(1-a)V]^{1-b} \quad (\text{as } wL = aV \text{ and } rJ = (1-a)V)$$

$$= a^b (1-a)^{1-b} V.$$

Equating the two forms of $(wL)^b\ (rJ)^{1-b}$, we obtain:

$$V = a^{-b}(1-a)^{b-1} w^b r^{1-b} L^b J^{1-b}$$

$$= AL^b J^{1-b}.$$

Then the relation:

(1) $$V = a^{-b}(1-a)^{b-1} w^b r^{1-b} L^b J^{1-b}$$

holds true for every real number b.

Two remarks, before going on.

***Remark* 1.** This result is more general than Shaikh's, as it is valid for *every real number* b (and not only for the factor's part a). It can be deduced noting that for *any* homogeneous function $F(\cdot)$ of degree 1 we have

$$F(wL, rJ) = F(aV, (1-a)V) = V \cdot F(a, (1-a)),$$

and then: $$V = [F(a, (1-a))]^{-1} \cdot F(wL, rJ).$$

In Shaikh's case: $F\ (x_1, x_2) = x_1^b x_2^{1-b}$.

***Remark* 2.** (Geometrical proof.) Consider $V, L,$ and J as coordinates in a three dimensional space. The accounting identity $V \equiv wL + rJ$ restricts the data to a two dimensional plane. If the ratio wL/V is fixed, then they are further restricted to a half straight line from the origin (taking into account that all the numbers involved are positive). But on such a line, all homogeneous functions of degree 1 are equal to within a constant multiplicative factor.

A first order approximation of the error made when factors shares vary slightly

In order to get rid of the complications due to the factors *w* and *r*, we set:

$\pi = rJ$ (interests served) $W = wL$ (payroll) $V = \pi + W$ (product).

Suppose we have a set of data (W_i, π_i) and we want to approximate them by a Cobb-Douglas function:

$$V_i^* = CW_i^b \pi_i^{1-b}.$$

Setting:

$$a_i = \frac{W_i}{Q_i},$$

we get, as above, from the equalities $V_i = \pi_i + W_i$ and $a_i = \frac{W_i}{Q_i}$:

$$V_i = B(a_i)W_i^b \pi_i^{1-b} \quad \text{where} \quad B(a) = a^{-b}(1-a)^{b-1}.$$

So, given the relation (1), we choose

$$C = B(b),$$

and the relative error

$$\frac{V_i - V_i^*}{V_i^*} = \frac{B(a_i)W_i^b \pi_i^{1-b} - CW_i^b \pi_i^{1-b}}{CW_i^b \pi_i^{1-b}}$$

is:

$$\frac{B(a_i) - B(b)}{B(b)}.$$

Differentiating $B(\cdot)$, we get:

$$B'(a) = \left(-\frac{b}{a} + \frac{1-b}{1-a}\right)B(a),$$

which implies the important result:

$$B'(b) = 0.$$

Consequently, given the Taylor expansion:

$$B(a_i) - B(b) = (a_i - b)B'(b) + \frac{(a_i - b)^2}{2} B''(b) + \ldots,$$

the relative error is of second order with respect to $a_i - b$.

It is easy to compute $B''(s)$ in order to find the principal part of the error. In the case of the American industry from 1909 to 1949, studied by Solow and Shaikh, the relative error never exceeds 1%.

Let us insist, as Shaikh does, that all *this is sheer mathematics* and does not involve any economic assumption.

References

Felipe, J. and J. McCombie (2013), *The Aggregate Production Function and the Measurement of Technical Change*: *Not Even Wrong*. Edward Elgar.

Guerrien, B. and Gun, O. (2015), "Putting an end to the aggregate function of production... forever?", *Real-world Economics Review*, issue no. 73, 11; Dec 2015, pp. 99-109.

Shaikh, A. (1974), "Laws of Production and Laws of Algebra: The Humbug Production Function", *The Review of Economics and Statistics*, 56(1).

Solow, R. (1957), "Technical Change and the Aggregated Production Function", *Review of Economics and Statistics*, 39(3).

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