The Law of Value:
A Contribution to the Classical Approach to Economic Analysis

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Abstract

This thesis contributes to our understanding of the relationship between the material activity of human labour and the monetary forms of an economy by examining the theoretical foundations of the classical approach to economic analysis, in particular the objective costs-of-production approach to economic value.

The classical labour theory of value suffers from two related problems: David Ricardo's problem of an invariable measure of value and Karl Marx's transformation problem. This thesis proposes to resolve both problems by constructing a more general labour theory of value.

The more general theory provides a new perspective on related issues in the classical theory, including Marx's classification of money-capital as an irrational commodity, the meaning and significance of Piero Sraffa's standard commodity and Luigi Pasinetti's restriction of the labour theory to a normative role.

According to the classical account of capitalist competition the scramble for profit causes market prices to "gravitate" to natural prices. This thesis proposes a nonlinear dynamic model of classical gravitation in which prices and labour costs converge to a state of mutual consistency in equilibrium. The dynamic model, combined with a general labour theory of value, establishes a lawful relation between prices and labour costs, which reconstructs Marx's version of the classical "law of value".

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"Every child knows a nation which ceased to work, I will not say for a year, but even for a few weeks, would perish. Every child knows, too, that the masses of products corresponding to the different needs required different and quantitatively determined masses of the total labor of society. That this *necessity* of the *distribution* of social labor in definite proportions cannot possibly be done away with by a *particular form* of social production but can only change the *mode* of its *appearance*, is self-evident. No natural laws can be done away with. What can change in historically different circumstances is only the *form* in which these laws assert themselves. And the form in which this proportional distribution of labor asserts itself, in the state of society where the interconnection of social labor is manifested in the *private exchange* of the individual products of labor, is precisely the *exchange value* of these products.

Science consists precisely in demonstrating *how* the law of value asserts itself. So that if one wanted at the very beginning to "explain" all the phenomenon which seemingly contradict that law, one would have to present science *before* science."

Letter from Marx to Kugelmann, July 1868 (Marx and Engels, 1975).

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Chapter 1

Introduction

In this thesis I examine the theoretical foundations of the classical approach to economic analysis, in particular the objective costs-of-production approach to economic value. More specifically, I propose a generalisation of the classical labour theory of value, which avoids some of its longstanding problems, and I propose a formal analysis of the dynamics of capitalist competition, which reconstructs Marx's version of the classical law of value. The thesis therefore contributes to our understanding of the relationship between the material activity of human labour and the monetary forms of an economy.

Readers looking for a straightforward defence of the classical labour theory of value will be disappointed, since I take a critical approach that emphasises its well-known foundational problems and in fact sharpens the contradictions; however, readers looking for another rejection of the classical theory will also be disappointed, since I point out how to naturally resolve the problems. My ideal reader will fully internalise and accept the existence of the foundational problems in order to clearly understand why the problems need to be resolved and how they can be. Those readers with established sympathies or antipathies to the classical theory of value will find my conclusions counterintuitive since they contradict many longstanding and seemingly well-established theoretical positions on both sides of the debate.

My reference points are the eighteenth and nineteenth century authors Adam Smith (1723–1790), David Ricardo (1772–1823) and Karl Marx (1818–1883). All these classical authors propose some variant of a labour theory of value that are at least related by the foundational problems they generate (e.g., Smith's restriction of the labour theory of value to an "early and rude state of society" (Smith, [1776] 1994, p. 53), Ricardo's problem of an invariable measure of value, and Marx's transformation problem).

The marginal revolution of the last half of the nineteenth century displaced the classical approach with a subjective utility approach to economic value (Dobb (1973, Ch. 7), Pasinetti (1981, Introduction) and Henry (1990, Chs. 4–5)). Much of modern economic theory represents a continuation and development of the marginal approach. Nonetheless, a subset of economists, dissatisfied with the marginal approach, continued to pursue the classical approach.

My "modern classical political economy" reference points are Piero Sraffa (1898–1983) whose theoretical work in the twentieth century was "explicitly designed to reconstruct the classical theory of value and distribution" (Kurz and Salvadori, 2000, p. 14) which, as Sraffa pointed out, had been "submerged and forgotten since the advent of the 'marginal' method at the end of nineteenth century" (Sraffa, 1960, p. v); and Luigi Pasinetti (1930–) whose "structural economic dynamics" represents a continuation and development of the classical and Post-Keynesian approach to theories of economic value and distribution.

1.1 The classical approach

What is the classical approach? I will attempt to answer this question by briefly drawing a contrast between the classical and marginalist research programmes.

The classical and marginalist programmes express different visions of economic reality that reduce to a theoretical "hard core" (Lakatos, 1978, p. 48–49), which is the starting point for analysis and shapes the character of any extensions and generalisations (Pasinetti, 1986).

The classical approach begins with the production of reproducible commodities as its object of analysis. A commodity is reproducible if the size of the workforce is the only enduring constraint on its level of supply. For example, Ricardo ([1817] 1996, p. 18) observes that the "commodities, the value of which is determined by scarcity alone ... form a very small part of the mass of commodities daily exchanged in the market". In contrast:

> "By far the greatest part of those goods, which are the object of desire, are procured by labour, and they may be multiplied, not in one country alone, but in many, almost without any assignable limit, if we are disposed to bestow the labour necessary to obtain them" (Ricardo, [1817] 1996, p. 18).

The analysis of the production of reproducible commodities is part of the hard core of classical economics. The classical approach therefore postpones the analysis of non-reproducible commodities to a later stage. For example, Ricardo initially excludes land and "rare statues and pictures" for which "no labour can increase the quantity of such goods and therefore their value cannot be lowered by an increased supply". He introduces marginalist concepts later, in his celebrated theory of differential rent, to directly address the question whether the scarcity of land modifies his prior labour theory of value.[1]

The classical approach analyses long run "natural" states of the economy where the supply of reproducible commodities equals the effective demand for them. A natural state is a stable "centre of gravitation" caused by a competitive scramble for profit, where capitalists invest or withdraw capital from different sectors of production based upon differential profit signals. The classical authors claim that the scramble for profit causes profits to converge toward a general, uniform profit-rate where supply equals demand (Smith, [1776] 1994, Book 1, Ch. VII). Ricardo, for example considers short run states of under or over-supply of reproducible commodities, which therefore may be temporarily scarce or abundant, as "accidental and temporary deviations" from the natural state (Ricardo, [1817] 1996, p. 109).[2]

[1] Ricardo presents his labour theory of value in Chapter 1 of *On the Principles of Political Economy and Taxation* and then turns to rent in Chapter 2, where he immediately asks "whether the appropriation of land, and the consequent creation of rent, will occasion any variation in the relative value of commodities, independently of the quantity of labour necessary to production" and then introduces marginal concepts. Smith, in Book 1 of *The Wealth of Nations*, discusses the productive powers of labour, the division of labour, wages and profits before turning to rent. Marx, under Engels' arrangement, presents his theory of rent at the end of Volume 3 of *Capital*.

[2] E.g., Marx writes, in *Value, Price and Profit*, "Supply and demand regulate nothing but the temporary fluctuations of market prices. They will explain to you why the market price of a commodity rises above or sinks below its value, but they can never account for the value itself. Suppose supply and demand to equilibrate, or, as the economists call it, to cover each other. Why, the very moment

The principal method of coordination in the classical hard core is capitalist competition, which is a mechanism that allocates fungible productive capacity to meet demand.

Marginalist economics has a different hard core. The property of economic scarcity, rather than reproducibility, dominates. For example, in the traditional marginalist vision, market participants arrive at the market endowed with different commodity bundles, which are scarce because their quantity is a given, and exogenous, variable. The market participants have different preferences that define the subjective utility they obtain from consumption. The principal method of coordination is then market exchange, which is a mechanism that efficiently allocates the given scarce resources to meet demand. For example, the marginal approach normally analyses short run "market clearing" states of the economy that define sets of exchanges that maximise the utility of each participant given their budget constraints. Marginalist authors normally assume that the forces that equate supply and demand have operated to completion, and therefore an analysis of the process by which market might clear is absent (e.g., Katzner (1989); Varian (1992); Mas-Collel, Whinston, and Green (1995)).

The marginal approach postpones the analysis of production and the allocation of productive capacity. For example, early marginalist authors "typically begin with a consideration of consumer wants, then proceed to exchange, and discuss production and distribution last" (Hennings, 1986). And Lionel Robbins (1898–1984) "at the apex of the marginalist era" (Pasinetti, 1981, p. 10) defined economics as "the science which studies human behaviour as a relationship between ends and scarce means which have alternative uses" (Robbins, 1945, p. 16).[3]

these opposite forces become equal they paralyze each other, and cease to work in the one or other direction. At the moment when supply and demand equilibrate each other, and therefore cease to act, the market price of a commodity coincides with its real value, with the standard price round which its market prices oscillate. In inquiring into the nature of that value, we have therefore nothing at all to do with the temporary effects on market prices of supply and demand" (Marx, 1999).

[3]The "non-substitution theorem" states that, under certain technical conditions and given the distribution of income, then relative prices are independent of the pattern of consumer demand. The technical conditions imply that the scarcity of endowments is no longer significant. Kurz and Salvadori (1995, p. 26) note that the theorem "was received with some astonishment by authors working in the neoclassical tradition since it seemed to flatly contradict the importance attached

Such differences motivate Pasinetti (1986) to call the classical hard core a "production paradigm" and the marginalist hard core an "exchange paradigm".[4] Clearly, each approach captures important aspects of economic reality while neglecting others. And each approach has been extended to encompass the concerns of the other. But the extensions bear the stamp of the originating research programme and therefore often generate different theories (e.g., a conflict-based, class struggle theory versus a just-rewards, marginal productivity theory of distribution).

All theoretical work begins with a vision that emphasises primary and deprecates secondary phenomena. My starting point in this thesis is the classical approach. I therefore focus on the production, distribution and consumption of reproducible commodities in the context of gravitation toward 'natural' states of an economy where productive capacity is allocated to meet demand. I view the economy as a technical system that produces a surplus, i.e. an excess of commodities or 'net product', together with social rules that distribute the surplus in the form of functional income categories, such as wages and profits (Garegnani, 1987). The functional income categories define the major economic classes of society, such as workers and capitalists.

In this thesis I do not analyse non-reproducibles, such as land. However, I do analyse out-of-equilibrium market prices that indicate the temporary scarcity (or abundance) of reproducible commodities. The Marshallian scissors are therefore present. I employ technical tools that answer questions within the classical approach, such as linear production theory and nonlinear dynamic systems theory. I also apply a philosophical method of analysis to some of the problems of the classical theory since they are of a logical and conceptual nature. I do not define utility functions, rational agents, or pose mathematical optimisation problems. Neither do I assume that markets instantaneously clear. Instead I analyse how an economy responds in his-

to consumer preferences for the determination of relative prices". The reception of this theorem, proved in the 1950's, is an interesting example of how the analysis of production was postponed in the marginalist tradition.

[4]Pasinetti suggests that the classical and marginalist approaches are distinct Kuhnian paradigms, which implies they are incommensurable in some sense. However, I prefer to think of them as Lakatosian research programmes, with different hard cores, but overlapping theoretical peripheries, including areas of agreement.

torical time to mismatches between supply and demand. Hence, marginal quantities are present but they are transient gradients within a dynamic process that includes both out-of-equilibrium and equilibrium states.

1.2 The value question

The fundamental research problem I address is a simple question that I call "the value question": *What does the unit of account, e.g. £1, represent or measure?* I analyse my classical reference points with this specific question in mind.

Possible answers to the value question include "some specific thing", "many things" or "nothing". The history of economic thought has explored all these options.

Consider, for example, the different attitudes to the meaning of a price. Classical authors, such as Smith, Ricardo and Marx, attempt to reduce prices to an objective and absolute measure of real costs. Simplifying greatly, Smith ([1776] 1994) reduces the price of a commodity to the labour it commands in the market (i.e., the units of labour that may be purchased from its sale). Ricardo ([1817] 1996), dissatisfied with Smith's theory of value, attempts to reduce a commodity's price to its "difficulty of production" either measured in labour time or some other real cost of production. Marx ([1894] 1971), in response to problems with Ricardo's attempt, proposes that prices are transformed representations of the labour 'embodied' in commodities.

The modern classical authors also propose answers to the value question. Sraffa's somewhat cryptic answer is that prices are the labour commanded by a special bundle of commodities, the "standard net product", which has certain unique properties. Pasinetti (1981, p. 153), on the other hand, denies that a labour theory of value can "reflect the price structure that emerges from the operation of the market in a capitalist economy" and instead emphasises the normative, rather than descriptive, role of the labour theory of value.

Samuel Bailey ([1825] 1967), in contrast, claims that the structure of market prices merely reflects the ratios that commodities exchange in the marketplace. Prices are a function of supply and demand, and they lack any necessary relationship to real costs of production: "value denotes consequently nothing positive or intrinsic, but merely the relation in which two objects stand to each other as exchangeable commodities" (Bailey, [1825] 1967, p. 4–5). Prices, therefore, represent nothing.[5] Marginalists, such as William Stanley Jevons (1835–1882), basically agree but are tempted to relate equilibrium prices to psychological states of the human mind, such as the "ratio of final degrees of utility".[6] Twentieth century Arrow-Debreu-McKenzie general equilibrium theory (e.g., Debreu (1959)), which still constitutes the theoretical template of much of modern economic theory, also answers "nothing": the configuration of preferences, endowments and market exchange determines prices, and prices simply express exchange ratios – they are relative quantities that do not measure or represent some other thing.

The hard core of the classical and marginalist research programmes generate different, and seemingly diametrically opposed, answers to the value question. The classical authors attempt to relate the structure of natural prices to real costs of production, and therefore propose 'semantic' theories of value in which prices denote or represent some other substance, such as labour time. In contrast, the marginalist authors relate the structure of market prices to a given state of supply and demand, and therefore propose 'syntactic' theories of value in which prices are merely exchange ratios that lack semantic content.

The majority of current economic thought occurs within the marginalist programme and therefore the predominant attitude among economists today is value nihilism. "There is only price" and to seek something behind prices, to dig deeper, is simply a kind of confused essentialism. In consequence, to ask a modern economist the value question is akin to raising the issue of phlogiston with a modern physicist. It is anachronistic. Furthermore, even within the classical research programme the

[5] Samuel Bailey's pamphlet *A Critical Dissertation on the Nature, Measures and Causes of Value* [1825] 1967 criticised Ricardo's attempt to identify an objective and invariable measure of exchange-value.

[6] Jevons, in his *The Theory of Political Economy*, writes that "value in exchange expresses nothing but a ratio, and the term should not be used in any other sense" (Jevons, [1871] 1965, p. 78) and "there is no difficulty in seeing that, when we use the word Value in the sense of ratio of exchange, its dimension will simply be zero" (Jevons, [1871] 1965, p. 83). Prices, however, relate to utility since "the ratio of exchange of any two commodities will be the reciprocal of the ratio of the final degrees of utility of the quantities of commodity available for consumption after the exchange is completed" (Jevons, [1871] 1965, p. 95).

nihilist attitude prevails, particularly in the post-Sraffian literature that, in general, rejects the classical attempt to reduce the structure of natural prices to labour costs (e.g., Howard and King (1992, Pt. IV)).

In my view the nihilist attitude does not represent a sophisticated rejection of naive substance theories of value but instead signifies the continued existence of unresolved and fundamental theoretical problems that first manifested at the birth of modern economic science in the eighteenth and nineteenth centuries. In this thesis I diagnose and propose a solution to those problems, specifically a more general labour theory of value that reconstructs the classical approach to the value question.

To borrow Marx's arresting phrase, "every child knows" that marks on a ruler measure distance, or a thermometer's mercury column measures temperature, or a clock's hands represent time. And inquisitive minds, before they are socialised to stop worrying about such things, naturally ask the value question and enquire about the nature of the numbers they find stamped upon the goods they buy and the tokens they carry in their pockets. But unlike rulers, thermometers or clocks, few adults have a clear and distinct idea of the semantics of monetary phenomena, including economists. We therefore disappoint our children. Economic science once grappled with the value question but has subsequently educated itself to stop asking it. Yet monetary phenomena, from the humble penny to the most esoteric financial instruments, control our lives in the most fundamental, pervasive and intimate manner. I believe the value question is therefore important, both within economics and the social sciences generally, because economic value is ubiquitous yet remains something of a mystery. I hope this thesis contributes towards clarifying the nature of this key social phenomenon.

1.3 Objectives

The overall aim of this thesis is to analyse and critique the classical labour theory of value in order to decide to what extent it does, or does not, answer the value question.

My first objective, therefore, will be to critically examine early formulations of the classical theory of value, as exemplified in the work of Ricardo and Marx. I aim to map Ricardo and Marx's theory into a unified formal framework, in order to make precise statements about their shared logical structure. I follow the standard criticisms of the classical labour theory of value that conclude that it fails, on its own terms, to establish that "labour" is the substance of economic value. Problems can only be solved once they are completely understood. I therefore aim to deepen the standard criticisms by identifying the fundamental cause of the problems of the classical theory.

Once this task is completed I aim to establish whether the problems of the classical theory are insurmountable or merely a property of the particular kind of theory proposed by these classical authors. If the latter, I will attempt to resolve the problems.

Ricardo and Marx's versions of the labour theory of value, although closely related, are importantly different. Marx's version, for example, is inseparable from his project of criticising capitalist property relations. A further objective, therefore, will be to determine to what extent the problems and limitations of the classical theory affect the fundamental logical structure of Marx's economic theory as a whole. For example, many authors consider that Marx's theory of exploitation, which is based on his theory of value, is either invalidated or significantly modified due to the problems of the classical theory of value. I aim, therefore, to further understand Marx's answer to the value question within the overall context of his work, and also decide whether the difficulties that Marx encounters when formulating a unified theory of value and exploitation are insurmountable, or, again, merely a property of the particular kind of value theory he employs.

Modern classical authors, such as Sraffa and Pasinetti, have contributed to deepening our understanding of the labour theory of value. A further objective, then, is to understand the contribution of these authors, and in particular whether their work supplies new or different answers to the value question. In general, post-Sraffians argue that Sraffa's work reconstructs the classical theory of value and distribution while avoiding the problems of the labour theory of value; and followers of Pasinetti, for example, view the labour theory as a normative, rather than descriptive, theory. I aim to evaluate both these claims and, thereby, develop a critical understanding of both Sraffa and Pasinetti's attitudes to the value question.

In my studies of the literature on the labour theory of value I frequently encountered some confusion and difficulties regarding the ontological status of a "labour-value", in particular whether a labour-value measures the historical real cost of producing a commodity-type (i.e., the labour supplied at the time of production), or the current real cost (i.e., the labour now supplied to make commodities of this type). Clearly, to evaluate the classical answer to the value question requires a clear conception of the precise nature of that answer. A further objective, therefore, will be to identify why these competing conceptions arise, and formulate a consistent and clear interpretation of the meaning of a labour-value.

The classical labour theory of value is predicated on the relationship between natural prices and labour costs, where natural prices are stable, equilibrium prices that emerge from the dynamics of capitalist competition. In order to evaluate the classical answer to the value question I must, therefore, evaluate the classical account of capitalist competition. The classical account is an informal theory expressed in natural language. Natural language theories lack the concepts and inferential machinery to properly formulate and analyse complex causal chains with positive and negative feedback. To overcome this limitation I aim to translate the classical theory of competition into a formal, dynamic model. The main contentious issues in the theory of value do not manifest in single commodity models, and therefore I aim to develop a multi-commodity model of classical macrodynamics. My objective is to determine to what extent the classical account constitutes a successful and logically coherent account of the emergence of natural prices, and to what extent that account may need to be modified.

Marx's theory of value claims that prices represent labour costs in virtue of the causal regularities of generalised commodity production. A further aim, therefore, will be to evaluate Marx's claim within the causal framework of the dynamic model of capitalist competition.

My final objective will be to summarise the results of my investigations and return to the value question. I aim to suggest an answer to the value question, within the context of a classical approach to economic analysis, and also discuss to what extent the answer may be incomplete and require further development.

1.4 Outline

In this section I summarise the main arguments and conclusions of each chapter. Readers, if they wish, may gain a broad understanding of the contents of this thesis without having to engage each chapter in detail.

I've tried to ensure that the chapters are relatively self-contained and therefore may be read in any order. Inevitably this necessitates some repetition of the core arguments here and there. The benefit is that readers with specific interests in parts of this thesis do not have to first progress through ancillary material.

The essential contributions of the thesis are found in Chapter 2, on the conceptual problems of the classical theory of value, and Chapter 7, on the dynamics of the classical law of value.

1.4.1 A category-mistake in the classical labour theory of value

The earliest critics of the classical labour theory of value were the classical authors themselves. My thesis begins, in Chapter 2, with the problems of the labour theory identified by Smith, Ricardo and Marx. In particular, I examine two well-known problems of the classical labour theory of value: Ricardo's problem of an invariable measure of value and Marx's transformation problem.

Ricardo wished to identify an Archimedean standpoint, outside the marketplace, from which to measure the value of commodities. Although he knew of "no other criterion of a thing being dear or cheap but by the sacrifices of labour made to obtain it" (Ricardo (2005a) p. 397) his own arguments demonstrated that the profit component of natural prices appears to be unrelated to labour cost. Although "the great cause of the variation of commodities is the greater or less quantity of labour that may be necessary to produce them" there is another "less powerful cause of their variation" (Ricardo, 2005a, p. 404), which Ricardo suggested was "a just compensation for the time that profits were withheld" (Ricardo, [1817] 1996). In consequence, natural prices (the measurand) vary independently of real costs of production defined in terms of labour costs (the candidate measure of value). A measure that fails to vary with its measurand is not fit for purpose.

Ricardo grappled with this problem, and wrote a remarkable unfinished essay on the topic in the last weeks of his life, which finally concluded that "it must then be confessed that there is no such thing in nature as a perfect measure of value" (Ricardo, 2005a, p. 404). Ricardo retreated to proposing approximate, and therefore, imperfect measures of value, which minimise the discrepancies between the measure and measurand. But a ruler that, on theoretical grounds alone, fails to invariably measure length is not merely an imperfect empirical tool – it implies that one's theory of length is flawed.

Marx, who inherited Ricardo's problem, proposed a creative and novel resolution. He argued that natural prices are transformed, or distorted, labour costs due to capitalist property relations. Prices then appear to vary independently of labour costs because the transformation obscures the measurement relation. This explained Ricardo's difficulties. Marx argued, however, that the transformation is conservative and therefore the measurement relation continues to hold between macroeconomic aggregates. The conservation relation is essential to Marx's solution since it restores an invariable measure, and therefore avoids the conclusion that, on theoretical grounds alone, a labour theory of value is flawed. According to Marx, therefore, the "form of value" (natural prices) only appears to contradict the "substance of value" (labour costs) in virtue of the institutional peculiarities of capitalist production.

Marx warned his readers, however, that his solution contained the "possibility of an error" (Marx, [1894] 1971, p. 165) if a particular assumption of his argument was relaxed. Marx's critics promptly demonstrated that possibility and argued that, in general, Marx's transformation is not conservative and hence fails to establish the desired measurement relation. The quantitative incommensurability between labour-values and natural prices has consequences for other parts of Marx's theory, such as his theory of exploitation, which claims that "surplus-value", e.g. profits and interest income, is a money representation of the "surplus-labour" that workers supply to capitalists without payment.

Both Ricardo and Marx's problems directly undermine the idea that labour costs can in principle explain economic value. Chapter 2 formally presents these theoretical problems in the context of a linear production model of capitalist production. The formality imparts precise semantics to some of the key concepts of the labour theory of value, which helps identify a certain kind of logical error in the classical theory. I argue that both problems derive from the same conceptual error of supposing that total costs (e.g., natural prices) and technical costs (e.g., labour costs) are of the same logical type. More specifically, the classical authors attempt to explain the structure of total costs of production – which include both technical costs due to the material conditions of production (e.g., the cost of physical capital and labour inputs) and additional social costs due to the institutional conditions of production (e.g., the cost of money-capital, state imposed taxes, etc.) – in terms of the structure of technical costs of production alone. Ricardo and Marx therefore implicitly expect to discover a commensurate relationship between cost structures defined by incommensurate accounting conventions. I clam that this conceptual error is the underlying cause of the almost two hundred year history of the "value controversy".[7]

Once identified we can avoid the error. The key step is to define a new measure of labour cost – total labour costs – that generalise the classical measure to include real costs induced by the institutional conditions of production. I then sketch a more general labour theory of value that includes both total and classical (i.e., technical) measures of labour cost. The general theory applies the different measures in distinct, but complementary, theoretical roles, and in consequence separates issues normally conflated in the classical theories.

In Chapter 2 I explain how the more general theory has both an invariable measure of value and lacks a transformation problem. The main technical result is the theorem that natural prices are proportional to physical real costs of production measured in labour time. Hence, prices and labour costs, in appropriate equilibrium conditions, are "two sides of the same coin". The measurement relation, missing from the classical theory, is therefore established, which implies that labour costs can in principle explain economic value. The more general theory removes the pri-

[7] I take this helpful phrase from title of a collection of essays on the labour theory of value, *The Value Controversy* (1981), edited by Ian Steedman.

mary theoretical obstacle that has hindered the development of the classical theory of value from its inception.

I claim the classical error causes a deep theoretical fracture in the labour theory of value that splits nominal phenomena, such as the structure of natural prices, from real phenomena, such as physical costs of production. Its effects should therefore be pervasive. I supply evidence for this claim in the next three chapters by examining how the error further manifests in the work of Marx, Sraffa and Pasinetti. In each case I identify a theoretical problem or incompleteness caused by the unidentified conceptual error, and then resolve, or dissolve, the problematic from the perspective of a more general labour theory of value.

1.4.2 Marx's irrational commodity

Marx's theory of value remains, even to this day, the most ambitious and sophisticated attempt to establish a semantic relationship between the monetary unit of account and the material activity of human labour. Partisans of Marx's thought are naturally concerned to defend this scientific legacy. Unfortunately, many criticisms of Marx's theory of value are decidedly superficial; and it quickly becomes wearisome to encounter another screed on Marx's supposed errors. In consequence, the typical partisan is conditioned to view any criticism of Marx's theory with a jaundiced eye. My Chapter 3, which introduces a new and critical reading of the hard core of Marx's economic theory, is therefore likely to be regarded, at best, warily. In way of an apology and invite I offer the following quotation from Hegel's *Science of Logic*:

> "*Intelligent* reflection, to mention this here, consists, on the contrary, in grasping and asserting contradiction. Even though it does not express the Notion of things and their relationships and has for its material and content only the determinations of ordinary thinking, it does bring these into a relation that contains their contradiction and allows *their Notion to show or shine through* the contradiction. Thinking reason, however, sharpens, so to say, the blunt difference of diverse terms, the mere manifoldness of pictorial thinking, into *essential* difference, into *opposition*.

> Only when the manifold terms have been driven to the point of contradiction do they become active and lively towards one another, receiving in contradiction the negativity which is the indwelling pulsation of self-movement and spontaneous activity" (Hegel, 1969, p. 442).

The language may initially seem opaque but Hegel's methodological remarks here are highly sophisticated. Partisans of dialectical materialism should pay even closer attention to logical contradictions at the level of "ordinary thinking" for the reasons Hegel gives. Often, to make theoretical progress, we need to compress the "manifold terms" of a complex theory into an essential *logical* contradiction. The reduction to a logical contradiction may reveal a glimpse of an underlying process of change that the theory fails to adequately reflect.

I argue in Chapter 3 that the classical error, identified in Chapter 2, is the essential contradiction of the hard core of Marx's economic theory. Marx's theory is indissolubly bound to his Hegelian commitments and therefore the classical error takes a specific form in his work. This chapter, therefore, tackles philosophical themes normally absent from strictly economic interpretations of *Capital*.

Chapter 3 may also be a good starting point for readers familiar with Marx's work but less familiar with linear algebra since I explain the key distinction between classical and total labour costs without recourse to linear production theory.

The chapter begins by examining the surface features of Marx's relatively neglected theory of money-capital. This serves as an entry point to deeper issues.

The prices of ordinary commodities, in Marx's theory of value, are lawfully regulated by their labour costs. Money-capital, in contrast, is an "irrational" commodity, with the form of a commodity but not its substance, because its price, which is the interest rate quoted in capital markets, lacks a lawful relationship to labour costs. Money-capital, in consequence, is an exceptional or unique commodity; or, as Marx states, a "commodity *sui generis*" (Marx, [1894] 1971, Ch. 22).

Marx's description of money-capital as irrational summarises the opposing implications of his theory of economic value and his theory of surplus-value. Marx's theory of value suggests that money-capital is a *bona fide* commodity with a corresponding real cost, which is the labour supplied to bring it to market, whereas his theory of surplus-value suggests that money-capital is *sui generis* because the labour supplied to bring it to market is surplus-labour, which is an excess of labour, over-and-above that necessary to reproduce workers, supplied "*gratis*" (Marx, [1867] 1954, Ch. 18) to capitalists. Surplus-labour is an intrinsically costless net output, or 'something for nothing', which cannot constitute a real cost of production. Money-capital therefore belongs to the class of commodities that "have a price without having a [labour-]value", for example land or "conscience, honour, etc." (Marx, [1867] 1954, Ch. 3).

Marx, as a follower of Hegel, is committed to an ontology that admits dialectical contradictions. Marx can therefore accept that 'irrational' kinds exist in reality. A logical contradiction denotes an impossibility (e.g., a square circle); in contrast, a real, or dialectical, contradiction denotes the struggle of parts of a system to control a property of the system in incompatible ways (e.g., two teams in a game of tug-of-war that attempt to pull the rope in opposite directions). Real contradictions are the cause of change and motion. Hence, a system with real contradictions is logically possible but may ultimately be unstable and therefore transient on some time-scale.

Marx applies the Hegelian ontology to the sweep of human history. Marx's "materialist conception of history" (Marx and Engels, 1987, Pt. 1) aspires to explain the rise and fall of kinds of societies in terms of recurring real contradictions between the causal powers of human labour and the social institutions that organise those powers. Humans spontaneously learn from their material practice and embody their knowledge in the form of tools and machinery etc.; in consequence, their causal powers have a tendency to alter and improve. At certain historical junctures the social institutions become a "fetter" (Marx, 1993a, preface) that prevent the full realisation of those powers. The social institutions have become 'irrational', and ripe for abolition, from the counterfactual perspective of causal possibilities immanent within society. As Friedrich Engels, Marx's lifelong collaborator, states, "all that is real in the sphere of human history, becomes irrational in the process of time, is therefore irrational by its very destination, is tainted beforehand with irrationality"; in consequence, "all that exists deserves to perish" (Engels, 1976, Pt. 1).

Capitalism, as a contradictory social formation, necessarily throws up irrational kinds such as money-capital. According to Marx "the relations of capital assume their most externalised and most fetish-like form" (Marx, [1894] 1971, Pt. V, Ch. 24) in money-capital. Marx is therefore comfortable with money-capital constituting an exception to his theory of value.

The main proposition of Chapter 3 is that Marx's arguments for this conclusion commit a logical fallacy. In consequence, Marx mischaracterises the nature of money-capital and unnecessarily restricts the explanatory reach of the labour theory of value. My argument relies on a distinction between factual and counterfactual accounts of the labour process.

Marx's theory of surplus-value, as an application of historical materialism, is an irreducibly counterfactual theory that identifies economic possibilities that capitalist institutions prevent. I demonstrate that Marx implicitly builds his theory of surplus-value upon a comparison between an empirical state-of-affairs, specifically the labour process organised under the rubric of capitalist property relations, and a possible state-of-affairs, specifically the labour process organised under the rubric of post-capitalist property relations. Marx justifies his classification of surplus-labour as excess labour, supplied to capitalists 'for free', by noting that this labour would be unnecessary in a post-capitalist society. Workers, in such circumstances, would not need to supply surplus-labour in order to receive the real wage and reproduce themselves. In this precise counterfactual sense *surplus-labour is not a necessary cost of production*. I emphasise, therefore, that Marx's theory of surplus-value is a critical theory of capitalist production that rejects the cost structure engendered by capitalist property relations.

But what if we take a purely factual view of the labour process? Capitalists, of course, do not supply money-capital for free, either nominally or in real terms. Workers, when organised under the rubric of capitalist property relations, in fact supply surplus-labour in order to receive the real wage and reproduce themselves. In this precise factual sense, therefore, *surplus-labour is a necessary cost of production*.

So is the surplus-labour supplied to capitalists a cost of production or not? On the one hand, and following Marx, we may take a critical view of the capitalist labour process and observe that the supply of surplus-labour is historically contingent and therefore counterfactually unnecessary; on the other hand, we may take an empirical view of the labour process, which Marx does not do, and observe that surplus-labour, although historically contingent, is factually necessary in the empirical circumstances of a capitalist economy. Which view should we adopt?

We should adopt the viewpoint commensurate with our theoretical aims, such as whether we wish to critique or explain the cost structure engendered by capitalist property relations. Classical labour cost accounting, by excluding surplus-labour as a cost, yields a counterfactual measure of "difficulty of production" and therefore provides the quantitative basis for a critique of the cost logic of capitalism. Total labour cost accounting, introduced in Chapter 2, includes surplus-labour as a cost and therefore yields a factual measure of "difficulty of production" that provides the quantitative basis for an explanation of the cost logic of capitalism. The general labour theory of value, sketched in Chapter 2, includes both kinds of labour costs and applies them in the appropriate contexts. Marx's labour theory of value, in contrast, admits only the classical measure.

Marx's theory of value, when compared to his theory of surplus-value, has the distinct explanatory aim of establishing a semantic relation between money and labour time.[8] Marx's transformation, for example, proposes that natural prices are conservative transforms of the underlying labour costs.

Marx, unfortunately, attempts to explain a factual cost structure that includes surplus-value as a cost, that is natural prices,[9] in terms of a counterfactual cost structure that excludes surplus-labour as a cost, that is classical labour costs. But a factual cost structure cannot be explained in terms of a counterfactual cost structure. This is the fundamental logical contradiction in the hard core of Marx's economic theory.

Marx aims to construct a unified theory of value and exploitation. On the one hand, Marx employs his theory of surplus-value to reject the cost logic of capitalism;

[8]Marx's theory of value generates propositions such as: 'money as a measure of value, is the phenomenal form that must of necessity be assumed by the measure of value which is immanent in commodities, labour-time' (Marx, [1867] 1954, p. 97)).

[9]For instance, Marx specifically includes the interest-rate as an *ex ante* cost that forms a component of the natural price of commodities.

on the other hand, Marx employs his theory of value to explain that logic. A counterfactual measure of labour costs can satisfy only one of these aims. The classical error, in Marx's work, takes the form of a conflation of empirical and critical analyses.

The fundamental contradiction manifests as different surface problems. The most well known manifestation is the transformation problem, discussed in Chapter 2. Marx's theory of the irrational nature of money-capital, discussed in Chapter 3, is another, less well-known manifestation. The classical error, once again, splits nominal phenomena (in this case the interest rate) from real phenomena (in this case the labour supplied to bring money-capital to market). Hence, Marx classifies money-capital as irrational, with a price that lacks labour content.

A general labour theory of value provides a different perspective. Money-capital, in the general theory, is a fully-fledged commodity with a natural price proportional to total labour cost. Total labour cost accounting reveals the intimate relation between nominal and real cost structures induced by capitalist property relations. However, Marx's "critical analysis of capitalist production"[10], identifies the surplus-labour supplied by workers as unnecessary. Classical labour cost accounting, which measures the quantity of surplus-labour supplied and the degree of exploitation etc., reveals the exploitative relation between workers and capitalists. The general theory can therefore claim that money-capital is a rational commodity, with a price that has labour content, while simultaneously claim that money-capital is the product of social relations that are irrational from the perspective of historical materialism. The general theory relocates the irrationality of money-capital from its nature as a commodity to its nature as a social practice. In other words, we can simultaneously critique money-capital and include it within the explanatory reach of the labour theory of value.

Hegel (1969, p. 442) advised that "only when the manifold terms have been driven to the point of contradiction do they become active and lively towards one another, receiving in contradiction the negativity which is the in-dwelling pulsation of self-movement and spontaneous activity". The "self-movement and sponta-

[10]The subtitle of *Capital*.

neous activity", or process of change, revealed by the fundamental contradiction is the historically contested and changing definition of what should, and should not, constitute a necessary cost of production in human society. Marx employs a single definition of necessary cost. A theory with sufficient representational capacity to adequately reflect this historical process includes contested, and therefore multiple, definitions. This is what a more general labour theory of value provides.

1.4.3 Sraffa's incomplete reductions to labour

Piero Sraffa's work in the twentieth century significantly contributed to a revival of the classical approach to value and distribution. Chapter 4 examines Sraffa's answer to the value question.

Sraffa is acutely aware of the problems of the classical labour theory of value.[11] Sraffa demonstrates, in his major work *Production of Commodities by Means of Commodities*, that natural prices necessarily vary independently of classical labour costs (Sraffa, 1960, Ch. 3). Sraffa's "reduction to dated quantities of labour" (Sraffa, 1960, Ch. VI) represents natural prices as as a "sum of a series of terms when we trace back the successive stages of the production of the commodity" (Sraffa, 1960, p. 89). The costs of production at each 'stage' consist of the wages of labour and the interest on the money-capital advanced compounded over the 'duration' of the advance. Natural prices are therefore reducible to a sum of wage and interest income. Sraffa's reduction equation makes it particularly clear that natural prices can change due to an alteration in the wage or interest-rate, even though the labour supplied to produce commodities remains constant. Sraffa therefore rejects the idea that real costs, such as labour time, function as a measure of value. In consequence post-Sraffian scholarship is near unanimous in rejecting this aspect of classical theory, especially Marx's more emphatic assertions of the necessary link between prices and labour time.

In Chapter 4 I demonstrate that Sraffa's reduction equation is incomplete in the specific sense that some actual labour supplied during the "successive stages of the

[11] Sraffa was well-versed in classical theory; for example, he edited, with Maurice Dobb, *The Works and Correspondence of David Ricardo*.

production of the commodity" is missing. The key issue, as should be clear by now, is that classical labour costs, which Sraffa employs, do not include surplus-labour as a cost of production. In consequence, the surplus-labour supplied at each stage of production is missing from Sraffa's reduction equation.

In contrast, I construct the complete "reduction to dated quantities of labour" equation that includes the surplus-labour supplied at each stage of production. Natural prices, in this alternative but quantitatively equivalent representation, completely reduce to a sum of wage incomes only. The complete reduction equation makes it particularly clear that natural prices are always proportional to total labour costs, regardless of the distribution of income. Sraffa's rejection of the possibility of a labour theory of value is therefore based on an incomplete "reduction to dated quantities of labour". In this sense, Sraffa reproduces, or at least does not identify, the classical error.

Nonetheless, Sraffa constructs a subtle and refined objective theory of value, which reconstructs some aspects of the classical theory. In particular Sraffa proposes a partial solution to Ricardo's problem of an invariable measure of value.

Sraffa observes that natural prices are relative, rather than absolute, since they are under-determined up to an arbitrary choice of *numéraire*. For example, assume that the natural prices of a two-commodity economy are $p_1 = 1$ and $p_2 = 4$, if we choose the *numéraire* $p_1 = 1$; or $p_1 = 1/4$ and $p_2 = 1$, if we choose the *numéraire* $p_2 = 1$. In both cases the relative cost structure is identical. The choice of *numéraire* then fixes an absolute, although arbitrary, scale.

Sraffa notes the following problem: consider a change in the distribution of income (i.e. a change in the wage or profit-rate) that alters the structure of natural prices. For example, assume that prices change to $p_1 = 1$ and $p_2 = 2$, given our choice of *numéraire* $p_1 = 1$. Can we therefore assert that p_2 halved due to the change in the distribution of income?

No, because the change in the distribution of income alters the *entire* structure of relative prices, including the relative price of the *numéraire* commodity. For example, if we instead had chosen the *numéraire* $p_2 = 1$ then we might be tempted to assert that p_2 remained constant while p_1 doubled.

Sraffa notes “it is impossible to tell of any particular price-fluctuation whether it arises from the peculiarities of the commodity which is being measured or from those of the measuring standard” (Sraffa, 1960, p. 18). Sraffa’s problem is precisely Ricardo’s problem of finding an invariable measure of value, except restricted to the special case of changes in the distribution of income.

It is not always sufficiently appreciated that Sraffa’s problem only arises because the classical labour theory fails to explain the structure of natural prices. If that theory succeeded then natural prices would reduce to labour costs, and therefore labour costs would function as price-independent, absolute measure of value. The failure of the classical theory does not prompt Sraffa to adopt Bailey’s nihilist position that natural prices are merely exchange ratios (i.e., relative quantities) that do not denote, refer to, or measure some non-price substance. Instead, Sraffa, via a remarkable and often misunderstood argument, constructs an invariable measure that partially solves Ricardo’s problem.

The invariable measure is Sraffa’s celebrated “standard commodity”, which is a special collection of commodities with the peculiar property that its price is independent of the fluctuations in prices that accompany a change in the distribution of income. Chapter 4 explains, in formal terms, how Sraffa’s standard commodity functions as an Archimedean standpoint, outside the system of relative prices, from which to measure the objective value of commodities. Once we adopt the standard commodity as *numéraire* then we can be sure that any price fluctuations do not arise “from the peculiarities ... of the measuring standard”.

After this breakthrough Sraffa then delivers something like a punchline to an elaborate theoretical joke. Sraffa reduces his standard commodity to the (variable) quantity of labour that can be purchased by it. (The quantity is variable because the price of the standard commodity, although independent of prices, nonetheless varies with the distribution of income). Sraffa explains how we can adopt this variable quantity of labour as the *numéraire* without needing to specify the composition of the standard commodity. The standard commodity, therefore, is “a purely auxiliary construction” (Sraffa, 1960, p. 31), a mere step in an argument towards the conclusion that a scalar quantity of labour, rather than a heterogeneous collection of commodities, is an invariable measure of value.

Sraffa's argument reconstructs, in attenuated form, aspects of the classical theory of value, specifically the attempt to measure a given physical surplus in terms of labour costs and relate how that quantity of labour breaks down into wage and profit income. However, as Sraffa notes, this invariable measure is not a real cost of production but "equivalent to something very close to the standard suggested by Adam Smith, namely 'labour commanded' " (Sraffa, 1960, appendix. D).

A general labour theory of value, which admits both classical and total labour costs, provides an entirely different perspective of Sraffa's problematic, and clarifies the meaning of Sraffa's argument.

I prove, in Chapter 4, that Sraffa's "variable quantity of labour" is the total labour cost of the standard commodity. Sraffa's invariable measure of value is therefore a proxy or indirect reference to the total labour costs introduced in Chapter 2. In consequence, Sraffa's invariable standard is not merely a "labour commanded" but is also a "labour-embodied" measure of value that denotes a real cost of production. Sraffa, implicitly and without knowing, refers to total labour cost, which is the external standard of natural prices missing from the classical theory.

Sraffa's remarks that some properties of his argument are "curious" (Sraffa, 1960, p. 37), especially "that we should be enabled to use a standard without knowing what it consists of". The mystery lessens once we realise that Sraffa's argument is highly indirect: the standard commodity is a bridge from the premise that labour costs cannot measure natural prices to the conclusion that a "quantity of labour" is nonetheless an invariable measure. The bridge can be thrown away, as Sraffa's analysis demonstrates, because the premise is mistaken. Sraffa's argument is a rather large hint that an invariable measure of value exists, which is not a composite, but rather a single substance. Sraffa's remarkable construction of the standard commodity therefore partially identifies the total labour costs that natural prices denote. In consequence, Sraffa's original problem of choosing an invariable *numéraire* disappears since we immediately possess a real cost standard outside the market and its system of relative prices.

I conclude that Sraffa's reconstruction of classical economics is incomplete since it fails to reconstruct a measurement relation between natural prices and real costs of production. The perspective of a more general labour theory of value is required for the complete reconstruction.

1.4.4 Pasinetti's vertically-integrated subsystems and Marx's transformation problem

Most interpreters of Sraffa believe his analysis implies that the labour theory of value is, at best, incomplete, or worse, logically incoherent (e.g., Samuelson, 1971; Lippi, 1979; Steedman, 1981) since they take it as conclusively demonstrating that natural prices and labour costs are fundamentally incommensurate. Luigi Pasinetti, a pupil of Sraffa, offers a different interpretation. He proposes a "separation thesis"[12] (Pasinetti, 2007, Ch. IX) that orders the study of economic systems into a pre-institutional or "'natural' stage of investigation", concerned with "the foundational bases of economic relations", followed by an "institutional stage" (Pasinetti, 2007, p. 276), which is "carried out at the level of the actual economic institutions" (Pasinetti, 2007, p. 275). Pasinetti's attitude to the labour theory of value is shaped by this separation.

Pasinetti argues, in a series of works (e.g., Pasinetti (1981, 1988, 1993)), that the labour theory of value is a powerful analytical tool at the pre-institutional stage of investigation, and therefore provides "a logical frame of reference" with "an extraordinarily high number of remarkable, analytical, and normative, properties" (Pasinetti, 1988, p. 132). For example, Pasinetti (1988) analyses the pre-institutional cost structure of a non-uniformly growing economy. He generalises the labour theory by proving that this economy's natural prices are proportional to a more general measure of labour costs, which he calls the hyper-integrated labour coefficients, that include the labour supplied to produce net investment goods.

At the institutional stage of analysis, however, even this more general labour theory breaks down. Pasinetti proposes a "complete generalisation of Marx's 'transformation problem'" (Pasinetti, 1988, p. 131) by proving that the natural prices engen-

[12]Pasinetti actually calls his proposal a "separation theorem", but since no proof is involved I prefer to name it a thesis.

dered by capitalist property relations, in the context of his non-uniformly growing economy, are not proportional to the hyper-integrated labour coefficients. Pasinetti (1981, p. 153) concludes that “a theory of value in terms of pure labour can never reflect the price structure that emerges from the operation of the market in a capitalist economy”.

Pasinetti therefore restricts the labour theory of value to a normative role that provides a ‘natural’ or ideal standard from which to analyse the institutional setups of actual economic systems. Pasinetti’s attitude echoes Adam Smith’s restriction of the labour theory to an “early and rude state of society” that precedes the “accumulation of stock” (Smith, [1776] 1994, p. 53).

Chapter 5 critically examines Pasinetti’s argument. I note that Pasinetti’s proposal of generalising the concept of labour costs to apply to more general economic situations is an important conceptual advance over the classical theory. However, I argue that Pasinetti’s restriction of the labour theory to a purely normative role is unwarranted.

Pasinetti’s generalisation of Marx’s transformation problem reproduces the classical error at a higher level of generality. Pasinetti’s hyper-integrated coefficients are pre-institutional labour costs that ignore additional real costs engendered by capitalist property relations. Pasinetti necessarily encounters a transformation problem when he contravenes his own “separation thesis” and compares a nominal cost structure, which belongs to the institutional stage of analysis, with a real cost structure that belongs to a natural, or pre-institutional, stage of analysis.

The general theory, introduced in Chapter 2, dissolves Pasinetti’s transformation problem. I construct the total labour costs for Pasinetti’s economy, which generalise the hyper-integrated coefficients to include real costs induced by the institutional conditions of production. I prove that the natural prices of Pasinetti’s non-uniformly growing economy are proportional to its total labour costs. This result demonstrates that the concept of total labour cost applies to quite general economic situations. A suitably generalised labour theory of value, which includes both natural and institutional measures of labour cost and employs them in their appropriate contexts, is therefore neither incomplete or incoherent, nor restricted to a normative role, but spans both the natural and institutional stages of analysis.

The post-Sraffian 'surplus approach' to classical theory discards problematic elements of the classical theory of value, specifically the attempt to explain natural prices in terms of real costs measured in labour time. In contrast, Marxian authors normally defend this aspect of classical theory, either by counter-critique or creative re-interpretation of Marx's theory. The theory of value is the fundamental issue that separates the Marxian and Sraffian schools. In my view the Sraffian tradition has properly internalised the real problems of the classical theory of value but incorrectly rejected its essential elements, whereas the Marxian tradition has in general failed to fully internalise its real problems but has correctly retained its essential elements. Both sides of the debate share a conceptual framework that reproduces the classical error of expecting a commensurate relationship to obtain between cost structures defined by incommensurate accounting conventions.

Chapters 2 to 5 of this thesis contribute to dissolving this separation by revealing the possibility of a theoretical unification centred on the development of a more general labour theory of value. The general theory, developed in this thesis, both explains and resolves the recurring split of nominal from real phenomena that repeatedly manifests in the classical economic tradition, from Smith's restriction of the labour theory to pre-civilised times, Ricardo's struggles to find an invariable measure of value, Marx's attempt to build a unified theory of value and exploitation, Sraffa's incomplete reconstruction of the classical theory and Pasinetti's restriction of the labour theory to a normative role. All these theoretical problems or limitations can be better understood, and also resolved in a relatively straightforward manner, once we adopt the more general viewpoint.

1.4.5 Substance or field? A note on Mirowski

Chapter 6 serves as an interlude that introduces some necessary conceptual clarity prior to introducing a dynamic model in Chapter 7.

The meaning of a labour cost, especially as employed by Marx, is subtle and not always well understood, even when we simplify and drop Marx's modifier "socially necessary" by assuming that firms in each sector of production have homogeneous labour productivity (as assumed in this thesis). What does Marx mean, for example, by the phrase “embodied” or “congealed” labour?

Phillip Mirowski criticises Marx for holding two contradictory theories of value: a “substance” theory, where a commodity’s labour cost is the historical cost “embodied” at the time of production, and a “field” theory, where a commodity’s labour cost is its current cost of replacement, given the prevailing technical conditions of production. Mirowski claims that Marx constructs a contradictory theory of value that uses both incompatible principles, and therefore must choose between a substance *or* field theory of value. Chapter 6 is a short note that clarifies Marx’s concept of labour cost by criticising Mirowski’s thesis. I argue that Marx’s constructs a remarkably sophisticated substance *and* field theory of value.

1.4.6 The general law of value

The coordination of millions of independent production activities in a large-scale market economy is neither perfect nor equitable but nonetheless “one should be far more surprised by the existing degree of coordination than by the elements of disorder” (Boggio, 1995). The classical tradition developed a theoretical framework in which this surprising fact could be understood.

The classical authors, such as Smith and Marx, explained economic coordination as the unintended consequence of the self-interested decisions of economic actors engaged in competition (e.g., Smith’s “invisible hand” or Marx’s “law of value”). Capitalists, who seek the best returns on their investments, withdraw capital from unprofitable sectors and reallocate it to profitable sectors. The scramble for profit eliminates arbitrage opportunities until a general or uniform profit-rate prevails across the whole economy, at which point capitalists lack any incentive to reallocate their capital. Capitalist competition, according to the classical authors, is a mechanism that causes market prices of reproducible commodities to gravitate toward or around their natural prices (e.g., Smith ([1776] 1994), Book 1, Chapter VII).

My formal analyses of the classical theory, in Chapters 2 to 5, employed linear production theory to examine steady state or growing economies in natural price equilibrium. These models implicitly assume gravitation has operated to completion.

The final chapter of this thesis drops this assumption and applies the formal tools of dynamic systems theory to examine the value question in the more general context of classical macrodynamics.

Modern formal analyses of classical gravitation have yielded mixed results that have led some authors to question whether gravitation is possible in principle. Chapter 7 presents a multisector, nonlinear dynamic model of the classical process of gravitation with simultaneous changes in both prices and quantities. I theoretically demonstrate that the equilibrium of the dynamic model is formally equivalent to the steady state discussed in Chapters 2 and 4, and I numerically demonstrate, via computer simulation, that the equilibrium is locally asymptotically stable. The dynamics of capitalist competition cause prices and real costs to grope towards mutual consistency during convergence to equilibrium, at which point natural prices are proportional to total labour costs. I conclude that the classical theory, suitably formalised, constitutes a successful and logically coherent explanation of the homoeostatic kernel of capitalist competition.

The dynamic model has a Keynesian character since the economy does not operate at full capacity in equilibrium. Capitalist competition, in itself, does not generate incentives for capitalists to spontaneously coordinate their plans in order to achieve full employment. The dynamic model also has a distinctly classical character since key economic variables, such as the distribution of income, the level of employment and equilibrium prices, are not merely technical outcomes but crucially depend on how workers and capitalists react to their changing economic circumstances. For example, the key driver of income shares is the management of the interest-rate by capitalists in response to fluctuations in their stocks of money wealth.

Marx's "law of value" states that the market prices of reproducible commodities are lawfully regulated by labour costs. The dynamic model makes it particularly clear that the class struggle over the distribution of the surplus is an ineradicable joint cause of the gravitation of market to natural prices. In consequence, market prices are not solely regulated by labour costs. Institutions, in particular the distributional rules instantiated by capitalist property relations, also matter. However, the dynamic model also makes it particularly clear that the homoeostatic dynamics of capitalist competition instantiate a lawful regularity between market prices and real costs measured in labour time. In consequence, natural prices represent labour costs in virtue of a causal law that connects them, much as we might claim that the height of a mercury column represents temperature in virtue of the law of thermal expansion. The dynamic analysis of Chapter 7 therefore reconstructs Marx's argument, presented in Volume 1 of *Capital*, that economic value represents labour time in virtue of the causal regularities of generalised commodity production. My analysis supports Marx's distinctly classical proposition that "labour is the substance, and the immanent measure of value" (Marx, [1867] 1954, p. 503), i.e. that monetary phenomena in some sense refer to, express, or measure labour time.

I conclude the thesis by briefly discussing to what extent the value question has been answered, and I outline some open questions and directions for further research.

Chapter 2

A category-mistake in the classical labour theory of value

David Ricardo defines “natural prices” as stable exchange ratios that are independent of “accidental and temporary deviations” (Ricardo, [1817] 1996, p. 109) between supply and demand. And he defines reproducible commodities as those “that may be multiplied ... almost without any assignable limit, if we are disposed to bestow the labour necessary to obtain them” (Ricardo, [1817] 1996, p. 59). Ricardo (2005a) observed that the natural prices of reproducible commodities vary with the distribution of income whereas their real costs of production, measured in labour time, do not. In consequence, labour costs cannot fully explain the structure of natural prices. This explanatory gap creates two famous problems in the classical labour theory of value: Ricardo’s problem of an invariable measure of value and Karl Marx’s transformation problem. These problems are the major reason why modern economists consider the classical labour theory of value as, at best, incomplete, or worse, logically incoherent (e.g., Seton, 1957; Samuelson, 1971; Lippi, 1979; Steedman, 1981).

Nonetheless, dissatisfaction with economic foundations based on the “shallow and superficial framework of supply and demand concepts” (Foley, 2000, p. 2) has ensured a continued interest in the classical problems. Despite significant intellectual effort, however, the classical problems remain essentially insoluble (see Howard and King, 1989, chapter 2; Howard and King, 1992, chapter 14).

For “ordinary language philosophers” (Passmore, 1978, chapter 18), such as Gilbert Ryle ([1949] 1984) and Ludwig Wittgenstein (1953), the underlying cause of a long-lived and insoluble problem is often a hidden conceptual confusion or mistake. The problem is insoluble because the conceptual framework in which the problem is stated is itself faulty. The problem must therefore be deflated or dissolved by applying "conceptual analysis" (Sloman, 1978, chapter 4).

For instance, Ryle introduced the term "category-mistake" (Ryle, [1949] 1984, chapter 1) to denote the conceptual error of expecting some concept or thing to possess properties it cannot have. For example, John Doe may be a relative, friend, enemy or stranger to Richard Roe; but he cannot be any of these things to the "Average Taxpayer". So if "John Doe continues to think of the Average Taxpayer as a fellow-citizen, he will tend to think of him as an elusive an insubstantial man, a ghost who is everywhere yet nowhere" (Ryle, [1949] 1984, p. 18).

The argument of this essay is that the contradictions of the classical labour theory of value derive from a "theoretically interesting category-mistake" (Ryle, [1949] 1984, p. 19), specifically the mistake of supposing that classical labour-values, which measure strictly *technical* costs of production, are of the same logical type as natural prices, which measure *social* costs of production, and in consequence labour-values and prices, under appropriate equilibrium conditions, are mutually consistent. Since this supposition is mistaken, Ricardo's search for an invariable measure of value and Marx's search for a transformation between labour-values and prices, attempt to discover a commensurate relationship between concepts defined by incommensurate cost accounting conventions. They therefore seek an "elusive and insubstantial man" or "ghost".

The identification of a category-mistake allows a resolution of the classical problems by "giving prominence to distinctions which our ordinary forms of language make us easily overlook" (Wittgenstein, 1953, § 132).[1] Such distinctions can then solve, or more accurately, dissolve the problems.

This chapter therefore introduces a new distinction, lacking in the classical labour theory, between a "technical" and a "total" measure of labour cost, where technical

[1] Clearly, many of the concepts employed in classical economic theory depart from their roots in ordinary language. However, any system of concepts, whether ordinary or technically specialised, may embody conceptual errors.

labour cost corresponds to the classical concept and total labour cost includes additional real costs of production incurred in virtue of non-technical, or social, conditions of production, such as production financed by a capitalist class. The more refined conceptual framework separates theoretical concerns that are conflated in the classical theory. For example, labour-values apply to distribution-independent questions about an economy, such as the productivity of labour over time or the quantity of "surplus labour" supplied by workers to capitalists (i.e., technical issues or questions in the theory of labour exploitation), whereas total labour-values apply to distribution-dependent questions, such as the relationship between nominal prices and the actual labour time required to produce commodities (i.e., issues in the theory of economic value). The classical problems dissolve by generalising the classical labour theory to apply both concepts of labour cost in the appropriate contexts. In consequence, I sketch, in an initial and incomplete manner, a new theoretical object: a more general labour theory of value with an invariable measure of value and without a transformation problem.

The structure of this chapter is as follows. The next three sections specify how the classical problems manifest in the simplest possible case – that of a capitalist economy in steady-state equilibrium. The subsequent section then introduces the concept of a "total labour cost", in contradistinction to the classical concept, by applying conceptual analysis to the concept "labour-value". The following three sections formally define total labour costs in the case of steady-state equilibrium. The final three sections explain how the new distinction dissolves the classical problems.

2.1 The definition of "labour value"

Since the seminal contribution of Ladislaus von Bortkiewicz ([1907] 1975), the transformation problem is normally defined in terms of properties of simultaneous equations.[2] I therefore begin by translating the classical concept of "labour value" into linear production theory (e.g., see Kurz and Salvadori, 1995). The formality imparts precise semantics to our key concepts, which helps identify the conceptual

[2]For examples of alternative interpretations, see Elson (1979) and Fine and Saad-Filho (2004, p. 133).

mistake.

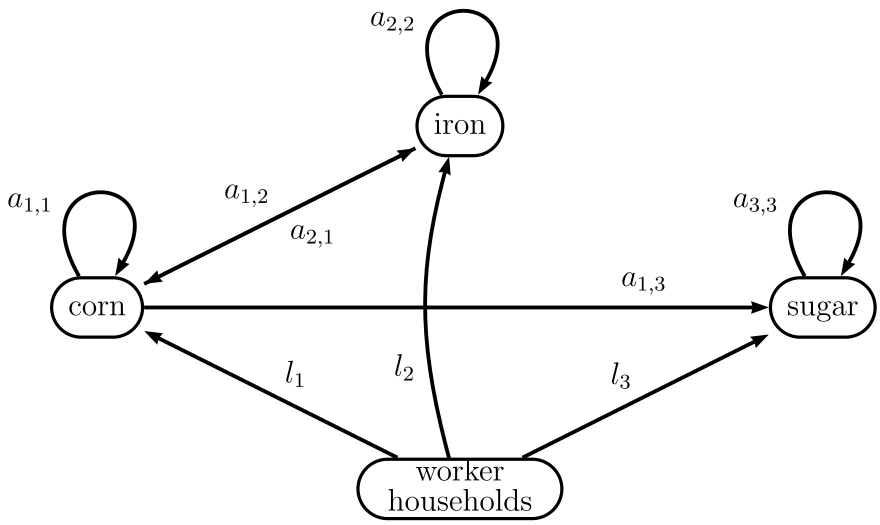
Assume $n \in \mathbb{Z}^{+}$ sectors that specialise in the production of one commodity type. The technique is a non-negative $n \times n$ input-output matrix of inter-sector coefficients, $\mathbf{A} = [a_{i,j}]$. Each $a_{i,j} \geq 0$ is the quantity of commodity i directly required to produce one unit of commodity j. Assume (i) $\mathbf{A}$ is fully connected, (ii) $\mathbf{I} - \mathbf{A}$ is of full rank and (iii) there exists a row vector[3], $\mathbf{x} \in \mathbb{R}^{n}_{+}$, such that $\mathbf{x} > \mathbf{A}\mathbf{x}$, i.e. the technique is productive. The $1 \times n$ vector, $\mathbf{l} = [l_i]$, are direct labour coefficients, where each $l_i > 0$ is the quantity of labour directly required to output 1 unit of commodity i. Figure 2.1 depicts an example three-sector technique both as a matrix and weighted directed graph.

The total "coexisting labour" (see Hodgskin, 1825; Marx, 2000, chapter 21, section 3; Perelman, 1987, chapter 5) supplied to *reproduce* commodity i is the direct labour operating in sector i *plus* the indirect labour operating in other sectors of the economy that is simultaneously supplied, in parallel, to replace *all* the commodity inputs used-up during the production of 1 unit of commodity i.

Marx, following the Ricardian socialist, Thomas Hodgskin, illustrated the concept of "coexisting labour" by contrasting it to "antecedent labour":

> '[Raw] cotton, yarn, fabric, are not only produced one after the other and from one another, but they are produced and reproduced *simultaneously*, alongside one another. What appears as the effect of antecedent labour, if one considers the production process of the individual commodity, presents itself at the same time as the effect of coexisting labour, if one considers the *reproduction process* of the commodity, that is, if one considers this production process in its continuous motion and in the entirety of its conditions, and not merely an isolated action or a limited part of it. There exists not only a cycle comprising various phases, but all the phases of the commodity are simultaneously produced in the various spheres and branches of production.' (Marx, 2000, Pt. 3, Ch. XXI)

[3] All vectors in this thesis are row vectors. The transpose operator, $\mathbf{x}^{\mathrm{T}}$, converts a row into a column vector.



$$\begin{bmatrix} \mathbf{A} \\ \mathbf{l} \end{bmatrix} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & 0 \\ 0 & 0 & a_{3,3} \\ l_1 & l_2 & l_3 \end{bmatrix}$$

Figure 2.1: A technique for an example 3-sector economy depicted as a directed graph and a matrix.

Commodities vary in their “difficulty of production” (e.g., Ricardo ([1817] 1996, p. 106)) because they require different quantities of coexisting labour for their reproduction. The classical labour theory of value is founded on this objective cost property of commodities, i.e. their “labour-value”.

To calculate a labour-value we vertically integrate over the technique (e.g., Pasinetti (1980)). For example, production of unit i uses-up direct labour l_i plus the bundle of input commodities $\mathbf{A}^{(i)}$ (i.e., column i of matrix $\mathbf{A}$). This used-up input bundle is replaced by the simultaneous expenditure of indirect labour $\mathbf{lA}^{(i)}$ operating in other sectors. But this production itself uses-up another bundle of input commodities $\mathbf{AA}^{(i)}$, which is also replaced by the simultaneous expenditure of an additional amount of indirect labour $\mathbf{lAA}^{(i)}$. To count all the coexisting labour, v_i, we continue the sum; that is,

$$\begin{aligned} v_i &= l_i + \mathbf{l}\mathbf{A}^{(i)} + \mathbf{l}\mathbf{A}\mathbf{A}^{(i)} + \mathbf{l}\mathbf{A}^2\mathbf{A}^{(i)} + \ldots \\ &= l_i + \mathbf{l}(\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \ldots)\mathbf{A}^{(i)} \\ &= l_i + \mathbf{l}(\sum_{n=0}^{\infty} \mathbf{A}^n)\mathbf{A}^{(i)}. \end{aligned} \tag{2.1}$$

This infinite sum converges since the technique is productive (see Lancaster, 1968, chapter 6). The vector of labour-values, from equation (2.1), is then

$$\mathbf{v} = \mathbf{l} + \mathbf{l}(\sum_{n=0}^{\infty} \mathbf{A}^n)\mathbf{A} = \mathbf{l}\sum_{n=0}^{\infty} \mathbf{A}^n.$$

An alternative representation of the infinite series $\sum \mathbf{A}^n$ is the Leontief inverse $(\mathbf{I} - \mathbf{A})^{-1}$. Hence, $\mathbf{v} = \mathbf{l}(\mathbf{I} - \mathbf{A})^{-1}$; that is:

Definition 1. *"Classical labour-values",* $\mathbf{v}$*, are given by*

$$\mathbf{v} = \mathbf{v}\mathbf{A} + \mathbf{l}. \tag{2.2}$$

This equation was probably first written down by Dmitriev (1868 – 1913) who translated the classical concept of 'labour embodied' into a mathematical formula (Nuti, 1974; Dmitriev, 1974). Dmitriev's formula is now standard (e.g., Sraffa (1960); Samuelson (1971); Pasinetti (1977); Steedman (1981)).

Now we've defined labour-values let us turn to two famous contradictions of the classical labour theory of value.

2.2 Ricardo's problem of an invariable measure of value

Consider a tree A that is twice the height of a tree B. At a later date tree A is three times the height of tree B. Assume we only know the *relative* change in heights. Does this change indicate that tree A has increased in size, tree B has decreased in size, or some combination of these causes? To answer this question we need an *absolute* measure of height that is *invariable* over time.

The "metre" is such an invariable standard. We measure the absolute height of tree A and B in metres, both before and after the change. Then we can unambiguously determine the cause of the variation in relative heights.

The definition and adoption of the metre by post-revolutionary France in 1793 was accompanied by much theoretical debate and reflection (Roncaglia, 2005, p. 192). Ricardo, a contemporary of these events, recognised that an objective theory of economic value requires an analogous invariable standard of measurement.

Market prices – whether stated in terms of exchange ratios between commodities or in terms of a money-commodity – cannot function as a standard because prices merely indicate relative values:

> If for example a piece of cloth is now the value of 2 ounces of gold and was formerly the value of four I cannot positively say that the cloth is only half as valuable as before, because it is possible that the gold may be twice as valuable as before. (Ricardo, 2005a, p. 289)

The cause of an altered exchange ratio might be due to an alteration in the absolute value of the standard itself. Picking a market price to measure absolute value is analogous to picking the height of a specific tree to function as an invariable standard of length. Between measurements the chosen tree might grow (or get cut down in size).

Perhaps we shouldn't try to find a standard? This is not an option because, lacking an invariable standard, the theory of value collapses into subjectivity, leaving "every one to chuse his own measure of value" (Ricardo, 2005a, p. 370). In consequence, public statements about objective value, such as "commodity A is now less valuable than one year ago", would, strictly speaking, be nonsense.

Ricardo states that if we had "possession of the knowledge of the law which regulates the exchangeable-value of commodities, we should be only one step from the discovery of the measure of absolute value" (Ricardo, 2005b, p. 315). Ricardo therefore looks beyond exchange ratios in the marketplace to seek a regulating cause that might constitute a “standard in nature” (Ricardo, 2005a, p. 381).

Ricardo claims that the natural price of a reproducible commodity is regulated by its “difficulty of production” measured in labour time (e.g., Ricardo, [1817] 1996, chapter 4). In conditions of constant “difficulty of production” market prices gravitate toward or around their natural prices due to profit-seeking behaviour, which reallocates capital to high-profit sectors and away from low-profit sectors.

Such natural prices, or “prices of production” (Marx, [1894] 1971, ch. 9), are equilibrium prices with uniform profit-rates, which we can define as,

$$\mathbf{p} = (\mathbf{pA} + \mathbf{l}w)(1 + r), \tag{2.3}$$

where $\mathbf{p}$ is a vector of prices (measured, say, in pounds sterling), w is a wage rate (pounds per hour), and r is a uniform rate of profit or percentage interest-rate on the money invested to fund the period of production. Equation (2.3) states that the production price p_i of commodity-type i has three components: (i) the cost of the input bundle, $\mathbf{pA}^{(i)}$, paid to other sectors of production, (ii) the wage costs, $l_i w$, paid to workers in sector i, and (iii) the profits, $(\mathbf{pA}^{(i)} + l_i w)r$, received by capitalists, as owners of firms in this sector, on the money-capital they advance to pay input and direct labour costs (collectively, the cost-price).

Now if “difficulty of production”, measured in units of labour, in fact regulates natural prices then, in theory, we can measure (absolute) labour-values to unambiguously determine the cause of variations in (relative) prices. We would have identified a “standard in nature” and Ricardo could “speak of the variation of other things, without embarrassing myself on every occasion with the consideration of the possible alteration in the value of the medium in which price and value are estimated” (Ricardo, [1817] 1996, p. 80).

In fact, in some special cases labour-values do vary one-to-one with natural prices. For instance, Adam Smith ([1776] 1994, p. 53) restricts the applicability of a labour theory of value to an “early and rude state of society” that precedes the “accumulation of stock”, where profits are absent and “the whole produce of labour belongs to the labourer”. In these circumstances a natural price is simply the wage bill of the total coexisting labour supplied to produce the commodity; that is,

Proposition 1. $r = 0$ *implies* $\mathbf{p} = w\mathbf{v}$.

Proof. Set $r = 0$ into price equation (2.3) to get $\mathbf{p} = \mathbf{pA} + \mathbf{l}w$ or $\mathbf{p} = w\mathbf{l}(\mathbf{I} - \mathbf{A})^{-1}$. Since $\mathbf{v} = \mathbf{l}(\mathbf{I} - \mathbf{A})^{-1}$ the conclusion follows. □

So prices are proportional to labour-values with constant of proportionality w. Hence (relative) prices vary one-to-one with (absolute) labour-values.

Ricardo notes that if the ratio of 'fixed capital' (i.e., the input bundle) to 'circulating capital' (i.e., the real wage bundle for "the support of labour") is identical in all sectors then production prices are proportional to labour-values (Ricardo, [1817] 1996, p. 31). Define $\bar{\mathbf{w}} = (1/\mathbf{lq}^{\mathrm{T}})\mathbf{w}$ as the real wage bundle consumed per unit of labour supplied, where $\mathbf{q} = [q_i]$ is the scale of production or gross product. Then Ricardo's ratio, in terms of labour-values, is

$$k = \frac{\mathbf{vA}^{(i)}}{\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}} l_i},$$

where $\mathbf{vA}^{(i)}$ is the labour-value of the input bundle and $\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}} l_i$ is the labour-value of the real wage consumed by workers in sector i. Marx would later call this ratio the technical or organic "composition of capital" (Marx, [1894] 1971, Ch. 8). A uniform organic composition of capital implies price-value proportionality; that is,

Proposition 2. $\mathbf{vA} = k\ \mathbf{v}\bar{\mathbf{w}}^{T}\mathbf{l}$ *implies* $\mathbf{p} = \alpha\mathbf{v}$*, where* $\alpha = w(1+r)/(1 - k\mathbf{v}\bar{\mathbf{w}}^{T} r)$.

Proof. Write price equation (2.3) in series form: $\mathbf{p} = (\mathbf{pA} + \mathbf{l}w)(1+r) = w(1+r)\mathbf{l}(\mathbf{I} - \mathbf{A}(1+r))^{-1} = w(1+r)\mathbf{l}\sum_{n=0}^{\infty}\mathbf{A}^n(1+r)^n$. Let $k' = k\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}}$. Given uniformity, $\mathbf{vA} = k'\mathbf{l}$, and therefore,

$$\mathbf{p} = w(1+r)\frac{1}{k'}\sum_{n=0}^{\infty}\mathbf{vA}^{n+1}(1+r)^n. \tag{2.4}$$

Given uniformity and the definition of labour-value, $\mathbf{v} = \mathbf{vA} + \mathbf{l}$, then $\mathbf{vA} = (k'/k' + 1)\mathbf{v}$. Hence $\mathbf{vA}^2 = (k'/k' + 1)\mathbf{vA} = (k'/k' + 1)^2\mathbf{v}$ and by induction, $\mathbf{vA}^n = (k'/(k' +$

$1))^n\mathbf{v}$. Substitute into price equation (2.4) to get

$$\mathbf{p} = \left(w \sum_{n=0}^{\infty} \frac{(k')^n (1+r)^{n+1}}{(1+k')^{n+1}} \right) \mathbf{v}.$$

Given that $k'r < 1$ then the infinite sum converges to $(1+r)/(1-k'r)$. $\square$

Proposition 2 confirms Ricardo's thesis. Again, in these special circumstances, production prices vary in lock-step with labour-values. Ricardo therefore claims that "the quantity of labour bestowed on a commodity ... is under many circumstances an invariable standard" (Ricardo, [1817] 1996, p. 19).

But apart from 'many' special cases there exists an infinite number of cases where production prices fail to vary one-to-one with labour-values. The reason is simple: production prices, $\mathbf{p}$, are a function of the profit-rate, r, but labour-values, $\mathbf{v}$, are not. Hence a variation in the profit-rate alters prices but leaves labour-values entirely unchanged. As Ricardo (2005a) clearly identifies: price depends on the distribution of income (i.e., how the net product is distributed in the form of wage and profit income) but "difficulty of production", a purely technical measure of direct and indirect labour costs, does not; therefore, production prices have an additional degree-of-freedom unrelated to labour-values. In general, *the relative value of a commodity varies independently of its absolute value*.

This is very perplexing since it is analogous to discovering that the relative size of two trees can change even though their absolute sizes, measured in metres, remain unaltered. Such a discovery would imply the metre is not an invariable standard of size, or one's theory of size is flawed. Ricardo's problem of an invariable standard of value arises, therefore, because his labour theory of value cannot fully account for production prices. The profit component of price appears to be unrelated to any objective labour cost. Although "the great cause of the variation of commodities is the greater or less quantity of labour that may be necessary to produce them" there is another "less powerful cause of their variation" (Ricardo, 2005a, p. 404).

Ricardo understands the necessity for an invariable standard in his theoretical framework yet simultaneously understands the conditions that prevent this necessity from being met. Faced with a contradiction he is forced to draw the negative conclusion that there cannot be an invariable standard of value.

Now let us turn to a related problem in Marx's theory of value.

2.3 Marx's transformation problem

Marx ([1867] 1954) explicitly assumes prices are proportional to labour-values in Volumes I and II of *Capital*. On this basis profit is the money representation of the unpaid or "surplus labour" of the working class. But Marx must establish the generality of this proposition in the case of (non-proportional) production prices. He tackles the issue in unfinished notes published as Volume III of *Capital* (Marx, [1894] 1971).

Marx proposes that *aggregates* of labour-values and production prices are proportional, even though individual prices and labour-values diverge, and therefore total profit remains the money representation of total surplus labour.

Let us reproduce Marx's reasoning in terms of linear production theory. Define $\mathbf{q} = [q_i]$ as the scale of production or gross product and $\mathbf{w} = [w_i]$ as the real wage. The total labour supplied is therefore $\mathbf{lq}^{\mathrm{T}}$ and bundle $\bar{\mathbf{w}} = (1/\mathbf{lq}^{\mathrm{T}})\mathbf{w}$ is the real wage consumed per unit of labour supplied.

Marx defines the "surplus-labour" in sector i as the labour supplied in excess of the labour-value of the real wage consumed, i.e. $l_i q_i - l_i q_i \mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}}$. The "rate of surplus-value", or "degree of exploitation", for sector i, is then the ratio of surplus-labour to the labour-value of the real wage. Marx assumes, for simplicity, that the degree of exploitation is uniform across sectors,

$$\theta = \frac{l_i q_i - l_i q_i \mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}}}{l_i q_i \mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}}} = \frac{1 - \mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}}}{\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}}}.$$

A high (resp. low) θ implies capitalists receive a larger (resp. smaller) share of the fruits of the labour they employ.

Now, according to Marx, only "living labour" creates profit from production. Hence the profit produced in *each sector* depends on the labour directly employed in that sector (the "variable capital") but is independent of the scale and composition of the material inputs to that sector (the "constant capital"). What, then, is the profit-rate in each sector?

Marx considers an initial situation of prices proportional to labour-values. In these circumstances a sector's profit-rate is the ratio of surplus-labour to the sum of the labour-value of constant and variable capitals,

$$r_i = \frac{(1-\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}})l_i q_i}{\mathbf{v}\mathbf{A}^{(i)}q_i + \mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}}l_i q_i} = \theta \frac{1}{(\mathbf{v}\mathbf{A}^{(i)}/\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}}l_i)+1}.$$

In consequence, the profit-rates in each sector, r_i, are only equal if the "organic compositions" of capitals, that is the ratios $\mathbf{v}\mathbf{A}^{(i)}/\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}}l_i$, are also all equal (Marx, [1867] 1954, chapter 25, section 1). But they are not equal; hence, "in the different spheres of production with the same degree of exploitation, we find considerably different rates of profit corresponding to the different organic composition of these capitals" (Marx, [1894] 1971, p. 155).

Marx notes that his initial situation is unstable: "The rates of profit prevailing in the various branches of production are originally very different" (Marx, [1894] 1971, p. 158) but, during the formation of production prices, the different rates "are equalised by competition to a single general [uniform] rate of profit" (Marx, [1894] 1971, p. 158).

Marx proposes that production prices *conservatively redistribute* the surplus-labour amongst capitalist owners (in the form of commodities purchased with profit income), at which point, "although in selling their commodities the capitalists of various spheres of production recover the value of the capital consumed in their production, they do not secure the surplus-value [i.e., surplus-labour], and consequently the profit, created in their own sphere by the production of these commodities." (Marx, [1894] 1971, p. 158). The capitalists share the available pool of surplus-labour in proportion to the size of the money-capitals they advance rather than the size of the (value-creating) workforces they employ.

Marx provides numerical examples to demonstrate the redistribution of surplus-value. He computes a uniform (labour-value) profit-rate, r_v, by dividing the aggregate surplus-labour by the aggregate labour-value of constant and variable capital,

$$r_v = \frac{(1 - \mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}})\mathbf{l}\mathbf{q}^{\mathrm{T}}}{\mathbf{v}\mathbf{A}\mathbf{q}^{\mathrm{T}} + \mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}}\mathbf{l}\mathbf{q}^{\mathrm{T}}}. \tag{2.5}$$

Marx states that the (labour-value) profit-rate, r_v, is identical to the uniform (money) profit-rate, r, which obtains once production prices have formed. He defines 'prices of production' as the *initial* cost-price of a commodity, which is proportional to labour-value, marked-up by the uniform profit-rate, r_v. Let α be the constant of proportionality. Then we can write Marx's production prices as

$$\mathbf{p}^{\star} = \alpha\left(\mathbf{v}\mathbf{A} + \mathbf{l}(\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}})\right)(1 + r_v). \tag{2.6}$$

"Hence, the price of production of a commodity is equal to its cost-price plus the profit, allotted to it in per cent, in accordance with the general rate of profit, or, in other words, to its cost-price plus the average profit [i.e., r_v]" (Marx, [1894] 1971, p. 157).

Marx's production prices $\mathbf{p}^{\star}$ are not proportional to labour-values. So "one portion of the commodities is sold above its [labour-]value in the same proportion in which the other is sold below it. And it is only the sale of the commodities at such prices that enables the rate of profit for capitals [to be uniform], regardless of their different organic composition" (Marx, [1894] 1971, p. 157).

In Marx's view production prices scramble and obscure the source of profit in surplus-labour. But the labour theory of value continues to hold in the aggregate because the "transformation" from unequal profit-rates to production prices is conservative. Nominal price changes neither create nor destroy surplus-labour, but merely redistribute it.

Marx therefore claims that three aggregate equalities are invariant over the transformation: (i) the (money) profit-rate, r, is equal to the (labour-value) profit-rate, r_v; (ii) "the sum of the profits in all spheres of production must equal the sum of the surplus-values", (Marx, [1894] 1971, p. 173); and (iii) "the sum of the prices of production of the total social product equal the sum of its [labour-]value" (Marx,

[1894] 1971, p. 173) (here Marx assumes, for simplicity, that $\alpha = 1$).

And in fact these equalities hold. Marx's 'prices of production' are computed from the assumption that money and labour-value profit-rates are equal and therefore equality (i) is true by definition. Also, Marx's prices $\mathbf{p}^{\star}$ satisfy equalities (ii) and (iii):

Proposition 3. *Marx's 'production prices', $\mathbf{p}^{\star}$, satisfy (ii) the sum of profits is proportional to surplus labour, $\alpha(\mathbf{vA}+\mathbf{l}(\mathbf{v}\bar{\mathbf{w}}^{T}))\mathbf{q}^{T}r \propto \mathbf{lq}^{T}-\mathbf{vw}^{T}$, and (iii) the price of the gross product is proportional to its labour-value, $\mathbf{p}^{\star}\mathbf{q}^{T} \propto \mathbf{vq}^{T}$.*

Proof. Marx defines $r = r_v$. From equation (2.5), $(\mathbf{vAq}^{\mathrm{T}}+\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}}\mathbf{lq}^{\mathrm{T}})r = (1-\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}})\mathbf{lq}^{\mathrm{T}} = \mathbf{lq}^{\mathrm{T}} - \mathbf{vw}^{\mathrm{T}}$ (since $\bar{\mathbf{w}} = (1/\mathbf{lq}^{\mathrm{T}})\mathbf{w}$), which establishes (ii). Multiply equation (2.6) by $\mathbf{q}$ to yield $\mathbf{p}^{\star}\mathbf{q}^{\mathrm{T}} = \alpha(\mathbf{vAq}^{\mathrm{T}} + \mathbf{vw}^{\mathrm{T}}) + \alpha(\mathbf{vAq}^{\mathrm{T}} + \mathbf{vw}^{\mathrm{T}})r_v$. Now substitute for r_v, $\mathbf{p}^{\star}\mathbf{q}^{\mathrm{T}} = \alpha(\mathbf{vAq}^{\mathrm{T}} + \mathbf{vw}^{\mathrm{T}}) + \alpha(\mathbf{lq}^{\mathrm{T}} - \mathbf{vw}^{\mathrm{T}}) = \alpha(\mathbf{vAq}^{\mathrm{T}} + \mathbf{lq}^{\mathrm{T}})$. Multiply equation (2.2) by $\mathbf{q}$ and substitute $\mathbf{vAq}^{\mathrm{T}} + \mathbf{lq}^{\mathrm{T}} = \mathbf{vq}^{\mathrm{T}}$. Hence $\mathbf{p}^{\star}\mathbf{q}^{\mathrm{T}} = \alpha\mathbf{vq}^{\mathrm{T}}$, which establishes (iii). $\square$

Hence, production-prices and labour-values, although non-proportional, are nonetheless one-to-one in the aggregate. Profit, despite appearances, is a money representation of surplus-labour.

But the first critic of the transformation is Marx himself. He immediately observes that "the cost-price of a commodity equalled the *value* of the commodities consumed in its production" (Marx, [1894] 1971, p. 165). Marx's 'prices of production', defined by equation (2.6), are calculated on the basis of *untransformed* cost-prices, $\alpha(\mathbf{vA} + \mathbf{l}(\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}}))$, which are proportional to labour-value. But since this assumption is false "there is always the possibility of an error if the cost-price of a commodity in any particular sphere is identified with the [labour-]value of the means of production consumed by it" (Marx, [1894] 1971, p. 165). As Marco Lippi (1979, p. 47) remarks, "the magnitudes on the basis of which surplus-value has been redistributed – that is, capital advanced, measured in [labour-]value – are not identical to the prices at which elements of capital are bought on the market. He therefore admits that the prices previously calculated must be adjusted". However, Marx does not pursue the adjustment but instead remarks that "our present analysis does not necessitate a closer examination of this point" (Marx, [1894] 1971, p. 165).

Once we make this adjustment then production prices are not defined by Marx's equation (2.6) but by equation (2.3). Now Marx's three aggregate equalities do not hold, except in certain special cases. The transformation problem is then the general impossibility of satisfying Marx's conservation conditions. In fact, we can deduce:

Proposition 4. *Marx's three equalities are true only if the economy satisfies the special condition,* $\mathbf{v}\big(\mathbf{I}-(\mathbf{A}+\bar{\mathbf{w}}^T\mathbf{l})(1+r)\big)\,\mathbf{q}^T=0$.

Proof. (i) If total profit is proportional to total surplus-labour then

$$(\mathbf{pA}+\mathbf{l}w)\mathbf{q}^{\mathrm{T}}r=\alpha(1-\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}})\mathbf{l}\mathbf{q}^{\mathrm{T}}, \tag{2.7}$$

where α is the constant of proportionality. (ii) If the profit-rate equals the labour-value profit-rate substitute r from (2.5) to get

$$(\mathbf{pA}+\mathbf{l}w)\mathbf{q}^{\mathrm{T}}=\alpha(\mathbf{vA}\mathbf{q}^{\mathrm{T}}+\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}}\mathbf{l}\mathbf{q}^{\mathrm{T}}). \tag{2.8}$$

(iii) If the total price of the gross product is proportional to its labour-value then $\mathbf{p}\mathbf{q}^{\mathrm{T}}=\alpha\mathbf{v}\mathbf{q}^{\mathrm{T}}$. Price equation (2.3) implies that

$$(\mathbf{pA}+\mathbf{l}w)\mathbf{q}^{\mathrm{T}}(1+r)=\alpha\mathbf{v}\mathbf{q}^{\mathrm{T}}. \tag{2.9}$$

Substitute (2.9) into (2.8) to get $\mathbf{v}\mathbf{q}^{\mathrm{T}}=(\mathbf{vA}\mathbf{q}^{\mathrm{T}}+\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}}\mathbf{l}\mathbf{q}^{\mathrm{T}})(1+r)$, which can be rearranged into the form

$$\mathbf{v}\big(\mathbf{I}-(\mathbf{A}+\bar{\mathbf{w}}^{\mathrm{T}}\mathbf{l})(1+r)\big)\,\mathbf{q}^{\mathrm{T}}=0. \tag{2.10}$$

Hence Marx's equalities (i), (ii) and (iii), with a given constant of proportionality α, imply (2.10). □

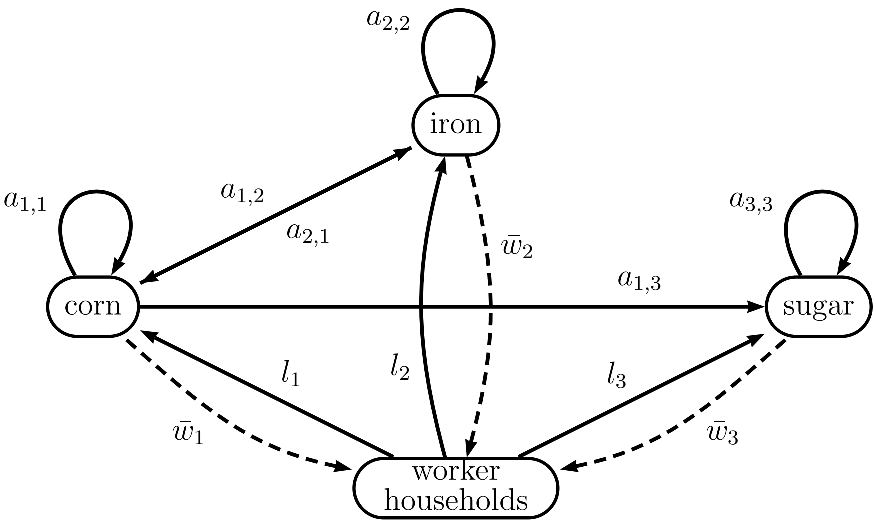
Proposition 4 specifies a macroeconomic constraint between labour-values, income distribution and the scale of production. Conditions that satisfy the constraint are zero profit, a uniform organic composition of capital, or a scale of production in certain special proportions (for further details see Abraham-Frois and Berrebi, 1997, chapter 6). But, in general, there is no economic reason why this macroeconomic constraint should hold, especially as income distribution and the scale of production vary independently of labour-values. In consequence, a conservative transformation does not exist and "there is no rigorous quantitative connection between the labour time accounts arising from embodied labour coefficients and the phenomenal world of money price accounts" (Foley, 2000, p. 17).

The transformation problem is the primary reason for the modern rejection of the logical possibility of a labour theory of value. The debate has generated a large literature spanning over one hundred years. Ian Steedman (1981) provides the definitive statement of the negative consequences for Marx's value theory. First, the theory is *internally inconsistent* because Marx "assumes that [r_v] is the rate of profit but then derives the result that prices diverge from [labour-]values, which means precisely, in general, that [r_v] is not the rate of profit" (Steedman, 1981, p. 31). Second, the theory is *redundant* because "profits and prices *cannot* be derived from the ordinary [labour-]value schema, that [r_v] is *not* the rate of profit and that total profit is *not* equal to surplus value" (Steedman, 1981, p. 48). Steedman notes, following Paul Samuelson (1971), that given a technique and a real wage (the "physical schema") one can determine (a) profits and prices and (b) labour-values. But, in general, there is "no way" of relating (a) and (b).

Despite Marx's efforts it appears that a theory of value based exclusively on labour-cost cannot account for price phenomena or the substance of capitalist profit.

2.4 Total labour costs

Having stated the major problems of the classical labour theory of value we can now turn to understanding why they exist. Clearly, prices and labour-values are incommensurate because a price depends on a profit-rate but a labour-value does not. But we need to dig deeper, and apply conceptual analysis to the concept "labour-value", to discover the fundamental reason why money costs and labour costs diverge. First, I will examine two related properties of labour-values, which are subtle and normally overlooked, in the context of an economy where capitalist profits are absent.



$$\begin{bmatrix} \mathbf{A} & \bar{\mathbf{w}}^{\mathrm{T}} \\ \mathbf{l} & 0 \end{bmatrix} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \bar{w}_1 \\ a_{2,1} & a_{2,2} & 0 & \bar{w}_2 \\ 0 & 0 & a_{3,3} & \bar{w}_3 \\ l_1 & l_2 & l_3 & 0 \end{bmatrix}$$

Figure 2.2: A social accounting matrix for an example 3-sector simple production economy depicted as a directed graph. This graph is identical to figure 2.1 apart from the addition of worker consumption $\bar{\mathbf{w}}$.

2.4.1 The independence of labour-values from the real wage

Figure 2.2 depicts an example economy where all household income takes the form of wages (cf. Marx's concept of "simple production"). There is no government or financial sector. The social accounting matrix therefore simply specifies the technique and the real wage consumed per unit of labour supplied, $\bar{\mathbf{w}}$.

Earlier, I described the computation of a labour-value as a procedure of vertical integration. If we perform this procedure in the context of a social accounting matrix we immediately notice that some input paths are ignored. Specifically, the real wage inputs to worker households, drawn as dashed arcs in figure 2.2, are not vertically integrated. So the labour supplied to produce the real wage, which maintains and reproduces the working class, is excluded as a component of the labour cost of producing commodities. Why is this coexisting labour not counted?

A labour-value is the answer to the question, "What is the total coexisting labour supplied to reproduce 1 unit of a commodity?" But it is not the answer to the question, "What is the total coexisting labour supplied to reproduce 1 unit of a commodity *and* reproduce the labour that reproduced that unit?" Measuring the cost of reproducing the very resource that serves as the measure of cost would be like measuring the height of a tree with a metre rod and including the length of the rod as part of the tree's height.

We can look at this another way. Any system of measurement defines a standard unit (e.g., the metre). We do not ask, "How many metres are in one metre?" since the measure of the standard unit is by definition a unit of the standard. In a labour theory of value the question, "What is the labour-value of one unit of direct labour?" is similarly ill-formed: the real cost of 1 hour of labour, *measured by labour time*, is 1 hour. No further reduction is possible or required. The self-identity of the measuring standard is a conceptual necessity in any system of measurement. So whether workers consume one bushel or a thousand bushels of corn to supply a unit of direct labour makes no difference to the labour-value of that unit of direct labour: an hour of labour-time is an hour of labour-time. In consequence, the procedure of vertical integration, when applied to a social accounting matrix, always terminates at labour inputs and does not further reduce labour inputs to the real wage.

For example, Marx notes that the expression 'labour-value of labour-power', where labour-power is the capacity to supply labour, denotes the "difficulty of production" of the real wage, which is the conventional level of consumption that reproduces the working class. In contrast, the expression 'labour-value of labour' is an oxymoron: "the value of labour is only an irrational expression for the value of labour-power". The expression, taken literally, is analogous to querying the colour of a logarithm (Marx, [1894] 1971) or the time on the sun (Pollock, 2004). "Labour is the substance, and the immanent measure of value, but *has itself no value*" (Marx, [1867] 1954, p. 503). In summary, labour-values, as a conceptual necessity, are independent of the scale and composition of the real income of workers.

2.4.2 Labour-values as total labour costs

Labour-values, then, exclude the reproduction costs of labour (i.e., the coexisting labour supplied to reproduce the real wage). In the context of a "simple production" economy the procedure of vertical integration therefore reduces *all* real costs (such as corn, iron and sugar) to quantities of direct labour *except* the cost of labour. Hence classical labour-values, in this context, are "total labour costs":

Definition 2. *The "total labour cost" of a commodity is (i) a measure of the coexisting labour supplied to reproduce it that (ii)* only *excludes the reproduction cost of labour.*

The classical proposition that equilibrium prices of reproducible goods are proportional to labour-values in an "early and rude state" (Smith, [1776] 1994) is not controversial. Indeed, even critics of a labour theory of value accept this (e.g., Samuelson, 1971; Steedman, 1981; Roemer, 1982). Natural prices are proportional to labour-values, that is $\mathbf{p} = w\mathbf{v}$ (see Proposition 1), because both accounting systems, that is money and labour costs, apply the same accounting convention: all commodities are reduced to a scalar measure of total cost – either total money or total labour cost. The accounting systems are dual or mutually consistent and therefore related by the price of labour, w.

Consequently, in a "simple production" economy the natural price of a commodity is the wage bill of the total coexisting labour supplied to produce it. Commodities that require more of society's labour-time to produce sell at higher prices in equilibrium.

Now let us introduce capitalist profit income and determine exactly why this simple relationship breaks down. We shall see that classical labour-values, in the context of capitalist production, no longer satisfy the definition of total labour costs.

2.5 Capitalist households

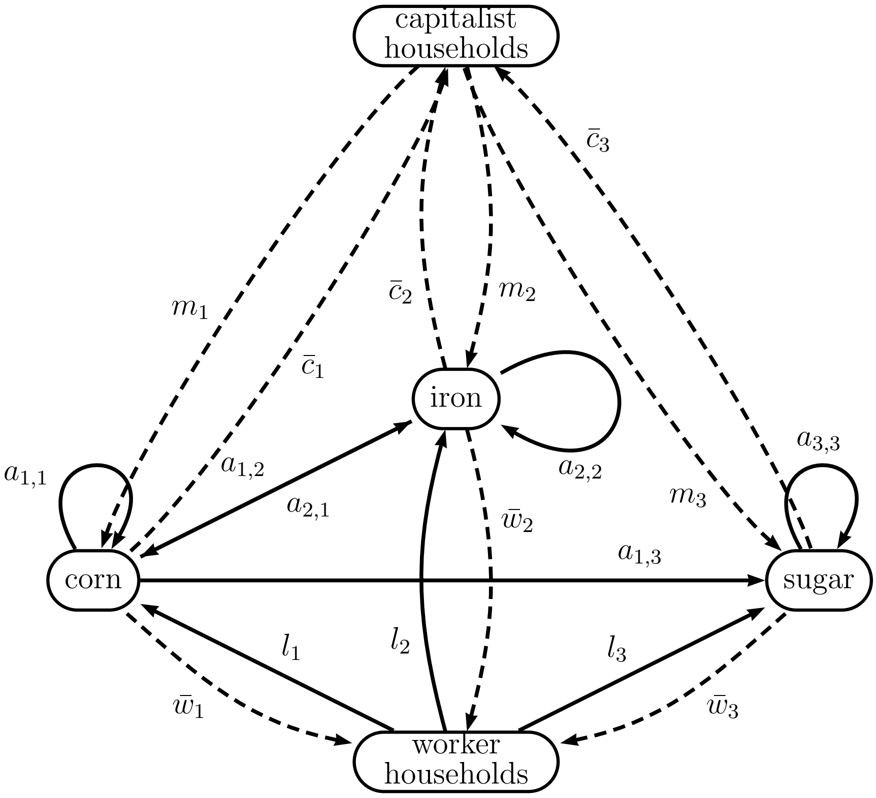
The natural prices of an economy with capitalist profit are production prices given by equation (2.3) where the profit-rate is uniform across all sectors. In this situation capitalists supply money-capital to firms to meet production costs and receive profit income proportional to their advance. This profit mark-up, or price of money-capital, r, forms a cost component of the production price.

Figure 2.3 depicts a social accounting matrix for a capitalist economy where capitalists spend all their profit income on personal consumption and therefore no capital accumulation takes place (cf. Marx's concept of simple reproduction (Marx, [1867] 1954, chapter 23)[4]). Simple reproduction is identical to our previous case of simple production apart from the addition of a capitalist household sector that funds the period of production by supplying money-capital to firms in each sector. The firms use the loan capital to purchase input goods and pay wages. The loan is repaid, with interest, at the close of the production period.[5] Capitalists purchase consumption goods with their interest income and workers purchase the real wage with their wage income. The social accounting matrix therefore also specifies the distribution of the net product in the form of the real wage and capitalist consumption.

Assume firms do not self-finance. Then the vector of cost prices, or money-capital requirement coefficients, $\mathbf{m} = [m_i]$, where $m_i = \mathbf{p}\mathbf{A}^{(i)} + l_i w$, denotes the quantity of money-capital supplied to produce unit outputs (see figure 2.3 and Vickers (1987)). A "quantity of money-capital" denotes a sum of loaned money (i.e., an outstanding principal) and the "supply of money-capital" denotes the supply of loan services, which includes loan management and actual transfers of money (at the opening of the production period). The total supply of money-capital is a function of the money-capital requirement coefficients and the scale of production, i.e. $\mathbf{m}\mathbf{q}^{\mathrm{T}}$. Note that the quantity of loaned money is not identical to the total stock of money in

[4]And also consult Trigg (2006, Ch. 3) for an analysis of the relationship between Marx's simple reproduction and Kalecki's principle that capitalists "earn what they spend".

[5]Linear production models are essentially static and therefore the 'production period' is unspecified. If we interpret the production period as 1 year then we imagine capitalist households advance their money-capital at the start of the year. At the end of the year the firms repay the loans at the annual rate of interest. In the next year the cycle starts again. Alternatively, if we interpret the production period as an infinitesimal duration then money-capital is continuously 'tied up' in production and earns a continuous income stream at the instantaneous rate of interest. Chapter 7 introduces a nonlinear dynamic model that explicitly considers time and embeds this linear production model as a special-case equilibrium point. In the dynamic model, a variable quantity of money-capital is continuously 'tied up' in production that earns a varying instantaneous rate of interest. Industrial capitalists continuously increase or decrease their borrowing requirements to fund their varying scale of output.



$$\begin{bmatrix} \mathbf{A} & \bar{\mathbf{w}}^{\mathrm{T}} & \bar{\mathbf{c}}^{\mathrm{T}} \\ \mathbf{l} & 0 & 0 \\ \mathbf{m} & 0 & 0 \end{bmatrix} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \bar{w}_1 & \bar{c}_1 \\ a_{2,1} & a_{2,2} & 0 & \bar{w}_2 & \bar{c}_2 \\ 0 & 0 & a_{3,3} & \bar{w}_3 & \bar{c}_3 \\ l_1 & l_2 & l_3 & 0 & 0 \\ m_1 & m_2 & m_3 & 0 & 0 \end{bmatrix}$$

Figure 2.3: A social accounting matrix for an example 3-sector capitalist economy depicted as a directed graph. This graph is identical to figure 2.2 apart from the addition of a capitalist household sector.

circulation since "the same mass of actual money can ... represent very different masses of money-capital" (Marx, [1894] 1971, p. 510). In other words, a given stock of money may service multiple loans.

Capitalist households receive a bundle of consumption goods $\mathbf{c}$. Figure 2.3 therefore also specifies capitalist consumption coefficients, $\bar{\mathbf{c}} = (1/\mathbf{mq}^{\mathrm{T}})\mathbf{c}$, which denote consumption per unit of money-capital supplied.[6] For example, $\bar{\mathbf{c}} = [10, 5]$ indicates that capitalists consume 10 bushels of corn and 5 kilos of sugar per £1 of money-capital supplied to production, where £1 is the unit of account. These coefficients are analogous to worker consumption coefficients, $\bar{\mathbf{w}} = (1/\mathbf{lq}^{\mathrm{T}})\mathbf{w}$, which denote worker consumption per unit of labour supplied. The economy's net product is then $\mathbf{n} = \mathbf{w} + \mathbf{c}$.

Assume, for simplicity, that the supply of money-capital does not incur direct labour costs, such as the labour of managing and servicing loans. So money-capital is not produced, like a unit of corn, but merely advanced. (Including the direct labour cost of the supply of money-capital would add a new kind of labour activity to our model, and corresponding wage income, but would not remove the fundamental difference between profit and wages: profit is received in virtue of the ownership of capital, whereas the wage is received in virtue of labour supplied.)

2.6 The divergence of technical and total labour costs

Now that we've specified a social accounting matrix for an economy with capitalist profit we can reconsider the process of vertical integration.

Production now additionally requires the supply of money-capital m_i (as shown by the dashed input edges from capitalist households to the system of production in Figure 2.3). Although the supply of money-capital, in this model, does not incur direct labour costs it does incur indirect labour costs. Capitalists do not advance money-capital for free, either nominally or in real terms. In parallel with the production of unit i, and the supply of money-capital m_i, capitalists consume commodity bundle $m_i\bar{\mathbf{c}}$. So, a quantity of coexisting labour, $m_i\mathbf{l}\bar{\mathbf{c}}^{\mathrm{T}}$, is indeed used-up during the supply of money-capital, specifically the coexisting labour that replaces the goods

[6] Note that capitalists fund their consumption with the interest from their money-capital, not the money-capital itself. Here we simply measure the rate of real consumption of capitalists relative to their supply of money-capital.

that capitalists consume.

The classical formula for labour-values (2.2) ignores this coexisting labour because the supply of money-capital to production is not part of the technique, and therefore is not included in the process of vertical integration (i.e., none of the dashed input arcs from capitalist households in Figure 2.3 are vertically integrated). Money-capital inputs are treated as an irreducible terminus. In consequence, classical labour-values do not count the labour supplied to replace capitalist consumption goods as part of the *ex ante* real costs of production.

Of course, we may measure the classical labour-value of any bundle of goods, including capitalist consumption goods. The point to notice, however, is that the classical measure ignores some actual labour supplied during the production of these goods.

Should this 'missing' labour be counted as a cost?

The classical authors exclude this labour as a real cost of production without fully recognizing the existence of a theoretical choice. They do not consider the possibility of alternative measures of labour cost.[7] However, the labour supplied to produce capitalist consumption goods is not a cost of reproducing labour and therefore necessarily excluded, as a conceptual necessity, from any definition of labour-value (as explained in section 2.4.1).

The answer depends, quite simply, on what we want to measure. And what we want to measure depends on the theoretical questions we pose and seek to answer.

For example, classical labour-values, as purely technical measures of labour costs, can answer questions about the productivity of labour over time independent of the distribution of income (see especially Flaschel, 2010, pt. 1). The reciprocal of a classical labour-value measures the quantity of the commodity produced by a unit of coexisting labour, independent of the wider institutional context in which this activity occurs.

But if we want to measure *total labour costs*, that is measure the actual labour sup-

[7] Marx, for example, excludes surplus-labour as a cost of production, on the grounds that it is an 'excess' or net product, and also, depending on the reading, classifies the labour supplied to produce capitalist consumption goods as "unproductive". Chapter 3 examines this issue in detail.

plied to reproduce commodities in the complete circumstances in which production takes place, then we cannot use classical labour-values. By definition total labour costs reduce *all* real costs to labour, except the cost of producing the real wage. But classical labour-values exclude the additional labour cost of producing capitalist consumption goods; hence, they do not measure total labour costs. This conclusion is merely a consequence of definitions.

Money-capital, in the circumstances of capitalist production, is not a technical input to production but nonetheless is an actual material prerequisite to production. Marx (1974, Ch. 18, pt. 2) describes money-capital as the "*primus motor* of every incipient business, and as its continual motor." And, in capitalist conditions, a commodity cannot be produced without capitalists supplying money-capital and workers simultaneously performing tributary or "surplus" labour to replace the commodities that capitalists consume. Classical labour-values, as a purely technical measure of labour cost, exclude this tributary labour as a real cost of production. A measure of total labour costs, by definition, must include it. Let us now do that.

2.7 Total labour costs: super-integrated labour values

Recall that coefficient $\bar{c}_i$ denotes the quantity of commodity i consumed by capitalists households per unit of money-capital supplied (e.g., 1 pound of sugar per £1 of money-capital supplied). And m_j is the money-capital supplied by capitalists to sector j per unit output of commodity j (e.g., £10 of money-capital per 1 tonne of iron). Note that the product, $\bar{c}_i m_j$ is therefore the quantity of commodity i consumed by capitalist households per unit output of commodity j (e.g., 10 pounds of sugar consumed per tonne of iron produced).

The social accounting matrix, in figure 2.3, therefore implicitly defines the $n \times n$ matrix of capitalist consumption coefficients,

$$\mathbf{C} = \bar{\mathbf{c}}^{\mathrm{T}} \mathbf{m} = [c_{i,j}],$$

where each $c_{i,j} = \bar{c}_i m_j$ is the quantity of commodity i consumed by capitalist households per unit output of commodity j. Matrix $\mathbf{C}$, in consequence, is a "capitalist consumption matrix" that specifies how the production of new commodities is synchronised with the consumption of existing commodities by capitalist households. Matrix $\mathbf{C}$ encapsulates the real costs of supplying money-capital to the different sectors of the economy. Note that matrix $\mathbf{C}$ is a physical input-output matrix that specifies relative material flows of commodities; for example, each element $c_{i,j}$ of $\mathbf{C}$ is measured in units identical to the corresponding element $a_{i,j}$ of the technique $\mathbf{A}$. We can therefore define the technique augmented by capitalist consumption as

$$\tilde{\mathbf{A}} = \mathbf{A} + \mathbf{C} = [\tilde{a}_{i,j}],$$

where each $\tilde{a}_{i,j} = a_{i,j} + c_{i,j}$ is the quantity of commodity i, including that consumed by capitalists, directly used-up per unit output of j.

We may now compute the total labour costs for this capitalist economy by vertically integrating over the technique augmented by capitalist consumption: Production of commodity i uses-up direct labour l_i and the bundle of input commodities $\mathbf{A}^{(i)} + m_i \bar{\mathbf{c}}^{\mathrm{T}} = \mathbf{A}^{(i)} + \mathbf{C}^{(i)}$, consisting of means of production and capitalist consumption goods. This bundle is replaced by the simultaneous expenditure of labour $\mathbf{l}(\mathbf{A}^{(i)} + \mathbf{C}^{(i)})$ operating in parallel, which itself uses-up input bundle $\tilde{\mathbf{A}}(\mathbf{A}^{(i)} + \mathbf{C}^{(i)})$. To count all the coexisting labour we continue the sum; that is,

$$\begin{aligned}
\tilde{v}_i &= l_i + \mathbf{l}(\mathbf{A}^{(i)} + \mathbf{C}^{(i)}) + \mathbf{l}\tilde{\mathbf{A}}(\mathbf{A}^{(i)} + \mathbf{C}^{(i)}) + \mathbf{l}\tilde{\mathbf{A}}^2(\mathbf{A}^{(i)} + \mathbf{C}^{(i)}) + \ldots \\
&= l_i + \mathbf{l}(\mathbf{I} + \tilde{\mathbf{A}} + \tilde{\mathbf{A}}^2 + \ldots)(\mathbf{A}^{(i)} + \mathbf{C}^{(i)}) \\
&= l_i + \mathbf{l}(\sum_{n=0}^{\infty} \tilde{\mathbf{A}}^n)(\mathbf{A}^{(i)} + \mathbf{C}^{(i)}).
\end{aligned}$$

The vector $\tilde{\mathbf{v}}$ of total coexisting labour supplied to reproduce a unit bundle $\mathbf{u} = [1]$ of commodities is

$$\tilde{\mathbf{v}} = \mathbf{l} + \mathbf{l}(\sum_{n=0}^{\infty} \tilde{\mathbf{A}}^n)(\mathbf{A} + \mathbf{C}) = \mathbf{l} \sum_{n=0}^{\infty} \tilde{\mathbf{A}}^n.$$

Rewrite the infinite series, such that $\tilde{\mathbf{v}} = \mathbf{l}(\mathbf{I} - \tilde{\mathbf{A}})^{-1}$; and therefore:

Definition 3. *"Super-integrated labour-values", $\tilde{\mathbf{v}}$, are*

$$\tilde{\mathbf{v}} = \tilde{\mathbf{v}}\tilde{\mathbf{A}} + \mathbf{l}, \tag{2.11}$$

where $\tilde{\mathbf{A}} = \mathbf{A} + \mathbf{C}$ is the technique augmented by capitalist consumption.

A super-integrated labour-value is a new measure of labour cost constructed by vertically integrating both the technique and the real cost of capitalist consumption, which satisfies the definition of a total labour cost in the context of simple reproduction.

Let us draw some contrasts between classical and super-integrated labour-values. The classical formula, $\mathbf{v} = \mathbf{vA} + \mathbf{l}$, is a property of the technique and measures technical labour costs. In contrast, the super-integrated formula, $\tilde{\mathbf{v}} = \tilde{\mathbf{v}}\tilde{\mathbf{A}} + \mathbf{l}$, is a property of the social accounting matrix[8], including the distribution of real income, and measures total labour costs.

Classical labour-values are the sum of direct labour, $\mathbf{l}$, plus indirect labour, $\mathbf{vA}$. Super-integrated labour-values are the sum of direct labour, $\mathbf{l}$, and indirect labour, $\tilde{\mathbf{v}}\mathbf{A}$, plus the 'super-indirect' labour, $\tilde{\mathbf{v}}\mathbf{C}$, which is tributary labour devoted to the production of capitalist consumption goods. In general, $\tilde{\mathbf{v}} > \mathbf{v}$. But in the absence of "profits on stock" the super-integrated labour-values reduce to classical labour-values.

Classical labour-values count all household consumption (whether workers or capitalists) as net output and therefore not a cost of production; in contrast, super-integrated labour-values count capitalist consumption as a real cost of production. Both schemes, of course, assign an *ex post* labour-value to the real income of capitalists, since this bundle of goods requires labour resources to produce it. However, in the classical scheme, the direct labour supplied to produce capitalist consumption is "surplus labour", i.e. supplied "gratis", and therefore, by definition, does not constitute an *ex ante* cost of production (e.g., see Marx, [1867] 1954, chapter 18; Marx,

[8] Specified in figure 2.3.

[1894] 1971, part V, chapter 32).

The definition of super-integrated labour-values does not provide or rely upon any theory of income distribution or profit and is independent of the possible reasons why workers and capitalists consume specific consumption bundles. However, in order to calculate super-integrated labour-values the distribution of real income must be given, in much the same manner that, in order to calculate production prices, the distribution of nominal income must be given.

Both classical and super-integrated labour-values are functions of real or 'physical' data alone, which we can operationalise without reference to monetary phenomena, and constitute entirely self-consistent labour-cost accounting schemes.[9] They measure different aspects of the same economy by applying different cost-accounting conventions to the analysis of the labour process. As we shall see, we need both measures to answer the full range of questions posed by a labour theory of value.

2.8 The category-mistake: conflating technical and total labour costs

Now that we've distinguished between technical and total labour costs we can understand the fundamental reason why money and labour costs diverge.

Money-capital has a price, the profit-rate, which is a 'mark up' component of the money cost of a commodity. Money-capital also has a real cost, which, in the case of simple reproduction, is capitalist consumption. Production prices, as total money costs, include the profit-rate as a money cost of production, and therefore prices depend on the distribution of nominal income. But classical labour-values, as technical labour costs, exclude the labour cost of money-capital as a real cost of production, and therefore labour-values are independent of the distribution of real income. In summary, the dual accounting systems apply different cost conventions and, in consequence, there cannot be a one-to-one relationship between prices and labour-values: in the classical framework the profit-rate component of money costs

[9] We can compute super-integrated labour-values given the technique, real wage, and the total labour supplied to production; see Appendix 9.1 for details.

refers to labour costs that are not counted.

The asymmetrical treatment of the commodity money-capital – present as a money cost in the price system but absent as a real cost in the labour-value system – is the fundamental reason for the divergence of money and labour costs. A quantitative mismatch necessarily arises if *total* money costs are compared to *partial* labour costs.

The classical contradictions of the labour theory of value are the manifestation of the category-mistake of supposing that technical costs are of the same logical type as total costs. Hence Ricardo's search for an invariable measure and Marx's transformation are theoretical attempts to find Ryle's "elusive and insubstantial man" or "ghost".

The classical category-mistake has been, and continues to be, the major obstacle toward a deeper understanding of the relationship between social labour and monetary phenomena. For example, it has directed theoretical attention toward the contradictions and away from the existence of a simple one-to-one quantitative relation between production prices and labour costs.

Definition 4. *A "steady-state economy" produces quantities,* $\mathbf{q} = \mathbf{q}\mathbf{A}^T + \mathbf{w} + \mathbf{c}$*, at prices,* $\mathbf{p} = (\mathbf{p}\mathbf{A} + \mathbf{l}w)(1+r)$*, where workers and capitalists spend what they earn,* $\mathbf{p}\mathbf{w}^T = \mathbf{l}\mathbf{q}^T w$ *and* $\mathbf{p}\mathbf{c}^T = (\mathbf{p}\mathbf{A} + \mathbf{l}w)\mathbf{q}^T r$*.*

Theorem 1. *The production-prices of a steady-state economy are proportional to super-integrated labour-values,* $\mathbf{p} = \tilde{\mathbf{v}}w$*.*

Proof. In a steady-state economy, $\mathbf{p}\mathbf{c}^{\mathrm{T}} = (\mathbf{p}\mathbf{A}\mathbf{q}^{\mathrm{T}} + \mathbf{l}\mathbf{q}^{\mathrm{T}}w)r$. Recall that cost prices $\mathbf{m} = \mathbf{p}\mathbf{A} + \mathbf{l}w$. Hence $r = \mathbf{p}\mathbf{c}^{\mathrm{T}}/\mathbf{m}\mathbf{q}^{\mathrm{T}} = \mathbf{p}\bar{\mathbf{c}}^{\mathrm{T}}$. Substitute $r = \mathbf{p}\bar{\mathbf{c}}^{\mathrm{T}}$ into price equation (2.3) to get $\mathbf{p} = (\mathbf{p}\mathbf{A} + \mathbf{l}w) + (\mathbf{p}\mathbf{A} + \mathbf{l}w)\mathbf{p}\bar{\mathbf{c}}^{\mathrm{T}} = (\mathbf{p}\mathbf{A} + \mathbf{l}w) + \mathbf{m}\mathbf{p}\bar{\mathbf{c}}^{\mathrm{T}} = \mathbf{p}\mathbf{A} + \mathbf{p}\bar{\mathbf{c}}^{\mathrm{T}}\mathbf{m} + \mathbf{l}w = \mathbf{p}(\mathbf{A} + \bar{\mathbf{c}}^{\mathrm{T}}\mathbf{m}) + \mathbf{l}w = \mathbf{p}\tilde{\mathbf{A}} + \mathbf{l}w$. Hence $\mathbf{p} = \mathbf{l}(\mathbf{I} - \tilde{\mathbf{A}})^{-1}w = \tilde{\mathbf{v}}w$, by equation (2.11). □

In consequence, in a steady-state economy, the production-price of a commodity is the wage bill of the total coexisting labour supplied to reproduce it. Commodities that require more labour time to produce sell at proportionally higher prices in equilibrium. Natural prices – whether in an "early and rude state" or in our late and civilised times – vary one-to-one with total labour costs.[10]

How general is this proposition? The definition of total labour cost applies to any social accounting matrix. Hence, in more complex models, total labour costs include additional real costs of production, over and above capitalist consumption. For example, in Chapter 5 we shall see that total labour-values, in circumstances of expanded reproduction with proportionate or non-proportionate growth, are "vertically super-integrated labour coefficients" that additionally include the labour cost of supplying the net investment goods required to expand the scale of production. The natural prices of growing economies are therefore also proportional to total labour costs.

Many possible generalisations remain unexplored, however. For example, the robustness of such equivalence theorems have yet to be tested in the context of (i) more complex social accounting matrices, which include capitalist savings, a public sector, credit money etc., (ii) production with fixed capital, and (iii) systems of joint production. In Chapter 7 I investigate a nonlinear dynamic model of classical macrodynamics where market prices gravitate to natural prices proportional to the super-integrated labour-values.

Now we've identified the category-mistake, and introduced a distinction between classical and total labour costs, we can finally return to the classical problems.

[10]Theorem 1 supports David Laibman's proposal that labour-values, in the institutional conditions of capitalism, are the labour quantities implicitly defined by natural prices (Laibman, 2002). Laibman argues that "*if* there is a substratum of labour value lying behind the money exchange values [of natural prices] on the surface, there *must* be a scalar coefficient linking them". Laibman then adopts this assumption and derives a non-classical definition of labour-value that happens to be proportional to the super-integrated labour coefficients. The argument of this chapter is substantially in the same spirit as Laibman's proposal, except I do not asssume there is such a scalar; instead I first identify the actual concrete labour supplied, in the different sectors of the economy, which is ommitted by the classical measure of labour cost.

2.9 Dissolution of the problem of an invariable measure of value

Ricardo conflates two concepts of "difficulty of production" that we can now distinguish.

Classical labour-values, $\mathbf{v}$, measure "difficulty of production" independent of an economy's institutional structure and distributive rules. A classical labour-value, v_i, is therefore a *counterfactual* measure of the total coexisting labour that would be supplied to reproduce commodity-type i if workers did not perform tributary labour during the production of commodities.

Super-integrated labour-values, $\tilde{\mathbf{v}}$, measure "difficulty of production" dependent on an economy's institutional structure and distributive rules. A super-integrated labour-value, $\tilde{v}_i$, is therefore an *actual* measure of the total coexisting labour supplied to reproduce commodity-type i given that workers perform additional tributary labour during the production of commodities.

Ricardo wished to reduce the structure of natural prices (relative value) to "difficulty of production" (absolute value) measured in terms of some real cost basis, such as labour costs. Classical labour-values are an invariable measure of absolute value independent of the distribution of income and therefore we can use them to say, without "embarrassment" or equivocation, that "commodity A is now less valuable than one year ago" in the strictly technical sense that commodity A requires less labour resources to reproduce than it once did. But it is a category-mistake to hope or expect, as Ricardo did, that this standard can also explain the structure of natural prices.

Super-integrated labour-values, in contrast, explain the structure of natural prices in terms of objective quantities of coexisting labour supplied to produce commodities (Theorem 1). Hence they provide that all-important one-to-one relation, required by a labour theory of value, between absolute values, measured in terms of labour time, and relative prices.

The point is this: classical labour-values answer distribution-independent questions about the technical "difficulty of production" of commodities, whereas super-integrated labour-values answer distribution-dependent questions about the actual "difficulty of production" of commodities. In consequence – and on condition we apply the appropriate concept of "difficulty of production" in each case – we can justifiably make public statements about changes in objective value, independent of the distribution of income *and* simultaneously claim that relative values covary with absolute values, and thereby explain the structure of natural prices in terms of labour costs. Ricardo's belief in another "less powerful cause" of the variation of relative values, which is unrelated to labour costs, is caused by the category-mistake. Ricardo's problem therefore dissolves.

2.10 Dissolution of the transformation problem

Marx employs classical labour-values to address issues in the theory of exploitation (e.g., how many hours do workers supply in excess of the time required to produce their real wage?) and, in addition, issues in the theory of economic value (e.g., what does the nominal unit of account, such as £1, "express" or measure? what is the "substance" of profit? etc.) The distinction between classical and total labour-values permits us to separate these concerns and therefore avoid the transformation problem while preserving Marx's analysis of the capitalist labour process.

Let $\mathbf{n} = \mathbf{w} + \mathbf{c}$ be the net product of the economy, where $\mathbf{c}$ is the consumption bundle of capitalists. The total working day equals the classical labour-value of the net product, $\mathbf{l}\mathbf{q}^{\mathrm{T}} = \mathbf{v}\mathbf{n}^{\mathrm{T}}$:

Proposition 5. *The total labour supplied equals the classical labour-value of the net product,* $\mathbf{l}\mathbf{q}^{T} = \mathbf{v}\mathbf{n}^{T}$.

Proof. Since $\mathbf{q} = \mathbf{q}\mathbf{A}^{\mathrm{T}} + \mathbf{n}^{\mathrm{T}}$ it follows that

$$\mathbf{v}(\mathbf{I} - \mathbf{A})\mathbf{q}^{\mathrm{T}} = \mathbf{v}\mathbf{n}^{\mathrm{T}}. \tag{2.12}$$

But $\mathbf{v} = \mathbf{l}(\mathbf{I} - \mathbf{A})^{-1}$. Replace $\mathbf{v}$ on the LHS of (2.12) to get $\mathbf{l}\mathbf{q}^{\mathrm{T}} = \mathbf{v}\mathbf{n}^{\mathrm{T}}$. □

Marx splits the working day into necessary labour, $\mathbf{v}\mathbf{w}^{\mathrm{T}}$, which is the part 'technically necessary' to reproduce workers, and surplus labour, $\mathbf{v}\mathbf{n}^{\mathrm{T}}-\mathbf{v}\mathbf{w}^{\mathrm{T}}$ $(=\mathbf{v}\mathbf{c}^{\mathrm{T}})$, which is an additional part appropriated by capitalists. Marx's normative point, among other things, is that production could occur without the performance of this surplus labour, and yet workers could continue to consume the same real wage.

Super-integrated labour-values, by definition, include surplus labour as a cost of production. In consequence, they do not split the working day into necessary and surplus parts. In terms of total labour costs the whole working day, $\mathbf{l}\mathbf{q}^{\mathrm{T}}=\tilde{\mathbf{v}}\mathbf{w}^{\mathrm{T}}$, is 'socially necessary' to reproduce workers given that the real wage cannot be produced without the simultaneous performance of surplus labour for capitalists:

Proposition 6. *The total labour supplied equals the super-integrated labour-value of the real wage,* $\mathbf{l}\mathbf{q}^{T}=\tilde{\mathbf{v}}\mathbf{w}^{T}$.

Proof. In a steady-state economy, $\mathbf{l}\mathbf{q}^{\mathrm{T}}w=\mathbf{p}\mathbf{w}^{\mathrm{T}}$. Use Theorem 1 to substitute for $\mathbf{p}$ and the conclusion follows. □

We can therefore restate Marx's concept of "surplus labour" in terms of super-integrated and classical labour-values. Surplus labour is the difference between (i) the labour time socially necessary and (ii) the labour time technically necessary to reproduce workers, i.e. $\tilde{\mathbf{v}}\mathbf{w}^{\mathrm{T}}-\mathbf{v}\mathbf{w}^{\mathrm{T}}$ (since $\tilde{\mathbf{v}}\mathbf{w}^{\mathrm{T}}=\mathbf{l}\mathbf{q}^{\mathrm{T}}=\mathbf{v}\mathbf{n}^{\mathrm{T}}$).

Splitting the working day this way is both logical and illuminating, regardless of any relationship it may have to the price system, since it provides the quantitative basis for a normative critique of capitalist production. But it is a category-mistake to hope or expect, as Marx did, that a technical, and therefore partial, measure of surplus labour has a one-to-one relation with a total measure of money profit. Money profit, in fact, has a one-to-one relation with total surplus labour, $\tilde{\mathbf{v}}\mathbf{n}^{\mathrm{T}}-\tilde{\mathbf{v}}\mathbf{w}^{\mathrm{T}}$, not Marx's surplus labour, $\mathbf{v}\mathbf{n}^{\mathrm{T}}-\mathbf{v}\mathbf{w}^{\mathrm{T}}$:

Proposition 7. *Money profit,* $(\mathbf{p}\mathbf{A}+\mathbf{l}w)\mathbf{q}^{T}r$*, is proportional to total surplus labour,* $\tilde{\mathbf{v}}\mathbf{n}^{T}-\tilde{\mathbf{v}}\mathbf{w}^{T}$.

Proof. In a steady-state economy, $\mathbf{p}\mathbf{c}^{\mathrm{T}}=(\mathbf{p}\mathbf{A}+\mathbf{l}w)\mathbf{q}^{\mathrm{T}}r$. Hence we need to demonstrate $\mathbf{p}\mathbf{c}^{\mathrm{T}}\propto\tilde{\mathbf{v}}\mathbf{n}^{\mathrm{T}}-\tilde{\mathbf{v}}\mathbf{w}^{\mathrm{T}}$. Theorem 1 implies $\mathbf{p}\mathbf{c}^{\mathrm{T}}=\tilde{\mathbf{v}}\mathbf{c}^{\mathrm{T}}w$. And $\tilde{\mathbf{v}}\mathbf{c}^{\mathrm{T}}w=(\tilde{\mathbf{v}}\mathbf{n}^{\mathrm{T}}-\tilde{\mathbf{v}}\mathbf{w}^{\mathrm{T}})w$

by the definition of **n**. □

In the context of the transformation problem, the Marxist tradition in general has accepted divergence of production prices from labour-values but defended conservation of labour-value in price, whereas critics have also accepted divergence but denied conservation of labour-value in price. But both sides of the argument are mistaken: once we measure in terms of total labour costs there is no divergence and there is aggregate conservation. Production prices represent total labour costs, i.e. super-integrated labour-values, and therefore capitalist profit is a money representation of labour time.

Corollary 1. *All Marx's equalities obtain when labour-values measure total labour costs, specifically (i) the profit-rate equals the labour-value profit-rate, (ii) total profit is proportional to surplus labour, and (iii) total production price is proportional to total labour-value.*

Proof. This is a trivial consequence of Theorem 1, i.e. the proportionality of production prices and total labour costs. □

In consequence, the standard criticisms of the classical labour theory of value do not apply: super-integrated labour-values are not internally inconsistent, since the money profit-rate equals the labour-value profit-rate, nor redundant, since production prices can be derived from labour-values by scaling by the money wage w. Hence a theory of value based exclusively on labour cost can account for price phenomena: total labour costs and prices are "two sides of the same coin". The transformation problem therefore dissolves.

This conclusion, it should be emphasised, undermines the basis for the claim that a labour theory of value must be logically incoherent because prices and classical labour-values are quantitatively incommensurable in linear production models (e.g., Samuelson, 1971; Lippi, 1979; Steedman, 1981).

2.11 Conclusion

The classical labour theory of value commits the category-mistake of supposing that classical labour-values, which measure strictly technical or material costs of production, are of the same logical type as natural prices, which measure non-technical or social costs of production, and therefore labour-values and prices, under appropriate equilibrium conditions, are mutually consistent. This category-mistake is the cause of Ricardo's problem of an invariable measure of value and Marx's transformation problem.

This essay has drawn a new distinction, lacking in the classical theory, between a "technical" and a "total" measure of labour cost, where a total labour cost includes additional real costs incurred in virtue of non-technical conditions of production, such as production financed by money-capital. Classical labour-values, in this more refined conceptual framework, apply to distribution-independent questions about an economy, such as the productivity of labour or measuring the surplus-labour supplied by workers; whereas total labour-values apply to distribution-dependent questions, such as the relationship between nominal prices and the actual labour time required to produce commodities (i.e., issues in the theory of economic value). The classical problems dissolve by generalising the classical labour theory to apply both concepts in the appropriate contexts.

The category-mistake has misdirected theoretical attention toward the contradictions and away from the fact that a commodity's natural price is the wage bill of the total coexisting labour supplied to produce it (Theorem 1). By ridding ourselves of longstanding conceptual confusions we discover the logical possibility of a new theoretical object: a more general labour theory of value with an invariable measure of value and without a transformation problem.

In the next chapter we examine how the category-mistake further manifests in Marx's critical analysis of capitalist production, and also take the opportunity to more closely examine the relationship between money-capital and Marx's theory of value.

Chapter 3

Marx's irrational commodity

The philosophical foundation of Karl Marx's "critique of political economy"[1] is Hegelian dialectics, which is "in its essence critical and revolutionary" (Marx, [1867] 1954, p. 29). According to Friedrich Engels, Marx's lifetime collaborator, the Hegelian dialectic implies that "all that is real in the sphere of human history, becomes irrational in the process of time, is therefore irrational by its very destination, is tainted beforehand with irrationality"; in consequence, "all that exists deserves to perish" (Engels, 1976, Pt. 1).

Marx, as a follower of Hegel, is committed to an ontology that admits irrational kinds that are essentially contradictory. A logical contradiction denotes an impossibility (e.g., a square circle); in contrast, a real, or dialectical, contradiction denotes the struggle of parts of a system to control a property of the system in incompatible ways (e.g., two teams in a game of tug-of-war that attempt to pull the rope in opposite directions; see also Marx's discussion of elliptical motion (Marx, [1867] 1954, Ch. 3, Sec. 2)). Real contradictions are the cause of change and motion. A system with real contradictions is therefore logically possible but may ultimately be unstable and therefore transient on some time-scale.

Marx, in his magnum opus, *Capital*, applies this "revolutionary method of thinking" (Engels, 1976, Pt. 1) to demonstrate that capitalism "deserves to perish". He identifies real contradictions of capitalist production – such as perpetual class conflict between workers and capitalists over the production and distribution of surplus-labour (Marx, [1867] 1954, Pt. 3–4) – and nascent social forms – such as worker

[1] The subtitle of Capital, Volume 1.

co-operatives and socialised property – that together imply that capitalism, on a historical time scale, is in the "process of becoming" something else, specifically a higher mode of production that transcends the contradictions (e.g., Jossa (2005)). Capitalism, therefore, is an "irrational" social system from the perspective of real possibilities immanent within capitalism itself.

All tools have their strengths and weaknesses. Hegelian dialectics, in Marx's hands, shaped the greatest critique of capitalism ever written. However, the "revolutionary method of thinking" can suffer from its own biases: since it expects to discover irrational properties in its object of analysis it is prone to misidentifying subjective irrationalities, that is mistakes in thinking, as objective irrationalities. One can be too eager to find the irrational in the real.

Hegel explains that identifying the logical contradictions within a theory is an essential part of dialectical analysis:

> "*Intelligent* reflection, to mention this here, consists, on the contrary, in grasping and asserting contradiction. Even though it does not express the Notion of things and their relationships and has for its material and content only the determinations of ordinary thinking, it does bring these into a relation that contains their contradiction and allows *their Notion to show or shine through* the contradiction. Thinking reason, however, sharpens, so to say, the blunt difference of diverse terms, the mere manifoldness of pictorial thinking, into *essential* difference, into *opposition*. Only when the manifold terms have been driven to the point of contradiction do they become active and lively towards one another, receiving in contradiction the negativity which is the indwelling pulsation of self-movement and spontaneous activity" (Hegel, 1969, p. 442).

The language may initially seem opaque but Hegel's methodological remarks here are highly sophisticated. Often, to make theoretical progress, we need to compress the "manifold terms" of a complex theory into an essential *logical* contradiction at the level of "ordinary thinking". The contradiction may then reveal a glimpse of an underlying process of change that the theory fails to adequately reflect.

In this paper I critically analyse Marx's theory of money-capital, which is a specific example of Marx identifying an irrational kind. This serves as an entry point to a deeper analysis that aims to identify the fundamental logical contradiction of Marx's political economy.

The structure of this chapter is as follows. First, I establish Marx's problematic of money-capital, which is his intriguing but *prima facie* illogical proposition that money-capital simultaneously is, and is not, a commodity. According to Marx, money-capital has the form of a commodity (i.e., an exchange-value) but lacks the substance of a commodity (i.e., a real cost of production measured in labour time). Second, I investigate whether the irrationality of money-capital is caused by a real contradiction, which expresses the inherently contradictory nature of capitalism, or alternatively by a logical contradiction, which expresses a contradiction in Marx's cognition of capitalism. I argue the latter, specifically that Marx's theory of money-capital is the surface manifestation of a fundamental logical contradiction, which is Marx's attempt to explain the cost structure engendered by capitalist property relations in terms of a counterfactual cost structure defined by the absence of those property relations. Third, I then explain how a more general labour theory of value, which admits and distinguishes between counterfactual and factual accounts of the labour process in capitalism, overcomes the contradiction. I argue that a logically consistent application of the "revolutionary method of thinking" implies that money-capital is a rational commodity, with both form and substance, that is, nonetheless, the product of irrational social relations that deserve to perish. I conclude by identifying the underlying process of change that Marx's theory fails to adequately reflect.

3.1 What is money-capital?

In Marx's theory "capital" is "self-expanding value" with multiple "forms of existence" (e.g., Marx (1974, Ch. 4)). Capital, when "in the state or form of money" (Marx, 1974, Pt. 1, Ch. 1), is money-capital. Money functions as capital, as "self-expanding value", when it participates in the social practice that Marx calls the "circuit of money-capital". In this social practice, the participants use money not merely as a means of exchange but also as a principal sum that is loaned to production for

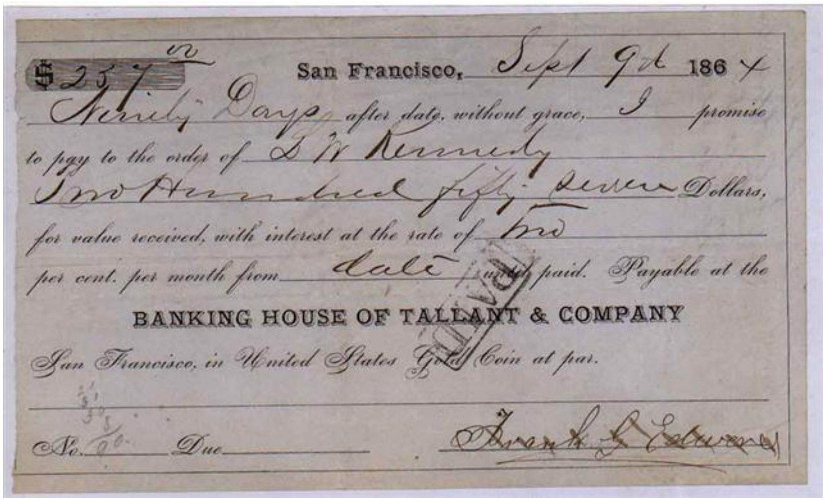


Figure 3.1: A material record in the circuit of money-capital: a promissory note issued in 1864. The borrower promises to repay $257 plus interest at 2% per month after 90 days.

a period of time in exchange for an interest payment.

The circuit of money-capital, especially in the context of a developed market for money-capital, engenders specific social roles in the division of labour, such as the distinction between "money-capitalists", who lend their money-capital at interest to finance production, and industrial capitalists who, as owners and managers of firms, borrow money-capital to finance their production plans with the expectation of earning profit-of-enterprise[2], which is a return in excess of the cost of borrowing (see especially Marx ([1894] 1971, Part V)). The circuit also features specific activities, such as the work of selling, arranging and servicing loans, and additional material forms, over-and-above money, such as loan contracts, loan accounts and promissory notes, for instance notes issued by a lender to a borrower (see figure 3.1).

Interest and profit-of-enterprise derive from different kinds of property claims. A finance capitalist owns stocks of outstanding loans and therefore maintains a property claim on all principals plus interest (plus any loan collateral). A claim terminates upon repayment of the loan. In contrast, the industrial capitalist, as owner of

[2]Marx ([1894] 1971, Ch. 36) writes, "when a man without fortune receives credit in his capacity of industrialist or merchant, it occurs with the expectation that he will function as capitalist and appropriate unpaid labour with the borrowed capital".

the firm, has a residual claim on the firm's net income, and therefore is liable for both profit and loss after all costs are deducted from revenue, including the cost of borrowing money. This claim terminates when ownership is transferred or the firm dissolves.

According to Marx the interest rate, or price of money-capital, is set in the market for loanable funds (Marx, [1894] 1971, Ch. 22). In commercial practice the interest rate is "assumed to be given beforehand, before the process of production begins, hence before its result, the gross profit, is achieved" (Marx, [1894] 1971, Ch. 23). The interest rate, or price of money-capital, is an *ex ante* cost of production whereas profit (or loss) is an *ex post* residual.

At an abstract stage of analysis, for example in the absence of the private ownership of land, then total profit breaks down into two different kinds of profit income: interest and profit-of-enterprise. The industrialist deducts the interest due on the borrowed money-capital from their total profit and distributes it to the money-capitalist. Interest, then, is a deduction from the total profit (Marx, [1894] 1971, Ch. 22).

Now that we've summarised Marx's description of money-capital as a social phenomenon we can turn to his theory of money-capital. To understand that theory we first need to present some key elements of Marx's general theoretical framework, specifically his theory of economic value, and his theory of surplus-value.

3.2 Exchange-value regulated by labour time

The starting point of Marx's theory of economic value is the production of reproducible commodities. Marx presents the primary features of commodities in the first three sections of Volume 1 of *Capital*. A commodity is a *use-value*, i.e. a thing of utility, because it "satisfies human wants of some sort or another" (Marx, [1867] 1954, Ch. 1). A commodity has an *exchange-value*, which is its rate of exchange with other commodities in the market; e.g., "a quarter of wheat is exchanged for x blackling, y silk, or z gold, etc." (Marx, [1867] 1954, Ch. 1). And a commodity has a *labour-value* that is the average quantity of vertically-integrated labour (or "coexisting labour" (Hodgskin, 1825); (Marx, 2000, Ch. 21, Sec. 3)) supplied to produce 1 unit and replace the means of production used-up (what Marx ([1867] 1954, Ch. 3) calls "socially necessary labour time"[3]). A labour-value is a function of the current methods of production, i.e., the prevailing know-how, technology and so forth, and denotes a quantity of "abstract labour" or "homogeneous human labour" (Marx, [1867] 1954, Ch. 1, Sec. 1), which is the universal human capacity to perform work in general.

The causal regularities of commodity production, which operate "behind the backs" (Marx, [1867] 1954, Ch. 1, Sec. 2) of the participants, instantiate a "law of value" (e.g., see Marx ([1867] 1954, Ch. 11) and Marx ([1867] 1954, Ch. 19)), which is the tendency, given constant methods of production, for exchange-values to "gravitate" toward or around their labour-values. The law of value distributes the available social labour to different branches of production according to market demand (see Marx and Engels (1975, p. 196) and c.f. Smith's ([1776] 1994) "invisible hand"). Hence, a quarter of wheat tends toward equality with "x blackling, y silk or z gold" because they have the same labour-value. Marx therefore claims that exchange-value "represents" or "expresses" abstract labour (Marx, [1867] 1954, Ch. 1, Sec. 2), much as the height of a mercury column, in virtue of the law of thermal expansion, refers to and measures ambient temperature.

Although Marx predominantly uses physical examples of commodities (e.g., wheat, silk, gold etc.) non-physical commodities, such as non-tangible services (e.g. acting or clowning (Marx, 2000, Ch. IV, Sec. 3)), may equally have a use-value, exchange-value and a labour-value.

Marx's theory of value, specifically his analysis of the commodity and the related proposition that *exchange-value refers to and is regulated by its labour cost*, is the first key to understanding Marx's theory of money-capital. Next we examine the second key.

[3]Marx introduced the modifier "socially necessary" in order to generalise over heterogeneous labour productivity within a sector of production.

3.3 The costless nature of surplus-value

Marx describes the labour process, within a capitalist firm, as an activity that simultaneously (i) transfers the labour-value of used-up means of production to the output, (ii) reproduces the labour-value of the real wage consumed by workers and (iii) produces or creates surplus-labour, which is the substance of capitalist profit or "surplus value". (For the full account see Chapters 6 to 10 of volume 1 of *Capital*).

For example, consider a single working day in a closed economy during which workers supply a total of L hours of labour. In terms of Marx's labour-value accounting the workers 'transfer' the labour-value of the inputs to the output and add an increment of L hours of their newly supplied labour. The net result is simply the addition of L hours of labour-value, or one working day. Assume that workers consume a real wage with a labour-value of x hours, where $x < L$. The day therefore splits into two parts: a *necessary* part, which is the x hours supplied to reproduce the real wage, and a *surplus* part, which is the $L - x$ surplus hours distributed to capitalists, ultimately in the form of consumption and net investment goods (Marx, [1867] 1954, Ch. 9).

Labour-power is fructiferous – it yields a surplus in excess of the cost to maintain it: "Labour-power ... not only reproduces its value ... but simultaneously produces a surplus-value, a value not existing previously and not paid for by any equivalent" (Marx, [1867] 1954, Ch. 11). The "prolongation of the working day" (Marx, [1867] 1954, Ch. 15, Sec. 3(A)) is a gift to the capitalist – a surplus-value provided "*gratis*" (Marx, [1867] 1954, Ch. 18).

The category of surplus-value forms the basis of Marx's theory of exploitation and exposé of the wage system. The supply of labour in return for the money wage seems to be a equal exchange "for common advantage" between two parties "constrained only by their free will" (Marx, [1867] 1954, Ch. 6). But Marx points out that, in terms of labour-time, the exchange is unequal. Workers supply a day of labour but receive only a part of that day in payment.

Surplus-value is "not paid for by any equivalent", and intrinsically costless, because no equivalent is supplied or exchanged in order to create it (Marx, [1867] 1954, Ch. 24, Sec. 1). Marx, in a footnote in Chapter 18 of Volume 1 of *Capital*, explicitly connects his concept of surplus-value to the earlier Physiocratic concept of the agricultural surplus or 'bounty of nature'. Of course, we can measure the *ex post* labour-value of the goods and services that 'materialise' surplus-labour. But this labour-value did not constitute an *ex ante* real cost of production because the labour supplied was a surplus – and therefore costless. In contrast, the labour supplied during the necessary part of the day has a real cost, viz. the real wage consumed by workers.

Marx's theory of surplus-value and the proposition that *surplus-value is costless* is the second key to understanding Marx's theory of money-capital. Let's now review that theory.

3.4 The irrational commodity

The existence of capital in money form transforms capital into a commodity: "It is not until capital is money-capital that it becomes a commodity, whose capacity for self-expansion has a definite price quoted every time in every prevailing rate of interest" (Marx, [1894] 1971, Pt. V, Ch. 24). Money-capital, therefore, has an exchange-value.

Marx calls money the "universal use-value" (Marx, 1993a, Ch. 1) because it functions as a universal means of exchange. But money-capital is endowed with an additional use-value over-and-above a means of exchange: "It is this use-value of money as capital – this faculty of producing an average profit – which the money-capitalist relinquishes to the industrial capitalist for the period, during which he places the loaned capital at the latter's disposal" (Marx, [1894] 1971, Ch. 21).

But although money-capital is both a use-value and an exchange-value, and therefore has commodity-like properties, it is not a *bona fide* commodity but rather *sui generis*:

> "We have seen that interest-bearing capital, although a category *which differs absolutely from a commodity*, becomes a commodity *sui generis*, so that interest becomes its price, fixed at all times by supply and demand

> like the market-price of an ordinary commodity." (Marx, [1894] 1971, Ch. 22)

Money-capital is an exceptional or quasi-commodity. Marx offers three different, but related, reasons for excluding money-capital from the class of commodities proper.

3.4.1 The interest-rate is a growth rate of money

Marx ([1894] 1971, Ch. 21) observes that the price of money-capital is a growth rate that "expresses the self-expansion of money-capital". For example, an interest rate of 10% per annum represents the potential of a sum of money, say £100, to expand to £110 in one year. Marx observes that a growth rate of money is a self-referential concept:

> "Capital manifests itself as capital through self-expansion ... *The surplus-value or profit produced by it – its rate or magnitude – is measurable only by comparison with the value of the advanced capital* ... If, therefore, price expresses the value of the commodity, then interest expresses the self-expansion of money-capital and thus *appears* as the price paid for it to the lender" (Marx, [1894] 1971, Ch. 21) (my emphasis).

The price of an ordinary commodity is defined in terms of non-price quantities, specifically money-cost per physical unit; e.g., £2 per bushel of corn. In contrast, the price of money-capital is entirely defined in terms of money value; e.g., 10 pence per £1 of money-capital. In consequence, the price of money-capital is a dimensionless ratio of two money magnitudes, i.e. an interest rate, and therefore only "appears" to be a price.

According to Marx, therefore the price of money-capital has a "purely abstract and meaningless form" (Marx, [1894] 1971, Ch. 21) with self-referential semantics quite unlike the denotational semantics of ordinary prices that, in contrast, refer to, or denote, an external substance:

> "Interest, signifying the price of capital, is from the outset *quite an irrational expression*. The commodity in question has a double value, first a

> value, and then a price different from this value, while price represents the expression of [labour-]value in money. Money-capital is nothing but a sum of money, or the value of a certain quantity of commodities fixed in a sum of money ... *How, then, can a sum of value have a price besides its own price, besides the price expressed in its own money-form?*" (Marx, [1894] 1971, Ch. 21) (my emphasis).

In summary, the price of a *bona fide* commodity denotes the unit cost of some non-monetary thing. Money-capital is *sui generis* because its price is a dimensionless growth-rate that does not refer beyond monetary phenomena.

3.4.2 The interest-rate is a purely nominal cost

Real capital, such as raw material inputs, is produced, used-up and replaced by human labour. In contrast, money-capital, as a sum of money loaned out as capital, is already existing money that returns to the lender with interest. Money-capital is not produced but circulates. Loanable capital therefore does not have a "difficulty of production" (Ricardo, [1817] 1996) and, in consequence, lacks a labour-value.

It therefore follows that the price of money-capital – the interest-rate – is purely nominal since there is no real cost that its price could refer to:

> "If we want to call interest the price of money-capital, then it is an *irrational form of price quite at variance with the conception of the price of commodities. The price is here reduced to its purely abstract and meaningless form*, signifying that it is a certain sum of money paid for something serving in one way or another as a use-value; *whereas the conception of price really signifies the [labour-]value of some use-value expressed in money*" (Marx, [1894] 1971, Ch. 21) (my emphasis).

The price of a *bona fide* commodity denotes a labour-value whereas the price of money-capital does not have a corresponding labour cost to refer to. Its price, therefore, is an "absurd contradiction":

> "Price, after all, is the [labour-]value of a commodity ... *A price which*

> *differs from [labour-]value in quality is an absurd contradiction.*" (Marx, [1894] 1971, Ch. 21) (my emphasis).

Money-capital is therefore *sui generis* for a further reason – it has a use-value and an exchange-value, but not a labour-value.

3.4.3 The interest rate is a lawless variable

Since money-capital lacks a labour-value its price does not bear a lawful relationship to the methods of production. In consequence, there is no "natural" interest-rate for the market rate to gravitate toward:

> "The average rate of interest prevailing in a certain country – as distinct from the continually fluctuating market rates – cannot be determined by any law. In this sphere there is no such thing as a natural rate of interest in the sense in which economists speak of a natural rate of profit and a natural rate of wages" (Marx, [1894] 1971, Ch. 22).

Instead, competitive haggling between finance and industrial capitalists in the market for loanable funds regulates the interest-rate. In consequence, the interest rate is "arbitrary and lawless" (Marx, [1894] 1971, Ch. 21) since it depends on an irreducibly subjective conflict between financiers and industrialists: "wherever it is competition as such which determines anything, the determination is accidental, purely empirical, and only pedantry or fantasy would seek to represent this accident as a necessity" (Marx, [1894] 1971, pg. 363).

The market price of a *bona fide* commodity is lawfully regulated by its labour-value. The market price of money-capital, in contrast, is regulated by a lawless distributional conflict that lacks any connection to real cost.

Hence, the final reason why money-capital is *sui generis* is that its price is not "regulated by the immanent laws of capitalist production" (Marx, [1894] 1971, Ch. 21). Money-capital is an irrational commodity because it is an exception to the law of value.

3.5 The rational commodity

The law of value regulates the price of a *bona fide* commodity, which gravitates towards its real cost of production measured in labour time. Money-capital, in contrast, is *sui generis* because its price is a purely nominal, self-referential growth-rate that fluctuates according to a distributional intra-class conflict that is arbitrary and lawless. Money-capital has the form of a commodity but lacks its substance.

Marx's analysis of money-capital, which appears mainly in volumes II to IV of *Capital*, was assembled by Friedrich Engels and others from unfinished notes. Marx's analysis is unfinished work-in-progress compared to the more precise structure and refined concepts and prose of Volume 1. Subject to this caveat I will now identify some problems with Marx's analysis.

3.5.1 On Marx's claim that the interest-rate is a growth rate of money

Marx argues that the interest-rate denotes the intrinsic "self-expansion of money-capital", rather than an extrinsic cost. But Marx's argument ignores a salient fact: the interest rate denotes the unit cost of a commodity, just like any price.

For example, consider the commodity 1 kilo of butter that costs £5 to buy. Its price is expressed in units of nominal cost (i.e., £5) per unit commodity (i.e., 1 kilo of butter), which has dimensions 'nominal cost per unit commodity'. Now consider a loan of one month maturity offered in the capital market at 5% interest. This interest-rate is equivalent to a price of 5 pence per £1 of money-capital. The price of money-capital is therefore also expressed in units of nominal cost (i.e., 5 pence) per unit commodity (i.e., £1 of money-capital) and conforms to the same dimensional form of an ordinary price. It just so happens that, in the case of money-capital, the numerator and denominator are of the same type and its price may also be conveniently expressed as a dimensionless interest-rate.

Note that the duration of the loan is an essential property of commodities of the type money-capital. Actual capital markets offer a range of money-capital commodities at different prices that reflect the term structure of interest rates (e.g., 5 pence per £1 loaned for 1 month, 7 pence per £1 loaned for 6 months etc.) In all cases the interest-rate denotes the extrinsic cost of purchasing quantities of money-capital in the capital market.

In summary, the fact that the price of money-capital is normally expressed as an interest-rate does not imply it is essentially self-referential or unusual. The interest-rate simply denotes the cost of a commodity that, as an additional property, has a substantial nature that we measure in units of account.

3.5.2 On Marx's claim that the interest-rate is a purely nominal cost

According to Marx, labour may be 'embodied' in any kind of material object or activity that is a use-value: "[labour-]Value is independent of the particular use-value by which it is borne, but it must be embodied in a use-value of some kind" (Marx, [1867] 1954, pg. 183) and "it is a matter of complete indifference what particular object serves this purpose" (Marx, [1867] 1954, pg. 196). Marx explicitly notes that both money and money-capital are distinct kinds of use-values. In consequence, both money and money-capital, as use-values, qualify for inclusion in the class of things that may 'embody' labour-value.

However, Marx is careful to exclude money. Although labour-value may inhere in any use-value "we leave out of consideration its purely symbolical representation by tokens" (Marx, [1867] 1954, Ch. 8). The reason is simple: an amount of money, say £1, refers to labour-value but is not itself a labour-value, just as 1°C refers to temperature but is not itself temperature.

The existence of commodity-money can obscure this essential point. Consider a gold coin. The coin has a labour-value, which is the labour required to mine the metal and coin it. But the coin's nominal value, stamped on its surface, is a symbolic quantity that lacks any necessary connection to the money-value or labour-value of the gold that bears it. This separation of "name and substance, nominal weight and real weight" begins as soon as coins are debased during circulation such that "the function of gold as coin becomes completely independent of the metallic value of that gold" (Marx, [1867] 1954, Ch. 3). So we must distinguish 'money', in the sense of a nominal representation of economic value, such as £1, from 'money' in the sense of the physical bearer of that representation, such as ounces of gold, paper bills, bytes of memory etc. The representation and its vehicle are materially conjoined but functionally distinct. Marx's point is that money, in the sense of a nominal unit of account, is not a labour-value but is a "purely symbolical representation" of labour-value. In order to avoid confusion in what follows, therefore, I will use the term 'money' to refer to fiat money without intrinsic value.

Money, then, cannot have a labour-value. But does the exclusion of money on these grounds also apply to money-capital? The exclusion certainly applies to a *quantity* of money-capital, which is a sum of money (e.g., £1,000 of loan capital) that constitutes a "purely symbolical representation" of economic value. But the exclusion does not apply to the *commodity* money-capital, which is produced by a complex of activities (e.g., loan arranging, servicing and repayment) and concomitant representational vehicles (e.g., money, promissory notes, loan contracts etc.) that are entirely distinct and not reducible to a "purely symbolical representation" of economic value. Since money-capital is a use-value and labour can be 'embodied' in any kind of thing or service – including, presumably, the activity of supplying money-capital – then money-capital cannot be excluded from the class of commodities with a labour-value on the same grounds as money *simpliciter*.

What labour-value could money-capital have? What coexisting labour must be supplied to bring it to market? In capitalist economies labour is supplied to service loans; for example, in the hyper-financialised economies of, say, the UK and US of the last three decades, a significant share of the total working day has been devoted to the reproduction of credit relations. The production of money-capital incurs a real cost of production, specifically the activities of selling, arranging and servicing loans performed by workers employed in financial enterprises.

Marx, depending on the reading, classifies the labour of servicing loans as unproductive, part of the "*fraux frais* of production", which are costs incurred due to the specific character of capitalist production (e.g., bookkeeping labour that maintains a record of stockholders and their claims), or costs incurred due to "circulation" (e.g., the labour of sales and marketing etc.) According to Marx, unproductive labour is a deduction from surplus-value and therefore does not have the "same absolute character of necessity, and the same rank" (Marx, [1894] 1971, Ch. 20) as properly productive labour.

Marx's distinction between productive and unproductive labour is either unclear or not fully resolved (e.g., see Rubin (1973, Ch. 19)). For the purposes of our argument we can entirely avoid this interpretative issue by assuming that the supply of money-capital incurs zero direct labour costs (i.e., no labour is supplied to administer the circuit of money-capital).

Observe instead that capitalists do not supply money-capital unless part of the working day is devoted to producing goods for their consumption. The necessaries, and luxuries, of life are a necessary condition of the supply of money-capital. For example, Marx states: "If an untowardly large section of capitalists were to convert their capital into money-capital, the result would be a frightful depreciation of money-capital and a frightful fall in the rate of interest; many would at once face the impossibility of living on their interest, and would hence be compelled to reconvert into industrial capitalists" (Marx, [1894] 1971, Ch. 23). Money-capitalists, and capitalists in general, cannot live on air. The reproduction of finance capitalists incurs a variable, but non-zero, labour cost.

In Marx's theory labour-power is a commodity with a real cost of production equal to to the labour-value of the real wage. *Prima facie* the real cost of the supply of money-capital is, at least, the labour-value of the consumption goods that allow money-capitalists to "live on their interest". The supply of money-capital incurs this real cost.

Marx claims that the price of money-capital is purely nominal because it lacks a labour-value. But a possible candidate for the labour cost of money-capital is the labour supplied to reproduce the class of money-capitalists.

3.5.3 On Marx's claim that the interest-rate is a lawless variable

Marx argues that the interest-rate is "lawless" and belongs to the "realm of accident" because it is regulated by a distributional conflict. Marx is surely correct that the interest-rate is not fixed by the technical methods of production; however, this does not imply that it is lawless and bears no relation to labour costs. A distributional conflict, for example, need not lack lawful regularities.

Marx himself predicates his theory of surplus-value on just such a distributional conflict, specifically the split of the working day into necessary and surplus parts, which is a function of the labour-value of labour-power, i.e. the labour supplied to produce the real wage. Marx ([1867] 1954) writes, "in contradistinction therefore to the case of other commodities, there enters into the determination of the value of labour-power a historical and moral element"; in other words, the scale and composition of the real wage is a distributional variable. Short-term distributional conflicts can alter the longer-term "historical and moral element" of the renumeration of labour (Green, 1991). Nonetheless, Marx does not classify the wage-rate as "lawless" nor does he consider the foundation of his theory of surplus-value to be arbitrary or capricious. Indeed, Marx is keen to emphasise that labour-power is a *bona fide* commodity. If Marx applied the same standard of classification to the causes of the interest-rate – specifically the supply and demand for loanable funds, and the historically formed consumption claims of money-capitalists – then he would also not classify the interest-rate as "lawless".

Class conflict, it seems, determines the real cost of reproducing workers and fixes a 'natural price' for the wage-rate. But class conflict does not determine the real cost of reproducing money-capitalists or fix a natural rate of interest that regulates the price they charge, or attempt to gain, for their money-capital.

Marx is aware of this incongruity. In the context of discussing the gravitation of the wage-rate to the natural price of labour he notes that interest constitutes an exception: "If supply and demand coincide, they neutralise each other's effect, and wages equal the value of labour-power. But it is different with the interest on money-capital" (Marx, [1894] 1971, Ch. 21). Marx goes on to explain that the difference is due to the absence of a stable real cost of money-capital. Marx's description of the interest-rate as "lawless" therefore ultimately derives from the absence of a regulating labour-value rather than from its intrinsically financial or distributional nature.

3.6 Marx's problem of money-capital

Marx claims that money-capital is *sui generis*. But our analysis of aspects of Marx's own theory, especially his theory of economic value, imply precisely the opposite, specifically that money-capital has a unit price that is causally related to the real cost of reproducing money-capitalists at their conventional level of consumption (in much the same manner that Marx would claim that the wage rate is causally related to the "value of labour-power"). On this basis, money-capital has a use-value, exchange-value *and* a labour-value, and is therefore endowed with all the properties of a *bona fide* commodity. So why does Marx insist on the 'irrationality' of money-capital?

Recall that Marx's theory of surplus-value splits the working day into necessary and surplus-labour. Interest, as a deduction from profit, is a claim on the surplus-labour supplied by workers. In consequence, although capitalist consumption goods have an *ex post* labour-value, since they require labour to produce them, this labour cannot constitute an *ex ante* real cost of production, since the labour is surplus, provided 'for free' and without cost.

The consumption of surplus-labour, on the part of capitalists, is therefore unproductive: "the commodities the capitalist buys for his private consumption are not consumed productively, *they do not become factors of capital*; just as little do the services he buys for his consumption, voluntarily or through compulsion (from the state, etc.), for the sake of their use value. *They do not become a factor of capital*" (Marx, 1994a) (my emphasis).[4] Since these commodities are not a "factor of capital" they do not form part of the *ex ante* real costs of production.

Marx's theory of surplus-value therefore implies that *the labour supplied to produce capitalist consumption goods is surplus-labour and therefore cannot constitute a real cost of production*. Marx's theory of surplus-value is founded on the asymmetrical treatment of the costs of reproducing the different classes of society: workers' consumption is a necessary cost but capitalists' consumption is not.

In contrast, Marx's theory of value implies that money-capital is a *bona fide* com-

[4]See also Volume 3 of *Capital*, Part V, Ch. 32, 'Money capital and real capital'.

modity with an *ex ante* cost. But Marx does cannot pursue this logic since his theory of surplus-value implies that money-capital cannot have an *ex ante* real cost. Marx resolves the contradiction by classifying money-capital as *sui generis* with a price that is a pure form without substance.

Money-capital, in Marx's theory, belongs to the class of commodities that "have a price without having a [labour-]value", for example land or "conscience, honour, etc.", which have prices that are "imaginary, like certain quantities in mathematics" (Marx, [1867] 1954, Ch. 3) such as the square root of minus 3 (Marx, 2000, Addenda, Sec. 5). An "imaginary" or "irrational" price is the exception to the rule that "money is *nothing but* the value-form of commodities" (Marx, [1867] 1954, Ch. 3, Sec. 1) (my emphasis) in the sense of representing or expressing labour-value.

I take the foregoing analysis as having established a problematic of money-capital in Marx's political economy, which in summary form is the proposition that money-capital both is, and is not, a commodity, where both the affirmation and its negation find their support, respectively, in Marx's theory of value and theory of surplus-value.

Marx's "revolutionary method of thinking" (Engels, 1976, Pt. 1) expects to identify irrational kinds. Capitalism, after all, is a social system riven with real contradictions that throws up contradictory, irrational and fetishistic social forms. Marx attempts to capture this reality in thought. The irrationality of money-capital, according to Marx's dialectical analysis, is therefore a manifestation of the ultimately contradictory nature of capitalism.

Is Marx correct in this assessment? Or is there something amiss in his cognition of capitalism? In other words, is the contradictory nature of money-capital real or logical?

To answer this question we need to situate Marx's economic theory within his broader scientific project of the "materialist conception of history" (Marx and Engels, 1987, Pt. 1).

3.7 The empirical-normative content of historical materialism

"Historical materialism" (Engels, 1970, Ch. 3), or the "materialist conception of history" (Marx and Engels, 1987, Pt. 1), aspires to explain the succession of kinds of societies in human history in terms of a recurring real contradiction between the causal powers of labour (the "forces of production") and the economic organisation of labour (the "social relations of production").

Humans spontaneously learn from their material practice. In consequence, the forces of production have a tendency to alter and improve. At certain junctures in history the forces of production develop to such an extent that the "material productive forces of society come into conflict with the existing relations of production" (Marx, 1993a). For example, the introduction of the manufacturing system (a force of production), which introduced a finer-grained, and therefore more productive, division of labour within a single workshop, dissolved the traditional trades and undermined the institutional power of the medieval guilds (relations of production).

The contradictions may drive social actors to instigate a period of social and political upheaval that, if successful, ultimately overthrows the existing social relations and establishes new relations consistent with the forces of production (Marx, [1867] 1954, Ch. 32). In sum, relatively high-frequency technical change drives relatively low-frequency institutional change. Marx's pithy aphorism, "The hand-mill gives you society with the feudal lord; the steam-mill, society with the industrial capitalist" (Marx, 1992), summarises the main idea.

Marx's "critical analysis of capitalist production" is replete with normative statements. However, he avoids comparing capitalism to a subjective standard or utopian ideal. Instead, he applies the perspective of historical materialism to identify the social contradictions of the capitalist system.

Marx documents a class conflict between workers and capitalists over the distribution of the economic surplus (Marx, [1867] 1954, Ch. 10). The combination of workers who are causally responsible for the production of the surplus do not decide on its distribution. Instead, the "owners of the means of production" (Marx, [1867] 1954, Ch. 10) distribute the surplus in virtue of a property claim rather than causal responsibility. Capitalism, therefore, is founded on a contradiction between the forces of production, i.e. socialised labour, and its relations of production, i.e. private appropriation of the fruits of others' labour. The social relations are irrational because they fail to reflect the actual material conditions of production and also constitute a "fetter" (Marx, 1993a, preface) on the further development of the causal powers of labour. For example, Marx argues that capitalist exploitation causes regular economic crises, such as interruptions of production due to falls in profitability (Marx, [1894] 1971, Pt. 3), financial crashes due to the inability to realise the value of "fictitious" capital (Marx, [1894] 1971, Ch. 25), and also the relative immiseration of the workers (e.g., (Marx, [1867] 1954, Ch. 25)), which prevents the full realisation of their human capacities and powers.

At the same time, real possibilities immanent within capitalism indicate that the contradictions can be abolished (e.g., see Engels (1970, Ch. 3)). For example, Marx and Engels believe that capitalism is pregnant with a post-capitalist, or socialist, system of production in which profit income, that is income received in virtue of the ownership of "the means of production" (i.e., the firm), rather in virtue of labour supplied, has been abolished. For example, Marx argues that joint-stock companies, which indicate how ownership can be socialised, and worker co-operatives, which indicate how a firm can be owned by its working members, are transitional institutions that prefigure fully social and democratic forms of property (Marx ([1894] 1971, Ch. 27) and also Jossa (2005)).

Marx's normative statements ultimately derive from comparing capitalism to a post-capitalist system partially present within or implied by capitalism itself. For example, Marx and Engels (1987, Pt. 1, Sec. A) state that, "communism is for us not a state of affairs which is to be established, an ideal to which reality [will] have to adjust itself. We call communism the real movement which abolishes the present state of things. The conditions of this movement result from the premises now in existence." Marx and Engels therefore claim that their critique of capitalism is especially scientific, rather than moral or utopian, since it reveals, in thought, an actual historical trajectory (Engels, 1970). Slaughter (1975, p. 34), for instance, remarks that for Marx "it was not a question of 'criticising property', but of seeing that history itself had 'criticised' property, by eliminating first slave, then feudal property, and replacing them with capitalist private property".

Marx's "critical analysis of capitalist production" is therefore empirically grounded, because it identifies real contradictions, but also normative, since it argues that transcending the contradictions will result in a better society, where "better" denotes increased causal powers or capacities. I will call this dialectical perspective "empirical-normative" to distinguish it from the standard meaning of "normative".

In summary, the theoretical framework of Marx's "critique of political economy" is historical materialism, which has empirical-normative content. A strictly economic or non-dialectical reading of *Capital*, therefore, may fail to register that many of Marx's key concepts are neither purely empirical nor purely normative. We can see this in Marx's concept of "surplus-labour" and his account of the labour process.

3.8 Marx's empirical-normative analysis of the labour process

Marx's split of the working day does not merely quantitatively identify that workers get this many hours and capitalists get that (in the form of goods and services). Marx's theory of surplus-value in addition classifies a part of the day as a *necessary cost* and a surplus part as *unnecessary* and costless. On what grounds does Marx justify this classification? Why, for example, is it necessary that workers produce the real wage but not the real income of capitalists?

Marx argues that the real wage is necessary because in any viable economic system the labour force must reproduce itself. But the labour force must reproduce capitalists only in the specific, historical circumstances of capitalism. Here is the key passage:

> "That portion of the working-day, then, during which this reproduction [of labour-power] takes place, I call 'necessary' labour time, and

> the labour expended during that time I call 'necessary' labour. *Necessary, as regards the labourer, because independent of the particular social form of his labour; necessary, as regards capital, and the world of capitalists, because on the continued existence of the labourer depends their existence also*.
>
> During the second period of the labour-process, that in which his labour is no longer necessary labour, *the workman, it is true, labours, expends labour-power; but his labour, being no longer necessary labour, he creates no value for himself. He creates surplus-value which, for the capitalist, has all the charms of a creation out of nothing*. This portion of the working-day, I name surplus labour-time, and to the labour expended during that time, I give the name of surplus-labour." (Marx, [1867] 1954, Ch. 9) (my emphasis).

The asymmetry is clear: capitalists need workers but workers don't need capitalists. This proposition is not strictly empirical but counterfactual: Marx implicitly assumes the possibility of alternative "social forms" of labour (i.e., methods of organising production) in which workers do not supply additional labour to capitalists over-and-above that necessary to reproduce themselves.

The justification for Marx's asymmetrical treatment of the reproduction costs of workers and capitalists ultimately derives from the empirical-normative perspective of historical materialism. Marx identifies a real contradiction between workers and capitalists. Nascent social forms – such as democratic worker-owned firms that hire-in capital, rather than labour-power, and democratically distribute firm profit to working members, rather than absentee owners – indicate the real possibility of more democratic and equitable property forms that transcend the hiring of human beings, i.e. the capitalist wage system. For example, a post-capitalist economy of democratic worker-owned firms would lack the social role of capitalist, much like the feudal lord and slave-owner disappeared in earlier social transitions. In this historical sense, the property relations and functional income categories that constitute the capitalist class, and the labouring activities that produce that income, are unnecessary.

Marx's theory of surplus-value, and its split of the working day, predicts that if the social role of capitalist was abolished – but capital accumulation and the size of the workforce remained constant – then workers could knock-off early yet still consume the same real wage. The surplus-labour, which supported the hyper-consumption of a small class of capitalists, would no longer be necessary. Alternatively, workers could choose to continue to work a 'full' day, and supply the 'surplus' labour, but would distribute it to themselves at their collective discretion.

Marx's empirical-normative perspective implies that the necessity to supply consumption goods to capitalists is historically contingent. The surplus-labour supplied to capitalists is, counterfactually speaking, unnecessary post-capitalism. Marx's theory of surplus-value is therefore irreducibly counterfactual because it relies on a comparison between what is and what could be.

3.9 An empirical analysis of the labour process

Marx's theory of surplus-value rejects the cost logic implied by capitalist social relations. Let's now contrast Marx's empirical-normative account of the labour process with a strictly empirical, or factual, account, which takes this cost logic as an empirical given.

Consider again the single working day in a closed economy during which workers supply L hours of labour. Marx splits the working day into x hours of necessary labour, supplied to reproduce the real-wage, and $L-x$ hours of surplus-labour, which for simplicity we'll assume is devoted entirely to the production of capitalist consumption and net investment goods. A necessary condition for the reproduction of capitalist social relations is that capitalists receive a share of the surplus-labour, $y \leq (L-x)$, as real income. In consequence, in the actual circumstances of capitalist production, rather than the counterfactual circumstances that may prevail post-capitalism, the real wage is *not* reproduced after x hours of labour. If it were then workers could knock-off early, after supplying $L-y$ hours of labour, and yet still consume the real wage. But they cannot do this. In the actual circumstances of capitalist production workers supply, as a necessary condition of the production of the real-wage, additional 'surplus' or tributary labour for the capitalist class. This 'surplus' labour is, from a strictly empirical perspective, a necessary cost.

Of course, the scale and composition of the real wage, capitalist consumption and net investment all vary over time, and therefore the quantity of surplus-labour supplied to capitalists, y, also varies. The proposition that tributary labour, y, is a necessary cost of production does not imply a specific quantity of tribute is necessary, or that its quantity is fixed; instead, the proposition captures the empirical fact that production organised under the rubric of capitalist social relations necessarily incurs this sort of cost.

As an empirical fact, then, workers supply the whole working day of L hours in order to receive the real wage and reproduce themselves. And this fact is robust to changes in the distribution of income and the scale and composition of the real wage. A strictly empirical account of the labour process therefore identifies the labour-value of the real wage as L hours.

In contrast, Marx's empirical-normative account identifies the labour-value of the real wage as $x < L$ hours, which is the total coexisting labour that *would be* supplied to reproduce the real wage in circumstances without capitalist exploitation and the production of net investment goods.

Clearly, different choices regarding what is, and what is not, a necessary real cost of production generate quantitatively different measures of the labour-value of commodities, and therefore commodity bundles such as the real wage.

3.10 A contradiction

Should we consider the surplus-labour supplied to capitalists as a real cost of production or not? On the one hand, and following Marx, we may take a empirical-normative view of the capitalist labour process and observe that the supply of surplus-labour is historically contingent and therefore counterfactually unnecessary; on the other hand, we may take an empirical view of the labour process, which Marx does not do, and observe that surplus-labour, although historically contingent, is factually necessary in the empirical circumstances of a capitalist economy. Which view should we adopt?

We should adopt the viewpoint commensurate with our theoretical aims, such as whether we wish to critique or explain the cost structure engendered by capitalist property relations. Classical labour cost accounting, which Marx employs, excludes surplus-labour as a cost and yields a counterfactual measure of "difficulty of production" and therefore provides the quantitative basis for a critique of the cost logic of capitalism. Empirical labour cost accounting, by including surplus-labour as a cost, yields a factual measure of "difficulty of production" and therefore provides the quantitative basis for an explanation of the cost logic of capitalism.[5]

Marx's theory of surplus-value reveals important truths about capitalist exploitation. Workers, in a historical sense, do not need to supply tributary labour to a capitalist class. Marx's theory of value, in contrast, is not solely critical but also has a distinct explanatory aim, which is the attempt to explain economic value in terms of labour time.

The natural prices of a capitalist economy include a profit mark-up that gives capitalists the power to command tributary labour. For example, Marx specifically includes the interest-rate as an *ex ante* cost of production. Marx, unfortunately, attempts to explain a factual cost structure that includes surplus-value as a cost, that is natural prices, in terms of a counterfactual cost structure that excludes surplus-labour as a cost, that is classical labour costs. But a factual cost structure cannot be explained in terms of a counterfactual cost structure. This is the fundamental logical contradiction at the core of Marx's political economy.

Marx aims to construct a unified theory of value and exploitation. On the one hand, Marx employs his theory of surplus-value to reject the cost logic of capitalism; on the other hand, Marx employs his theory of value to explain that logic. A counterfactual measure of labour costs can satisfy only one of these aims.

This fundamental contradiction manifests as different surface problems. The most well known manifestation is the transformation problem, discussed in Chapter 2. It also causes Marx's problem of money-capital.

[5] The empirical labour cost accounting, mentioned here, is identical to the total labour costs introduced in Chapter 2. Total labour costs are empirical in the sense that they measure the actual labour supplied to reproduce commodities.

3.11 The nature of money-capital

We can now return to the question: is Marx's classification of money-capital as irrational the manifestation of a real or logical contradiction? Marx normatively argues that money-capital, as a social practice in which money-capitalists claim a share of surplus-labour, is an exploitative fetter on the forces of production and therefore "irrational" from the perspective of a society that has abolished this form of exploitation (much as we reject slavery or feudal bondage from the perspective of liberal capitalism). However, Marx additionally claims that money-capital *in fact* is "irrational" in the sense that it possesses irrational properties, such as a price "reduced to its purely abstract and meaningless form", which is an "absurd contradiction". It is this latter, specifically empirical, claim that concerns us here.

Marx acknowledges that labour is supplied in order to bring money-capital to market, for instance the labour supplied to produce the real income of money-capitalists. However, his counterfactual analysis of the labour process classifies this labour as surplus that, by definition, cannot constitute an *ex ante* real cost of production.

Money-capital therefore appears "irrational" – with a price but not a labour-value – because Marx compares its actual, *ex ante* nominal cost, i.e. the interest-rate, with its counterfactual, *ex ante* labour cost, which is zero.

In contrast, a strictly empirical analysis of the labour process classifies 'surplus' labour, such as the labour supplied to produce capitalist consumption or net investment goods, as a cost of production. An empirical analysis captures the fact that money-capital has an *ex ante* labour cost, which is non-zero.

In consequence, once we compare like with like, i.e. actual-nominal with actual-labour costs, then money-capital no longer appears irrational, or *sui generis*, but rather belongs to the class of commodities proper, a *bona fide* commodity, with a use-value, exchange-value *and* a labour-value.

We can now answer our question. Marx claims, as a matter of fact, that money-capital is irrational because it factually has a nominal cost but counterfactually lacks a labour cost. But counterfactual properties cannot be factual properties. Marx's claim therefore commits a logical fallacy.

Marx's problem of money-capital therefore dissolves, in a relatively straightforward manner, once we adopt the viewpoint of a more general labour theory of value that includes, yet distinguishes between, both factual and counterfactual accounts of the labour process. The theory of surplus-value, which explains the phenomenon of capitalist exploitation, requires Marx's empirical-normative perspective that views the reproduction of a capitalist class as historically contingent and therefore unnecessary. The theory of value, which Marx employs to explain the phenomenon of exchange-value in the circumstances of capitalist social relations, requires an empirical perspective that views the reproduction of a capitalist class as necessary.

In this general theory, money-capital is a *bona fide* commodity with a price and a labour-value; while, simultaneously, in the context of Marx's "critique of political economy", money-capital is the product of exploitative social relations. The irrationality of money-capital is therefore relocated from its nature as a commodity to its nature as a social practice. We therefore avoid stating Marx's extravagant contradiction that money-capital both is, and is not, a commodity, and instead state the more transparent proposition that money-capital is a commodity that expresses a social relationship that deserves to perish.

3.12 Conclusion

Marx's theory of money-capital, which states the intriguing but contradictory proposition that money-capital is an irrational kind because it both is, and is not, a commodity suffers from the characteristic bias of the "revolutionary method of thinking" (Engels, 1976, Pt. 1): it misidentifies a logical contradiction in thought for a real contradiction of reality. Marx's designation of money-capital as irrational ultimately derives from the fundamental logical contradiction of his theory of political economy, which is his attempt to explain a factual cost structure predicated on capitalist social relations in terms of a counterfactual cost structure predicated on the abolition of those relations. This contradiction prevents Marx from developing a fully successful and unified theory of value and exploitation.

Once identified we can transcend the contradiction by generalising Marx's theory to include both factual and counterfactual accounts of the labour process in capitalism. The general theory unifies the predominately explanatory aims of Marx's theory of value with the predominately critical aims of his theory of surplus-value. Money-capital, in this general setting, is a rational commodity, with a price and a labour cost, and therefore does not constitute an exception to the law of value, that nonetheless expresses social relations that are irrational from the perspective of historical materialism.

We began this chapter with Hegel's advice that the "manifold terms" of a theory should be "driven to the point of contradiction". The presence of a logical contradiction may indicate a hidden real contradiction, or process of historical change, which the theory fails to adequately reflect.

The "self-movement and spontaneous activity", or process of change, indicated by the fundamental contradiction of Marx's political economy, is the historically contested and changing definition of what should, and should not, constitute a necessary cost of production in human society. Marx employs a single definition of necessary cost. A theory with sufficient representational capacity to adequately reflect this historical process includes contested, and therefore multiple, definitions. This is what the more general labour theory of value provides.

We now leave the world of classical political economy and turn our attention to modern classical economics, beginning in the next chapter with Sraffa's reconstruction of the classical labour theory of value.

Chapter 4

Sraffa's incomplete reductions to labour

Piero Sraffa's work in the twentieth century significantly contributed to the revival of the classical approach to value and distribution. In this chapter I examine Sraffa's attitude to the problems of the classical theory of value, specifically how he engages with Ricardo's problematic of an invariable measure of value. First, I review Sraffa's rejection of the existence of a "simple rule" that links natural prices and labour costs. I argue that Sraffa's reduction is based on an incomplete analysis of labour costs. Second, I review Sraffa's construction of the standard commodity, which partially resolves Ricardo's problematic. I argue that Sraffa's standard commodity is, in effect, an indirect or proxy reference to the total labour costs introduced in Chapter 2. Total labour costs reveal the "simple rule" that links natural prices and labour costs. I conclude, therefore, that Sraffa's project to reconstruct classical economics is incomplete and that we need to adopt the perspective of a more general labour theory of value, which admits multiple measures of labour cost, in order to complete it.

4.1 Sraffa's concept of surplus

Sraffa (1960), in Part 1 of his book, *Production of Commodities by Means of Commodities* (PCMC), describes an economy in terms of sets of simultaneous equations. His work therefore belongs to the tradition of linear production theory (Gale, 1960; Pasinetti, 1977; Kurz and Salvadori, 1995), which includes notable precursors such as Quesnay's *Tableau Economique* (1758) and Marx's reproduction schemes in Volume 2 of *Capital* (see Marx (1974) and also Trigg (2006)).

In Chapter 1, "production for subsistence", Sraffa examines a multisector economic model, formally similar to a closed Leontief model, that "produces just enough to maintain itself" including the "necessaries for the workers" (Sraffa, 1960, p. 3). Sraffa notes the existence of a unique set of relative prices, defined by the technique, that "if adopted by the market" would make it "possible for the process to be repeated" (Sraffa, 1960, p. 3). At these subsistence prices the outputs of each sector can be exchanged to restore the original input distributions and, in consequence, the economy may reproduce itself at the same scale and in the same proportions.

Following Pasinetti (1977, ch. 5) we can write Sraffa's subsistence prices as $\mathbf{pA} = \mathbf{p}$, where $\mathbf{p}$ is a row vector of money prices per unit commodity and $\mathbf{A} = [a_{i,j}]$ is a $n \times n$ input-output matrix, where each $a_{i,j}$ is the quantity of commodity i used-up, as means of production and workers subsistence, to produce 1 unit of commodity j. This equation states that every commodity's cost of production, $\mathbf{pA}$, equals its selling price, $\mathbf{p}$. The prices constitute n unknown variables. Assume matrix $\mathbf{A}$ has a dominant eigenvalue of 1, which implies the economy can produce exactly what it consumes. Assume also that matrix $\mathbf{A}$ is of full rank and irreducible. Then we can solve the equation to yield $n-1$ relative prices. Since one degree-of-freedom remains undetermined the solution is a price ray.

In Chapter 2, "production with a surplus", Sraffa considers an economy that "produces more than the minimum necessary for replacement and there is a surplus to be distributed" (Sraffa, 1960, p. 6). Sraffa considers that the undistributed surplus is an excess output or net product, which can be distributed either as additional consumption for workers or capitalists, or additional investment for capital accumulation and economic growth.

Sraffa, in formal terms, assumes matrix $\mathbf{A}$ has a dominant eigenvalue less than 1 (e.g., see Pasinetti (1977, pp. 62–63)). The economy is then able to produce more than it consumes: more 'comes out' than 'goes in'. Given a physical output that exceeds the used-up physical inputs, and constant prices for the period under consideration, then necessarily output prices exceed input costs. So the original subsistence price equation now becomes the *inequality*, $\mathbf{pA} < \mathbf{p}$, which states that every commodity's cost of production is less than its selling price. Profit is now possible. The existence of a surplus breaks the equality of the original price equation and prices become under-determined. Sraffa remarks that "the system becomes self-contradictory" (Sraffa, 1960, p. 6) because the left and right-hand sides of the equation no longer balance.

The production of a surplus raises the "difficulty" (Sraffa, 1960, p. x) of specifying relative prices that make it "possible for the process to be repeated" (Sraffa, 1960, p. x). Sraffa adopts the classical point-of-view that repeatability implies a uniform profit-rate otherwise capitalists will reallocate their capital and thereby alter the relative quantities produced in each sector. (In Chapter 7 we formally analyse the dynamics of this process and demonstrate that a steady, repeatable state is indeed characterised by a uniform profit-rate).

Sraffa introduces the distributional variables, the scalar profit-rate, r, and wage-rate, w, to construct a new "natural price" (Sraffa, 1960, p. 9) equation,

$$\mathbf{pA}(1+r)+\mathbf{l}w=\mathbf{p}, \tag{4.1}$$

which restores the equality between output prices and input costs, where $\mathbf{l}=[l_i]$ is a vector of direct labour coefficients such that each l_i is the quantity of labour used-up to produce 1 unit of commodity i.[1]

Natural prices comprise (i) the cost of means of production, $\mathbf{pA}$, (ii) the profit on the money-capital advanced, $\mathbf{pA}r$, and (iii) the cost of labour, $\mathbf{l}w$. Prices $\mathbf{p}$ are positive if $0 \leq r \leq R = (1/\lambda) - 1$, where λ is the dominant eigenvalue of $\mathbf{A}$ and R is the maximum profit-rate of the economic system; see Pasinetti (1977, pp. 95–97).

Sraffa assumes a fixed scale and composition of output (Sraffa, 1960, p. v). He explores the space of possible natural prices by conjecturally varying w and r, which fix different shares of the given physical surplus that could be purchased by workers and capitalists. However, as Ravagnani (2001) notes, "Sraffa never introduces in

[1] This equation is identical to production prices defined by equation (2.3) in Chapter 2 except that wages are paid *post factum* rather than *ex ante*.

his analysis any specific assumption about the allocation of the physical surplus", i.e. Sraffa does not specify the actual commodity bundles distributed to the population. Ravagnani therefore argues that Sraffa's approach is not restricted to self-reproducing states but has more general applicability. All statements in this chapter are restricted to self-reproducing states with a given net output. I consider conjectural variations of both the real and nominal distribution of income. My arguments are therefore independent of this interpretative issue or any assumptions regarding returns to scale. In Chapter 7 I begin to generalise this analysis by including out-of-equilibrium adjustment processes and transient non-reproducing states.

4.2 The "reduction to dated quantities of labour"

Sraffa expresses his natural price equation (4.1) in the equivalent form of an infinite series[2], or "reduction equation" (Sraffa, 1960, p. 35):

$$\begin{aligned} \mathbf{p} &= \mathbf{l}w + \mathbf{lA}w(1+r) + \mathbf{lA}^2 w(1+r)^2 + \cdots + \mathbf{lA}^n w(1+r)^n + \ldots \\ &= \sum_{n=0}^{\infty} \mathbf{lA}^n w(1+r)^n, \end{aligned}$$

which he hypothetically interprets as a "sum of a series of terms when we trace back the successive stages of the production of the commodity" (Sraffa, 1960, p. 89). The reduction reveals how prices resolve into functional income categories, that is payments to workers and capitalists. The nth term is the production costs, in terms of wages and profit, incurred n 'years' prior to final output.

For example, in year $n = 0$, we imagine that capitalists sell unit outputs and pay workers $\mathbf{l}w$ in wages. In the previous year, $n = 1$, capitalists advanced $\mathbf{lA}w$ in wages to pay the labour that transforms means of production, $\mathbf{A}$, into unit outputs for sale the following year. The advanced wages are therefore 'tied up' in production for 1 'year'. The total costs incurred in year $n = 1$, then, are wages plus 1 'year' of profit on the advance, i.e. $\mathbf{lA}w(1+r)$. In general, wages advanced in year n do not return to the capitalist until n years later when outputs are sold. Investments of

[2] Simply rearrange the price equation and note that $(\mathbf{I} - \mathbf{A}(1+r))^{-1} = \sum_{n=0}^{\infty} \mathbf{A}^n (1+r)^n$ for $0 \leq r \leq R$.

different duration earn an equal return, or uniform profit-rate, by the application of compound interest. In consequence labour costs are "multiplied by a profit factor at a compound rate for the appropriate period" (Sraffa, 1960, p. 34). Sraffa's reduction is therefore a series of terms that specify the wages of "dated quantities of labour" (Sraffa, 1960, p. 34) plus profit compounded over the duration of investment, i.e. $\mathbf{l}\mathbf{A}^n w(1+r)^n$.

Define classical labour-values as $\mathbf{v} = \mathbf{v}\mathbf{A} + \mathbf{l}$, where $\mathbf{v} = [v_i]$ is a row vector and each v_i measures the direct (l_i) and indirect ($\mathbf{v}\mathbf{A}^{(i)}$) labour required to reproduce 1 unit of commodity i, as per definition 1 in section 2.1, Chapter 2. If capitalist profits are zero, i.e. $r = 0$, then the reduction equation yields $\mathbf{p} = \sum \mathbf{l}\mathbf{A}^n w$ and natural prices are a simple sum of wage costs. Prices are therefore proportional to labour-values, i.e. $\mathbf{p} = \mathbf{v}w$ (see Proposition 1, Chapter 2). Sraffa states, therefore, that when the surplus is entirely distributed as wages "the relative values [prices] of commodities are in proportion to their labour cost, that is to say to the quantity of labour which directly and indirectly has gone to produce them. *At no other wage-level do values* [prices] *follow a simple rule*" (Sraffa, 1960, p. 12) (my emphasis). In capitalist conditions, where profit is non-zero, natural prices do not simply vary with labour costs but also vary with the profit-rate. In consequence, natural prices are not proportional to classical labour-values, except in special cases (see Chapter 2).

Natural prices are an amalgam of labour costs and compound profits. Ricardo ([1817] 1996) therefore suggested that profit is "only a just compensation for the time that profits were withheld". Natural prices, it appears, are partially determined by a period of 'waiting' entirely unrelated to labour costs.

4.3 The complete reduction to dated quantities of labour

Sraffa's reduction equation does not exhaust the possible series representations of natural prices. For instance, consider "production with a surplus" from the point of view of quantities $\mathbf{q} = [q_i]$ rather than prices $\mathbf{p}$, where each q_i is the gross output of commodity i. Quantities satisfy the inequality, $\mathbf{q}\mathbf{A}^{\mathrm{T}} < \mathbf{q}$, which states that, for each commodity, the quantity used-up as inputs is less than the quantity output. A physical surplus is now possible dual to profits in the price system. To restore equality we must extend Sraffa's analysis and explicitly specify the distribution of real income as an exogenous variable.

The surplus, or net product, $\mathbf{n}$, is fixed once the gross output is given, where $\mathbf{n} = \mathbf{q} - \mathbf{q}\mathbf{A}^{\mathrm{T}}$. Assume the net product consists of the real wage, $\mathbf{w} = [w_i]$, and capitalist consumption bundle, $\mathbf{c} = [c_i]$, such that $\mathbf{n} = \mathbf{w} + \mathbf{c}$. The quantity equation is then

$$\mathbf{q}\mathbf{A}^{\mathrm{T}} + \mathbf{w} + \mathbf{c} = \mathbf{q}, \tag{4.2}$$

which describes a self-reproducing state where the physical surplus is consumed by workers and capitalists. Equation (4.2) is an open Leontief system where final demand consists of the consumption demands of workers and capitalists (see Pasinetti (1977, pp. 60–61)).

In a self-reproducing state, the distribution of nominal income, specified by the profit and wage-rate, w and r, is sufficient to purchase the real income, specified by $\mathbf{w}$ and $\mathbf{c}$. The distribution of real and nominal income are therefore necessarily linked. In fact, price equation (4.1) and quantity equation (4.2) imply

$$\mathbf{p}\mathbf{A}\mathbf{q}^{\mathrm{T}} r + \mathbf{l}\mathbf{q}^{\mathrm{T}} w = \mathbf{p}\mathbf{w}^{\mathrm{T}} + \mathbf{p}\mathbf{c}^{\mathrm{T}}, \tag{4.3}$$

which states that total profit, $\mathbf{p}\mathbf{A}\mathbf{q}^{\mathrm{T}} r$, and total wage income, $\mathbf{l}\mathbf{q}^{\mathrm{T}} w$, equals the cost of the net product, $\mathbf{p}\mathbf{w}^{\mathrm{T}} + \mathbf{p}\mathbf{c}^{\mathrm{T}}$. Assume further that workers and capitalists spend what they earn; in consequence,

$$\mathbf{l}\mathbf{q}^{\mathrm{T}} w = \mathbf{p}\mathbf{w}^{\mathrm{T}} \tag{4.4}$$

and

$$\mathbf{p}\mathbf{A}\mathbf{q}^{\mathrm{T}} r = \mathbf{p}\mathbf{c}^{\mathrm{T}}. \tag{4.5}$$

Equation (4.4) states that wage income equals the price of the real wage and equation (4.5) states that profit income equals the price of capitalist consumption. Together equations (4.4) and (4.5) link the distribution of real and nominal income.

Once we consider the distribution of income, in both nominal and real terms, important conclusions follow. Substitute $r = \mathbf{pc}^{\mathrm{T}}/\mathbf{pAq}^{\mathrm{T}}$ (from equation (4.5)) into Sraffa's price equation (4.1):

$$\begin{aligned}
\mathbf{p} &= \mathbf{pA}(1 + \frac{\mathbf{pc}^{\mathrm{T}}}{\mathbf{pAq}^{\mathrm{T}}}) + \mathbf{l}w \\
&= \mathbf{pA} + \frac{\mathbf{pc}^{\mathrm{T}}}{\mathbf{pAq}^{\mathrm{T}}}\mathbf{pA} + \mathbf{l}w \\
&= \mathbf{p}(\mathbf{A} + \frac{1}{\mathbf{pAq}^{\mathrm{T}}}\mathbf{c}^{\mathrm{T}}\mathbf{pA}) + \mathbf{l}w \\
&= \mathbf{pA} + \mathbf{pC} + \mathbf{l}w,
\end{aligned}$$

where matrix $\mathbf{C} = [c_{i,j}]$, such that

$$c_{i,j} = \frac{\mathbf{pA}^{(j)}}{\mathbf{pAq}^{\mathrm{T}}} c_i, \tag{4.6}$$

and $\mathbf{A}^{(j)}$ denotes the jth column of matrix $\mathbf{A}$. (Note that c_i denotes the ith component of the capitalist consumption bundle, $\mathbf{c}$, whereas $c_{i,j}$ denotes an element of the matrix $\mathbf{C}$).

What is matrix $\mathbf{C}$ in this equation? The meaning of each element $c_{i,j}$ becomes clearer if we multiply the numerator and denominator by the profit-rate,

$$c_{i,j} = \mathbf{pA}^{(j)} r \frac{c_i}{\mathbf{pAq}^{\mathrm{T}} r}.$$

The term $\mathbf{pA}^{(j)}r$ is the profit income generated by the sale of one unit of commodity j. The fraction $c_i/\mathbf{pAq}^{\mathrm{T}}r$ is the quantity of commodity i consumed by capitalists per unit of profit income. Each element $c_{i,j}$ is therefore the quantity of commodity i distributed to capitalists per unit output of commodity j. Matrix $\mathbf{C}$, in consequence, is a 'capitalist consumption matrix' that specifies how the production of new commodities is synchronised with the consumption of existing commodities by capitalist households. Note that matrix $\mathbf{C}$ is a 'physical' input-output matrix that specifies relative material flows of commodities; for example, each element $c_{i,j}$ of $\mathbf{C}$ is a quantity measured in units identical to the corresponding element $a_{i,j}$ of the technique $\mathbf{A}$.

Sraffa's price equation (4.1) therefore has the equivalent form

$$\mathbf{pA} + \mathbf{pC} + \mathbf{l}w = \mathbf{p}, \tag{4.7}$$

where the real distributional variable $\mathbf{C}$ has replaced the nominal distributional variable r. Equation (4.7) provides an alternative, but quantitatively equivalent, perspective on the cost components of natural prices. In this representation natural prices comprise (i) the cost of means of production, $\mathbf{pA}$, (ii) the cost of maintaining the capitalist class at a given level of consumption, $\mathbf{pC}$, and (iii) the cost of labour, $\mathbf{l}w$.

Write equation (4.7) as an infinite series[3] to yield the 'complete reduction to dated quantities of labour',

$$\begin{aligned} \mathbf{p} &= \mathbf{l}w + \mathbf{l}(\mathbf{A}+\mathbf{C})w + \mathbf{l}(\mathbf{A}+\mathbf{C})^2 w + \cdots + \mathbf{l}(\mathbf{A}+\mathbf{C})^n w + \ldots \\ &= \sum_{n=0}^{\infty} \mathbf{l}(\mathbf{A}+\mathbf{C})^n w. \end{aligned}$$

In this series the profit-rate component of natural prices has been replaced by the labour cost of producing capitalist consumption goods. The wage rate is the only nominal variable that appears in the reduction. The reduction is therefore 'complete' or 'total' in the specific sense that it reduces *all* costs to labour costs (as per definition 2 in Chapter 2).

The complete reduction reveals the additional labour supplied by workers to produce capitalist consumption goods at each "successive stage" of the production of commodities. In comparison, Sraffa's reduction is incomplete because it omits this labour. Sraffa's reduction and the complete reduction are merely different representations of the same natural prices. Sraffa's representation hides some labour per-

[3]The infinite series converges on condition that matrix $\mathbf{A}+\mathbf{C}$ is productive, i.e. has a dominant eigenvalue less than one. If this condition does not hold then the level of capitalist consumption exceeds what is possible to reproduce.

formed, because the profit-rate is unreduced, while the other representation reveals it, because the profit-rate is reduced.

The complete reduction equation immediately suggests the following more general definition of labour cost:

Definition 5. *The super-integrated labour-values are*

$$\tilde{\mathbf{v}} = \tilde{\mathbf{v}}\tilde{\mathbf{A}} + \mathbf{l}, \tag{4.8}$$

where $\tilde{\mathbf{A}} = \mathbf{A} + \mathbf{C}$ *is the technique augmented by capitalist consumption.*

The super-integrated labour-values measure total labour costs, i.e. the direct ($\mathbf{l}$), indirect ($\tilde{\mathbf{v}}\mathbf{A}$) *and* 'super-indirect' ($\tilde{\mathbf{v}}\mathbf{C}$) labour required to reproduce unit commodities, in circumstances of simple reproduction, where 'super-indirect' refers to the labour supplied to produce capitalist consumption.

Classical and super-integrated labour-values identify different properties of the same economy. For example, classical labour-values are 'technical' labour costs that allow productivity comparisons across time independent of the distribution of income (e.g., see especially Flaschel (2010, pt. 1)). The super-integrated labour-values, in contrast, are total labour costs that include the tributary or surplus labour supplied to capitalists as a cost of production. Both kinds of measures are required to answer the range of questions posed by a labour theory of value.

For example, an immediate consequence of the complete reduction equation is that natural prices are proportional to super-integrated labour-values.

Definition 6. *A "steady-state economy" produces quantities,* $\mathbf{q} = \mathbf{q}\mathbf{A}^T + \mathbf{w} + \mathbf{c}$*, at prices,* $\mathbf{p} = \mathbf{p}\mathbf{A}(1+r) + \mathbf{l}w$*, where workers and capitalists spend what they earn,* $\mathbf{p}\mathbf{w}^T = \mathbf{l}\mathbf{q}^T w$ *and* $\mathbf{p}\mathbf{c}^T = \mathbf{p}\mathbf{A}\mathbf{q}^T r$.

Theorem 2. *The production-prices of a steady-state economy are proportional to super-integrated labour-values,*

$$\mathbf{p} = \tilde{\mathbf{v}}w,$$

where $\tilde{\mathbf{v}}$ *are super-integrated labour-values.*

Proof. From equation (4.7) $\mathbf{p} = \mathbf{p}\tilde{\mathbf{A}} + \mathbf{l}w = \mathbf{l}(\mathbf{I} - \tilde{\mathbf{A}})^{-1}w$. From the definition of super-integrated labour-values, $\tilde{\mathbf{v}} = \tilde{\mathbf{v}}\tilde{\mathbf{A}} + \mathbf{l} = \mathbf{l}(\mathbf{I} - \tilde{\mathbf{A}})^{-1}$. Hence $\mathbf{p} = \tilde{\mathbf{v}}w$. □

Sraffa's statement that prices and labour cost follow a "simple rule" only in the special case of zero profit must therefore be qualified. The statement is correct for classical labour-values, which measure technical costs of production, but false for a more general measure of labour-value that additionally includes the cost of reproducing the capitalist class. The period of 'waiting', which seems to exclude the possibility that labour costs can explain the structure of natural prices, is merely an artifact of an incomplete reduction. Natural prices, at all levels of the profit-rate, represent total labour costs, which in the context of simple reproduction are the super-integrated labour-values. Sraffa's "reduction to dated quantities of labour" is incomplete and therefore fails to reveal this "simple rule".

The classical labour theory of value attempts to relate the structure of natural prices ('values') to real costs of production, especially labour costs. Sraffa, partly on the basis of his incomplete reduction, rejects this aspect of classical theory. Nonetheless he circumvents some of the problems of the classical labour theory of value in a remarkable but oblique manner.

4.4 The standard commodity

Consider situations A and B that share the same technology but differ in income distribution. Now, to consistently close the price system in both situations, we must specify a *numéraire* equation, $\mathbf{p}\mathbf{d}^{\mathrm{T}} = 1$, where $\mathbf{d}$ is an arbitrarily chosen commodity bundle (this formulation includes the special case of setting one price to be unity, i.e. $p_i = 1$). Sraffa then asks us to consider a measuring problem:

> "The necessity of having to express the price of one commodity in terms of another which is arbitrarily chosen as standard [i.e., the *numéraire*], complicates the study of the price-movements which accompany a change in distribution. It is impossible to tell of any particular price-fluctuation whether it arises from the peculiarities of the commodity which is being measured or from those of the measuring standard"

(Sraffa, 1960, p. 18).

Essentially this is Ricardo's problem of an invariable measure of value (see Section 2.2, Chapter 2) restricted to conjectural variations in the distribution of income.

Since we define the price of the *numéraire* to be constant what can Sraffa mean by a price-fluctuation that "arises from the peculiarities" of the *numéraire*? To answer this question I follow the path-breaking analysis of Sraffa's standard commodity provided by Bellino (2004) and its reformulation by Baldone (2006).

Prices in Sraffa's equation (4.1) are a function of the wage and profit-rate. In the two situations, A and B, we have prices $\mathbf{p}_A = f(w_A, r_A)$ and $\mathbf{p}_B = f(w_A, r_B)$. The wage and profit-rate, prior to the choice of *numéraire*, are independent variables. Define $\Delta\mathbf{p} = \mathbf{p}_B - \mathbf{p}_A$, $\Delta r = r_B - r_A$ and $\Delta w = w_B - w_A$. The change in price of an arbitrary commodity bundle $\mathbf{d}$, from situation A to B, is then

$$\Delta\mathbf{p}\mathbf{d}^{\mathrm{T}} = (1 + r_A + \Delta r)\Delta\mathbf{p}\mathbf{A}\mathbf{d}^{\mathrm{T}} + \Delta r\mathbf{p}_A\mathbf{A}\mathbf{d}^{\mathrm{T}} + \Delta w\mathbf{l}\mathbf{d}^{\mathrm{T}}. \tag{4.9}$$

This expression is stated by Baldone (2006). The derivation is as follows:

Proposition 8. *Consider (i)* $\mathbf{p}_A\mathbf{d}^T = \mathbf{p}_A\mathbf{A}\mathbf{d}^T(1 + r_A) + \mathbf{l}w_A$ *and (ii)* $\mathbf{p}_B\mathbf{d}^T = \mathbf{p}_B\mathbf{A}\mathbf{d}^T(1 + r_B) + \mathbf{l}w_B$. *Define* $\Delta\mathbf{p} = \mathbf{p}_B - \mathbf{p}_A$, $\Delta w = w_B - w_A$ *and* $\Delta r = r_B - r_A$. *Then*

$$\Delta\mathbf{p}\mathbf{d}^T = (1 + r_A + \Delta r)\Delta\mathbf{p}\mathbf{A}\mathbf{d}^T + \Delta r\mathbf{p}_A\mathbf{A}\mathbf{d}^T + \Delta w\mathbf{l}\mathbf{d}^T.$$

Proof. Subtract equation (i) from (ii):

$$\begin{aligned}
\Delta\mathbf{p}\mathbf{d}^{\mathrm{T}} &= \Delta\mathbf{p}\mathbf{A}\mathbf{d}^{\mathrm{T}} + \mathbf{p}_B\mathbf{A}\mathbf{d}^{\mathrm{T}}r_B - \mathbf{p}_A\mathbf{A}\mathbf{d}^{\mathrm{T}}r_A + \Delta w\mathbf{l}\mathbf{d}^{\mathrm{T}} \\
&= \Delta\mathbf{p}\mathbf{A}\mathbf{d}^{\mathrm{T}} + (\Delta\mathbf{p} + \mathbf{p}_A)\mathbf{A}\mathbf{d}^{\mathrm{T}}(\Delta r + r_A) - \mathbf{p}_A\mathbf{A}\mathbf{d}^{\mathrm{T}}r_A + \Delta w\mathbf{l}\mathbf{d}^{\mathrm{T}} \\
&= \Delta\mathbf{p}\mathbf{A}\mathbf{d}^{\mathrm{T}} + \Delta\mathbf{p}\mathbf{A}\mathbf{d}^{\mathrm{T}}\Delta r + \Delta\mathbf{p}\mathbf{A}\mathbf{d}^{\mathrm{T}}r_A + \mathbf{p}_A\mathbf{A}\mathbf{d}^{\mathrm{T}}\Delta r \\
&\quad + \mathbf{p}_A\mathbf{A}\mathbf{d}^{\mathrm{T}}r_A - \mathbf{p}_A\mathbf{A}\mathbf{d}^{\mathrm{T}}r_A + \Delta w\mathbf{l}\mathbf{d}^{\mathrm{T}} \\
&= (1 + r_A + \Delta r)\Delta\mathbf{p}\mathbf{A}\mathbf{d}^{\mathrm{T}} + \Delta r\mathbf{p}_A\mathbf{A}\mathbf{d}^{\mathrm{T}} + \Delta w\mathbf{l}\mathbf{d}^{\mathrm{T}}.
\end{aligned}$$

□

Equation (4.9) is informative: the presence of the term $(1+r_A+\Delta r)\Delta\mathbf{pAd}^{\mathrm{T}}$ tells us that, in general, the price of $\mathbf{d}$ changes partly due to changes in all other prices ($\Delta\mathbf{p}$) affecting the input cost of its means of production, i.e. $\Delta\mathbf{pAd}^{\mathrm{T}}$. In other words, the price of $\mathbf{d}$ fluctuates due to the transmission of relative price changes through its own "peculiarities of production" or technical input requirements. The price of commodity bundle $\mathbf{d}$ is affected by, rather than isolated from, changes in the prices of all other commodities.

In consequence, if we happen to choose $\mathbf{d}$ as *numéraire*, i.e. $\mathbf{pd}^{\mathrm{T}} = 1$, which implies $\Delta\mathbf{pd}^{\mathrm{T}} = 0$, then the alteration in prices from A to B must satisfy the following constraint

$$0 = (1+r_A+\Delta r)\Delta\mathbf{pAd}^{\mathrm{T}} + \Delta r\mathbf{p}_A\mathbf{Ad}^{\mathrm{T}} + \Delta w\mathbf{ld}^{\mathrm{T}}.$$

Bellino (2004) calls this constraint the "*numéraire* effect" because the choice of *numéraire* affects how prices fluctuate, given the change in income distribution, Δw and Δr. The *numéraire* itself imposes a constraint on $\Delta\mathbf{p}$ and therefore the standard in which prices are expressed affects the system it measures. Given an arbitrary *numéraire* $\mathbf{d}$ it's "impossible to tell of any particular price fluctuation whether it arises from the peculiarities of the commodity which is being measured or from those of the measuring standard" (Sraffa, 1960, p. 18). The choice of measuring standard affects the system it measures. This is Sraffa's measurement problem.

Sraffa therefore seeks a "standard capable of isolating the price-movements [due to changes in the distribution of income] of any other product *so that they could be observed as in a vacuum*" (Sraffa, 1960, p. 18) (my emphasis). The "vacuum" is an ideal situation that would remove the interfering effects of the *numéraire*'s own "peculiarities of production".

A measuring standard that is independent of the price changes that occur between situation A and B would create such a "vacuum". Although such a standard "would be no less susceptible than any other to rise or fall in price relative to other individual commodities; but we should know for certain that any such fluctuation would originate exclusively in the peculiarities of production of the commodity which was being compared with it, and not in its own" (Sraffa, 1960, p. 18).

The "standard commodity" is Sraffa's answer to the measuring problem. The standard commodity[4] is the bundle of commodities $\mathbf{b}$ that satisfies

$$\lambda \mathbf{b} = \mathbf{b}\mathbf{A}^{\mathrm{T}} \tag{4.10}$$

where λ is the dominant eigenvalue of technique $\mathbf{A}^{\mathrm{T}}$. The standard commodity $\mathbf{b}$ is therefore an eigenvector of $\mathbf{A}^{\mathrm{T}}$, and has the special property that, when multiplied by matrix $\mathbf{A}^{\mathrm{T}}$, it retains its proportions.

In economic terms, the production of "the various commodities [that constitute bundle $\mathbf{b}$] are produced in the same proportions as they enter the aggregate means of production [that is, $\mathbf{b}\mathbf{A}^{\mathrm{T}}$]", which "implies that the rate by which the quantity produced exceeds the quantity used up in production is the same for each of them" (Sraffa, 1960, p. 20). Hence, if we consider the price of the standard commodity,

$$\begin{aligned} \lambda \mathbf{p}\mathbf{b}^{\mathrm{T}} &= \mathbf{p}\mathbf{A}\mathbf{b}^{\mathrm{T}} \\ \lambda &= \frac{\mathbf{p}\mathbf{A}\mathbf{b}^{\mathrm{T}}}{\mathbf{p}\mathbf{b}^{\mathrm{T}}}, \end{aligned} \tag{4.11}$$

then, regardless of prices $\mathbf{p}$, the cost of production of the standard commodity, $\mathbf{p}\mathbf{A}\mathbf{b}^{\mathrm{T}}$, is always a constant fraction, λ, of its selling price. No matter how prices change this relationship always holds. In a sense, the "peculiarities of production" of the standard commodity transmit cost price changes to the price of the output in an especially 'balanced' and invariant manner, a property explicitly inspired by Ricardo's notion of an "average" commodity (Sraffa, 1960, p. 94).

But why does $\mathbf{b}$ constitute an invariable standard? Recall that Baldone's equation (4.9) describes the change in price of an arbitrary commodity bundle due to changes in income distribution. Let's now choose that arbitrary commodity to be Sraffa's

[4] For simplicity, and without loss of generality, I postpone discussion of the normalisation conditions that Sraffa imposes on his definition of the standard commodity.

standard commodity. Substitute (4.11) into (4.9):

$$\begin{aligned}\Delta \mathbf{p}\mathbf{b}^{\mathrm{T}} &= (1+r_A+\Delta r)\Delta \mathbf{p}\mathbf{A}\mathbf{b}^{\mathrm{T}} + \Delta r \mathbf{p}_A \mathbf{A}\mathbf{b}^{\mathrm{T}} + \Delta w \mathbf{l}\mathbf{b}^{\mathrm{T}} \\ &= (1+r_A+\Delta r)\lambda \Delta \mathbf{p}\mathbf{b}^{\mathrm{T}} + \Delta r \mathbf{p}_A \mathbf{A}\mathbf{b}^{\mathrm{T}} + \Delta w \mathbf{l}\mathbf{b}^{\mathrm{T}} \\ &= \frac{\Delta r \lambda \mathbf{p}_A \mathbf{b}^{\mathrm{T}} + \Delta w \mathbf{l}\mathbf{b}^{\mathrm{T}}}{1-\lambda(1+r_A+\Delta r)}, \quad \lambda(1+r_A+\Delta r) \neq 1, \end{aligned} \tag{4.12}$$

(the condition on the denominator is equivalent to the final profit-rate not reaching its theoretical maximum, i.e. $r_B \neq R$). The change in price of the standard commodity, in equation (4.12), is *independent of the change in prices*, $\Delta \mathbf{p}$, and only changes in virtue of the alteration in income distribution, Δr and Δw. The variation of other prices does not affect the variation of the price of the standard commodity; the only relevant variable is the change in income distribution itself. Due to its special "peculiarities of production" the standard commodity is isolated from the relative price changes that occur in the economy.

The standard commodity therefore meets Sraffa's requirement of invariance with respect to "price-movements which accompany a change in distribution". This is the fundamental meaning of the 'invariance' of the standard commodity: the "*numéraire* effect" is nullified and we have a measuring standard that does not affect the system it measures.

In general, the price of the standard commodity varies with income distribution.[5] The standard commodity's invariance is therefore completely different from the trivial 'invariance' of the *numéraire*, which, by construction, is constant (Bellino, 2004).

However, if we adopt the standard commodity as *numéraire* it confers a special property to the price system. Scale the standard commodity by a normalisation factor, $\alpha \mathbf{b}$, where $\alpha = (1-\lambda)/\mathbf{l}\mathbf{b}^{\mathrm{T}}$ (in fact, Sraffa reserves the term "standard commodity" for this normalised commodity bundle) and set the *numéraire* equation to

[5] Despite some claims in the literature (e.g., Baldone (2006) and also see Vienneau (2005) for a comprehensive review of claims regarding Sraffa's standard commodity) the price of the standard commodity is not invariant to changes in income distribution, except in special cases, such as an economy with gross output proportional to some scalar multiple of its standard commodity (i.e., an economy in Sraffa's "standard proportions").

$\alpha\mathbf{pb}^{\mathrm{T}} = 1$, then the maximum wage-rate is unity and

$$r = R(1-w), \tag{4.13}$$

where R is the maximum profit-rate:

Proposition 9. *The* numéraire *equation* $\alpha\mathbf{pb}^T = 1$*, where* $\alpha = (1-\lambda)/\mathbf{lb}^T$*, implies*

$$r = R(1-w).$$

Proof. The *numéraire* equation $\alpha\mathbf{pb}^{\mathrm{T}} = 1$ implies $\alpha\mathbf{pAb}^{\mathrm{T}} = \lambda$ by equation (4.11). Multiply Sraffa's price equation (4.1) by $\alpha\mathbf{pb}^{\mathrm{T}}$:

$$\begin{aligned}
\alpha\mathbf{pAb}^{\mathrm{T}}(1+r) + \alpha\mathbf{lb}^{\mathrm{T}}w &= 1 \\
\lambda(1+r) + \alpha\mathbf{lb}^{\mathrm{T}}w &= 1 \\
r &= \frac{1}{\lambda} - 1 - \frac{\alpha}{\lambda}\mathbf{lb}^{\mathrm{T}}w \\
r &= R(1-w),
\end{aligned}$$

by substituting for α and R in the last step. □

Equation (4.13) reveals a linear relationship between the profit and wage-rate: as r increases from 0 to its maximum value R then w decreases from its maximum value 1 to 0. The standard commodity has "render[ed] visible what was hidden" (Sraffa, 1960, p. 23), specifically the existence of a zero-sum distributional conflict between workers and capitalists that is logically independent of relative prices.[6]

4.5 The reduction to a "variable quantity of labour"

Sraffa (1960, p. 31) considers the standard commodity "a purely auxiliary construction" that can be "displaced" by "a more tangible measure for prices of commodities"

[6]Flaschel (2010, ch. 11) suggests we choose $\mathbf{pn}^{\mathrm{T}} = 1$, where $\mathbf{n}$ is the net product, as the *numéraire* equation. We can then study conjectural variations in income distribution in the context of fixed income. Flaschel concludes, therefore, that Sraffa's standard commodity is superfluous. However, Flaschel's choice does not nullify the "*numéraire* effect" nor reveal the existence of a fixed *physical*, i.e. non-price, magnitude that breaks down into profit and wage income.

which is "the quantity of labour that can be purchased by the Standard net product" (Sraffa, 1960, p. 32), or, to use Smith's terminology, the "labour commanded" (Smith, [1776] 1994) by the standard net product, i.e. its price divided by the wage rate. Denote this quantity of labour ω; then, from equation (4.13),

$$\omega = \frac{\alpha \mathbf{p}\mathbf{b}^{\mathrm{T}}}{w} = \frac{R}{R-r}. \tag{4.14}$$

Sraffa (1960, p. 32) writes:

> "all the properties of an 'invariable standard of value' ... are found in a variable quantity of labour, which, however, *varies according to a simple rule which is independent of prices*: this unit of measurement increases in magnitude with the fall of the wage, that is to say with the rise of the rate of profits, so that, from being equal to the annual labour of the system when the rate of profits is zero, it increases without limit as the rate of profit approaches its maximum value at R" (my emphasis).

(Sraffa normalises the total labour of the system to unity; and hence $r = 0$ implies ω equals the "annual labour"; but this normalisation is not central to the construction.)

By adopting the standard commodity as *numéraire* "in effect" (Sraffa, 1960, p. 32) we indirectly measure prices in terms of a variable quantity of labour, ω, which is independent of the price changes that accompany a change in income distribution.

Why does Sraffa displace the standard commodity with ω? Recall that, according to Sraffa's reduction equation, no "simple rule" exists between natural prices and labour costs. In consequence, classical labour-values cannot function as a price-independent, invariable standard of prices. However, Sraffa discovers, via the construction of the standard commodity, that in the specific case of changes in income distribution a (variable) quantity of labour is an invariable standard, and its variability follows a "simple rule".

Pasinetti (1977, p. 120) argues that the significance of the standard commodity is to "treat the distribution of income independently of prices" and "this possibility is not tied to the pure labour theory of value". Equation (4.13) specifies how a given 'physical' quantity, R, determined by the objective conditions of production, breaks down into wage and profit income. Eatwell (1975) notes that "this is consistent with the classical view that the determination of the distribution of income between wages and profits is logically *prior to*, and independent of, prices". Hence Sraffa's analysis preserves parts of the classical surplus approach to income distribution and separates it from the intractable contradictions of the labour theory.

Sraffa's further step, of reducing the standard commodity to a quantity of labour, also reclaims, in attenuated form, aspects of the classical theory of value, specifically the attempt to measure a given physical surplus in terms of a single substance, such as units of labour, and relate how that quantity of labour breaks down into wage and profit income. However, as Sraffa notes, this invariable measure is not a real cost of production but "equivalent to something very close to the standard suggested by Adam Smith, namely 'labour commanded' " (Sraffa, 1960, appendix. D).

4.6 The complete reduction to a "variable quantity of labour"

Sraffa's route to a "variable quantity of labour", ω, requires we specify the profit-rate. So ω is irreducibly defined in terms of nominal, or monetary, phenomena. Sraffa's reduction of the standard commodity is therefore incomplete in the sense that the "variable quantity of labour" does not denote a real cost of production; it remains a 'labour commanded' measure of value. However, we can go further and completely reduce ω to a real cost:

Theorem 3. *Sraffa's "variable quantity of labour", ω, is the total labour cost of the standard commodity, $\alpha\mathbf{b}$; that is,*

$$\omega = \alpha\tilde{\mathbf{v}}\mathbf{b}^T. \tag{4.15}$$

Proof. Substitute $\mathbf{p} = \tilde{\mathbf{v}}w$ into equation (4.14) and the conclusion follows. □

Sraffa's "variable quantity", therefore, denotes a real cost of production, specifically the the direct, indirect and super-indirect labour supplied to produce the standard commodity given a technique, $\mathbf{A}$, and capitalist consumption, $\mathbf{c}$. Sraffa's labour-commanded invariable measure of value is dual to a labour-embodied measure.

We can now explain Sraffa's observation that his "variable quantity of labour" varies from one to infinity as the profit-rate, r, varies from 0 to its maximum at R. Consider conjectural variations in the distribution of real income, which are dual to the distribution of nominal income. Given the net product $\mathbf{n} = \mathbf{w} + \mathbf{c}$ then vary capitalist consumption between its minimum $\mathbf{c} = \mathbf{0}$ (such that $\mathbf{w} = \mathbf{n}$) and its maximum, $\mathbf{c} = \mathbf{n}$ (such that $\mathbf{w} = \mathbf{0}$).[7] At $\mathbf{c} = \mathbf{0}$ total labour costs collapse to classical labour-values, and therefore $\tilde{\mathbf{v}} = \mathbf{v}$, and $\omega = 1$ (due to the choice of normalisation). As $\mathbf{c}$ increases the capitalist class consumes a greater share of the net product, or surplus, and the labour-time required to produce their consumption increases. In consequence, the *total* labour cost of the standard commodity also increases. In the limit, capitalist consumption exhausts the whole surplus, leaving zero consumption for workers, at which point the economy cannot reproduce. The total labour costs, $\tilde{\mathbf{v}}$, approach infinity, indicating no quantity of labour is sufficient to reproduce unit commodities. Total labour costs therefore reveal the underlying economic meaning of the variability of Sraffa's "variable quantity of labour".

4.7 Sraffa's proxy reference to total labour costs

Sraffa embarks on a search for an invariable standard due to the "necessity of having to express the price of one commodity in terms of another which is arbitrarily chosen as standard" (Sraffa, 1960, p. 18). The classical labour theory of value proposed to 'express' prices in terms of an external standard but, as Sraffa's reduction equation demonstrates, classical labour-values vary independently of prices and hence cannot be their measure. There is no "simple rule" that relates them. Prices, of "necessity", must be measured in terms of other prices because an external standard does not exist. In consequence, we must address the problems of an internal standard or *numéraire*.

[7] More formally, we consider a monotonically increasing sequence $(\mathbf{c}_n)_{n=1}^{k}$ such that $\mathbf{c}_n \leq \mathbf{c}_{n+1}$, where $\mathbf{c}_n \in \{\mathbf{c} : \mathbf{c} \leq \mathbf{n}\}$ for all $n \in [1, k]$, $\mathbf{c}_0 = \mathbf{0}$, and $\mathbf{c}_k = \mathbf{n}$.

Sraffa defines a standard commodity, which has a price that functions as a fulcrum, and uses it to 'reach outside' the price system to the variable quantity of labour it commands in the market, which "in effect" (Sraffa, 1960, p. 32) is an invariable standard. This quantity of labour varies with the distribution of income according to a "simple rule". Sraffa's remarkable argument therefore restores, in attenuated form, the classical idea of a physical surplus, measured in terms of labour, which breaks down into wage and profit income.

Sraffa's remarkable argument is a rather large hint that a standard exists, which is not a composite, but rather a single substance. Sraffa's argument, however, is premised on incomplete reductions. In consequence, Sraffa only partially solves Ricardo's problem of an invariable measure, and the full meaning of his solution remains somewhat opaque even to himself. The complete reduction, in contrast, lead to a better understanding of Sraffa's argument and also a complete solution to Ricardo's problem.

The complete reduction to dated quantities of labour reveals the "simple rule" that relates prices and total labour costs, where total labour costs generalise the classical measure to include the super-indirect labour, which is the labour supplied to produce the real income of capitalists (Theorem 2). Sraffa's "variable quantity of labour" is, in consequence, not merely a labour-commanded measure of value but in fact denotes a real cost of production, specifically the total labour cost of the standard commodity, which is the direct, indirect and super-indirect labour supplied to produce it (Theorem 3). Sraffa's "variable quantity" is therefore an indirect or proxy reference to total labour costs, which is the external standard of prices missing from the classical labour theory.

Sraffa (1960, p. 32) remarks, in the context of displacing the standard commodity, that "it is curious that we should thus be enabled to use a standard without knowing what it consists of" (i.e., the composition of the standard commodity need not be known). This "curious" property of Sraffa's argument is a symptom of its indirectness. The standard commodity is a bridge from the premise that labour costs cannot measure natural prices to the conclusion that a "variable quantity of labour" is nonetheless an invariable measure. The bridge can be thrown away, and Sraffa's own analysis suggests it can, because the premise is mistaken.

Total labour costs, $\tilde{\mathbf{v}}$, immediately allow us to "treat the distribution of income independently of prices" (Pasinetti, 1977, p. 120) because total labour costs are constitutively independent of prices and function as their measure. As soon as we possess an external standard then the requirement, and therefore the problem, of choosing an internal standard that nullifies the *numéraire* effect disappears: the "necessity" to express prices in terms of prices is not a necessity after all, but rather the artifact of an incomplete reduction. Total labour costs are entirely unaffected by "price-movements which accompany a change in distribution"; in consequence, the standard commodity, and the labour it commands, can be displaced by total labour costs, which have "all the properties of an 'invariable standard of value'" as defined by Sraffa.

Sraffa's standard commodity solves Ricardo's problem of an invariable measure in the restricted case of changes in the distribution of income. Ricardo, however, wished to find an objective measure of value that is invariant to both changes in technique and changes in the distribution of income. Sraffa's standard commodity, and the variable quantity of labour it commands, does not fully satisfy this requirement because every technique defines a different standard commodity and therefore different and incommensurate measures of value.

A more general labour theory of value, which admits both classical and super-integrated measures of labour cost, meets Ricardo's requirements, although not in the manner he would have expected (see Chapter 2). The super-integrated labour-values explain the structure of natural prices in terms of objective quantities of labour supplied to produce commodities. We can therefore state "commodity A is more valuable than commodity B" in the strictly objective sense that commodity A costs more than B because it requires more labour resources to produce. We can make such comparisons and claims regardless of any changes in technique or the distribution of income. However, the super-integrated labour-values are not strictly technical measures of "difficulty of production", since they include the real cost of producing non-wage income, and therefore vary with the distribution of real income. In consequence, they do not satisfy Ricardo's requirement to measure absolute value independently of the distribution of income. Classical labour values, in contrast, fulfil this requirement. But we cannot hope or expect, as Ricardo did, for classical labour-values to explain the structure of natural prices, and therefore function as their measure.

We need both kinds of measure of labour cost to answer the full range of questions that a theory of value poses. Classical labour-values answer distribution-independent questions about the technical "difficulty of production" of commodities, whereas super-integrated labour-values answer distribution-dependent questions about the actual "difficulty of production" of commodities. In consequence – and on condition we apply the appropriate concept of 'labour cost' in each case – we can justifiably make public statements about changes in objective value, independent of the distribution of income *and* simultaneously claim that relative values covary with absolute values, and thereby explain the structure of natural prices in terms of labour costs.

4.8 Conclusion

Sraffa's PCMC "was explicitly designed to reconstruct the classical theory of value and distribution" (Kurz and Salvadori, 2000, p. 14) which, as Sraffa pointed out, had been "submerged and forgotten since the advent of the 'marginal' method at the end of nineteenth century" (Sraffa, 1960, p. v). Sraffa demonstrates, via the remarkable construction of the standard commodity, that we can measure the physical surplus in terms of labour and relate that measure to actual money incomes. However, Sraffa's reconstruction does not identify or resolve the classical category-mistake. In consequence, Sraffa's reductions to labour – the reduction of natural prices to "dated quantities of labour" and the reduction of the standard commodity to a "variable quantity of labour" – are incomplete. Sraffa's theory, like its classical precursors, cannot sustain a concept of objective 'value' that reductively explains the structure of natural prices in terms of real costs of production. The post-Sraffian reconstruction of classical economics therefore dispenses with an essential aim of a theory of economic value, which is to explain what the unit of account might measure or refer to.

In Sraffa's theory natural prices are reduced to an amalgam, the sum of quantities of labour and compound profits. The "simple rule" that links total labour values, a physical real cost, to natural prices is absent. Sraffa's reconstruction of classical economics is therefore incomplete. To complete that reconstruction requires the perspective of a more general labour theory of value that admits both classical and total measures of labour cost. Sraffa's reduction of the standard commodity to the variable quantity of labour it commands can then be seen as an indirect or proxy reference to total labour cost, which is the external standard of prices missing from the classical theory.

Chapter 5

Pasinetti's vertically-integrated subsystems and Marx's transformation problem[1]

Piero Sraffa (1960, Ch. 3) demonstrated that the natural prices of reproducible commodities necessarily vary with the distribution of income due to the "inequality of the proportions in which labour and means of production are employed in the various industries" whereas the real costs of production of commodities, measured in terms of labour, do not. In consequence labour costs cannot fully explain the structure of natural prices. This explanatory gap creates various problems for the classical labour theory of value, most notably Karl Marx's transformation problem (e.g., see Seton (1957); Desai (1988); Hunt and Glick (1990)). Many authors interpret Sraffa's analysis to imply that the labour theory of value is, at best, incomplete, or worse, logically incoherent (e.g., Samuelson, 1971; Lippi, 1979; Steedman, 1981).

Luigi Pasinetti, a follower of Sraffa, offers a different interpretation. He proposes a "separation thesis"[2] (Pasinetti, 2007, Ch. IX) that orders the study of economic systems into a pre-institutional or "'natural' stage of investigation", concerned with "the foundational bases of economic relations" that reveal the fundamental constraints that any economic system must satisfy, followed by an "institutional stage" (Pasinetti, 2007, p. 276), which is "carried out at the level of the actual economic institutions"

[1]This chapter is dedicated to the memory of Angelo Reati.

[2]Pasinetti calls it a theorem; but since no proof is involved I prefer to call it a thesis.

(Pasinetti, 2007, p. 275), which identifies how the constraints manifest in specific institutional setups. Pasinetti's attitude to the labour theory of value is shaped by this separation.

Pasinetti argues, in a series of works (e.g., Pasinetti (1981, 1988, 1993)), that the labour theory of value, rather than being incomplete or incoherent, is a powerful analytical tool at the pre-institutional stage of investigation, and therefore "has to be taken as providing a logical frame of reference" with "an extraordinarily high number of remarkable, analytical, and normative, properties" (Pasinetti, 1988, p. 132).

For example, Pasinetti (1988) analyses the pre-institutional cost structure of a non-uniformly growing economy. Pasinetti constructs a "complete generalisation of the pure labour theory of value" (Pasinetti, 1988, p. 130) by proving that the economy's natural prices are proportional to the "physical quantities of labour" supplied to "vertically hyper-integrated subsystems" that include the production of net investment goods. In consequence, the labour 'embodied' in a commodity, suitably generalised, equals the labour it 'commands' in the market.

However, at the institutional stage of analysis the "pure labour theory of value" breaks down. Marx's "prices of production" (Marx, [1894] 1971, ch. 9) are the steady-state prices that correspond to an institutional setup in which capitalists reallocate their capital to seek higher returns until a uniform, general rate of profit prevails across all sectors of production. Pasinetti constructs a "complete generalisation of Marx's 'transformation problem'" (Pasinetti, 1988, p. 131) by proving that, in general, production-prices are not proportional to the labour supplied to the hyper-integrated subsystems. Pasinetti (1981, p. 153) concludes that "a theory of value in terms of pure labour can never reflect the price structure that emerges from the operation of the market in a capitalist economy".

Pasinetti therefore restricts the labour theory of value to a normative role that provides a 'natural' or ideal standard from which to analyse and critique the institutional setups of actual economic systems. Pasinetti's attitude echoes Adam Smith's restriction of the labour theory to an "early and rude state of society" that precedes the "accumulation of stock" (Smith, [1776] 1994, p. 53).

The argument of this chapter is that the "pure labour theory of value" also applies to the price structure of a capitalist economy. I solve Marx's transformation problem, and Pasinetti's generalisation of it, by extending Pasinetti's vertically integrated approach to encompass the institutional conditions of production. I construct vertically super-integrated subsystems that additionally include the production of capitalist consumption goods. I prove that production-prices, both in the special case of simple reproduction and the more general case of Pasinetti's non-uniform growth model, are proportional to the labour supplied to the vertically super-integrated subsystems. In consequence, the labour 'embodied' in a commodity equals the labour it 'commands', even in the circumstances of capitalist production. The transformation problems therefore dissolve once we consider the vertically-integrated subsystems induced by the specific institutional setup of a capitalist economy. A suitably generalised labour theory of value is therefore neither incomplete or incoherent, and need not be restricted to a normative role, but spans both the natural and institutional stages of analysis.

The structure of this chapter is as follows: Sections 5.1 to 5.3 summarise Pasinetti's model and his argument for restricting the labour theory of value to a normative role. Section 5.4 proves that Marx's production-prices are proportional to the labour supplied to the vertically super-integrated subsystems. Section 5.5 concludes by discussing the implications for the post-Sraffian reconstruction of classical economics.

5.1 Hyper-subsystems and their natural prices

Sraffa (1960, p. 89) proposed to decompose an integrated economic system into "as many parts as there are commodities in its net product, in such a way that each part forms a smaller self-replacing system the net product of which consists of only one kind of commodity. These parts we shall call 'subsystems'." A subsystem is a vertically-integrated 'slice' of the economy that produces a single commodity as final output and replaces the used-up means of production.

Pasinetti (1988) generalises Sraffa's approach to apply to a growing economy where each sector produces investment goods that increase the scale of production.[3]

[3]For a discussion of the genesis of the concept of vertical hyper-integration see Garbellini (2010,

Pasinetti defines growing "vertically integrated hyper-subsystems" that additionally include the production of investment goods. Pasinetti constructs a n-sector economy, which exhibits unbalanced growth, in terms of n hyper-subsystems. The total output of a hyper-subsystem is

$$\mathbf{q}_i(t) = \mathbf{q}_i(t)\mathbf{A}^{\mathrm{T}} + (g + r_i)\mathbf{q}_i(t)\mathbf{A}^{\mathrm{T}} + \mathbf{n}_i(t), \tag{5.1}$$

where $\mathbf{q}_i(t)$ is a vector of n quantities, $\mathbf{A} = [a_{i,j}]$ is a constant $n \times n$ input-output matrix, and $\mathbf{n}_i(t)$ is a zero vector except for the ith component, which is a scalar n_i that represents the final demand for commodity i. The total output of a hyper-subsystem, $\mathbf{q}_i(t)$, therefore breaks down into (i) replacement for used-up means of production, $\mathbf{q}_i(t)\mathbf{A}^{\mathrm{T}}$, (ii) additional investment in means of production, $(g + r_i)\mathbf{q}_i(t)\mathbf{A}^{\mathrm{T}}$, to meet increased demand for commodity i due to the growth rate, g, of the population and the per-capita growth rate, r_i, of consumption demand for commodity i (which may be positive or negative), and (iv) the final output, or net product, $\mathbf{n}_i(t)$, which is the quantity of commodity i consumed. The total labour supplied to the hyper-subsystem is

$$L_i(t) = \mathbf{l}\mathbf{q}_i^{\mathrm{T}}(t) = \mathbf{l}\left(\mathbf{I} - \mathbf{A}(1 + g + r_i)\right)^{-1}\mathbf{n}_i^{\mathrm{T}}(t), \tag{5.2}$$

where $\mathbf{l} = [l_i]$ is a vector of n direct labour coefficients.[4]

A hyper-subsystem includes the labour and means of production necessary for the production of its final output *and* the labour and net investment in means of production necessary for its expansion at the growth rate $(g + r_i)$. Pasinetti defines the trajectory of final demand as

$$n_i(t) = n_i(0)\mathrm{e}^{(g+r_i)t}, \tag{5.3}$$

i.e., $\frac{\mathrm{d}n_i}{\mathrm{d}t} = n_i(g + r_i)$, which drives the growth of the subsystem starting from its initial scale at $t = 0$. For notational convenience I now drop explicit time parameters. All

pp. 36–38).

[4]Equations (5.1) and (5.2) are identical to equations (2.5), (2.6), and (2.7) in (Pasinetti, 1988) except, in this chapter, we make the simplifying assumption that $\mathbf{B} = \mathbf{I}$.

subsequent algebraic statements therefore hold at an implicit time t. (I consider the implications of the trajectory of final demand in Appendix 9.3.3).

Pasinetti obtains the integrated economic system by composing the n hyper-subsystems. Define the total output of the integrated economic system as the sum of its n hyper-subsystems,

$$\mathbf{q} = [q_i] = \mathbf{q}\mathbf{A}^{\mathrm{T}} + g\mathbf{q}\mathbf{A}^{\mathrm{T}} + \left(\sum_{i=1}^{n} r_i \mathbf{q}_i\right)\mathbf{A}^{\mathrm{T}} + \mathbf{n}, \tag{5.4}$$

where $\mathbf{q} = \sum_{i=1}^{n} \mathbf{q}_i$, $\mathbf{n} = \sum_{i=1}^{n} \mathbf{n}_i$, and $L = \mathbf{l}\mathbf{q} = \sum_{i=1}^{n} L_i$, with standard restrictions on the eigenvalues of $\mathbf{A}$ and the feasibility of the growth rates.[5] Note that Pasinetti's model includes a steady-state economy as a special case (by setting $g = 0$ and $r_i = 0$ for all i).

Pasinetti defines natural prices that correspond to the pre-institutional stage of investigation. He stipulates that each hyper-subsystem has its own natural profit-rate, $\pi_i^\star$, which "is equal to the rate of growth of demand for the corresponding consumption good" (Pasinetti, 1988, p. 129); that is, we have n natural profit-rates, $\pi_1^\star, \pi_2^\star, \ldots, \pi_n^\star$, where $\pi_i^\star = g + r_i$, and, in consequence, n vectors of natural prices, $\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n$, one for for each hyper-subsystem, such that

$$\mathbf{p}_i = \mathbf{p}_i\mathbf{A} + \mathbf{p}_i\mathbf{A}\pi_i^\star + \mathbf{l}w, \tag{5.5}$$

where w is the wage rate.

Define $\mathbf{X}^{(i)}$ as the ith column and $\mathbf{X}_{(i)}$ as the ith row of $\mathbf{X}$. The price of commodity j in hyper-subsystem i therefore breaks down into (i) the cost of replacing used-up means of production, $\mathbf{p}_i\mathbf{A}^{(j)}$, (ii) the cost of net investment in additional means of production, $\mathbf{p}_i\mathbf{A}^{(j)}\pi_i^\star$, and (iii) the wage bill, $l_i w$.

In general, each commodity-type has a different natural price in each hyper-subsystem. The natural profit-rates "make possible the expansion of the production of each final good according to the evolution of its final demand", which, as Bellino (2009) explains, "provides a 'social' justification for profit" in terms of the structural

[5]Equation (5.4) is identical to equation (2.1) in (Pasinetti, 1989), except, again, we set $\mathbf{B} = \mathbf{I}$.

necessity for 'mark-up' rates that fund the growth of each hyper-subsystem. Profit and wages, in this natural system, perform different economic functions: profit is purchasing power that injects commodities back into the circular flow, for the expansion of the system, whereas wages are purchasing power that ejects commodities from the circular flow, for final consumption (Garbellini, 2010, pp. 29–30).

5.2 A complete generalisation of the pure labour theory of value

The concept of a vertically-integrated subsystem and the real cost of production of a commodity, measured in terms of labour, are closely connected. For example, Sraffa (1960, p. 13) defines the labour cost of commodity i as the total labour supplied to the subsystem that produces a single unit of i as final output. The standard equation for classical labour-values, $\mathbf{v} = \mathbf{vA} + \mathbf{l}$, immediately follows, since each labour-value, v_i, is the sum of the direct labour, l_i, supplied to sector i and the vertically integrated, indirect labour, $v_i\mathbf{A}^{(i)}$, supplied to other sectors of the economy that replace the used-up means of production (e.g., see Sraffa (1960), Samuelson (1971) and Pasinetti (1977)).

The natural prices, $\mathbf{p}_i$, of each hyper-subsystem, given by equation (5.5), vary with the natural profit-rate, $\pi_i^\star = g + r_i$, whereas classical labour-values do not. Classical labour-values, therefore, cannot fully explain the structure of the natural prices of Pasinetti's growing economy.

In the economy defined by (5.4), with the set of natural prices (5.5), wages are the only type of income. Capitalist profit, in the sense of income received in virtue of firm ownership rather than labour supplied, is absent at the pre-institutional stage of investigation. As Reati (2000, p. 497) notes, "the mere existence of wages could presuppose two social classes. However, on this point also Pasinetti's model is flexible, because nothing prevents us from considering a self-managed economy in which workers decide on the amount and allocation of a surplus". Pasinetti has therefore demonstrated that capitalist profit is not the essential cause of the divergence of natural prices and classical labour-values. In consequence, even in the absence of capitalist profit, a "transformation problem" arises: the natural prices cannot be reduced to "labour-values". Capitalist social relations are therefore merely a sufficient, not a necessary, condition for the divergence of natural prices and labour-values (this point was made, somewhat differently, by von Weizsäcker and Samuelson (1971), who demonstrate that the natural prices of a post-capitalist economy, which lacks capitalist profit income, necessarily deviate from classical labour-values).

Pasinetti constructs a more general definition of labour cost that corresponds to his more general definition of a subsystem. The "vertically hyper-integrated labour coefficients" generalise classical labour-values to include the labour supplied to produce net investment goods (the "hyper-indirect" labour):

Definition 7. *Pasinetti's vertically hyper-integrated labour coefficients,* $\mathbf{v}^{\star}$*, are*

$$\mathbf{v}_i^{\star} = \mathbf{l} + \mathbf{v}_i^{\star}\mathbf{A} + \mathbf{v}_i^{\star}\mathbf{A}(g + r_i), \tag{5.6}$$

which is the sum of direct, indirect and "hyper-indirect" labour.[6]

A hyper-integrated labour coefficient is therefore the total labour supplied to the hyper-subsystem. Note that, in conditions of zero growth, the hyper-integrated labour coefficients reduce to classical labour-values, i.e. $\mathbf{v}_i = \mathbf{l} + \mathbf{v}_i\mathbf{A}$.

Pasinetti then demonstrates that the natural prices of each hyper-subsystem are proportional to the vertically hyper-integrated labour coefficients:

$$\mathbf{p}_i = \mathbf{p}_i\mathbf{A} + \mathbf{p}_i\mathbf{A}\pi_i^{\star} + \mathbf{l}w = \mathbf{l}\big(\mathbf{I} - \mathbf{A}(1 + \pi_i^{\star})\big)^{-1} w = \mathbf{v}_i^{\star} w.$$

The natural prices, therefore, reduce to the total wage bill of each hyper-subsystem, i.e. the wages of the direct, indirect and hyper-indirect labour supplied to produce unit commodities.

Pasinetti (1988, p. 130) notes "this is a complete generalisation of the pure labour theory of value" that recreates Smith's "early and rude state" of society in which labour-embodied equals labour-commanded. Furthermore, "the analytical step that

[6]Equation (5.6) is identical to equation (2.9) in (Pasinetti, 1988) except $\mathbf{B} = \mathbf{I}$.

allows the achievement of this result is of course a re-definition of the concept of 'labour embodied', which must be intended as the quantity of labour required directly, indirectly *and* hyper-indirectly to obtain the corresponding commodity as a consumption good" (Pasinetti, 1988, pp. 131–132).

To summarise: if we relate classical labour-values to the natural prices of a hyper-subsystem we encounter a 'transformation problem': labour costs and nominal costs are incommensurate. The fundamental reason is simple: the natural prices of a hyper-subsystem include the cost of net investment as a component of the price of commodities, whereas classical labour-values exclude the labour cost of net investment as a component of the labour-value commodities. In this case, the dual systems of prices and labour-values adopt different, and incommensurate, cost accounting conventions.

This 'transformation problem' dissolves once we relate natural prices to the hyper-integrated labour coefficients. Hyper-integrated labour coefficients adopt the same accounting convention as the price system, and therefore include the labour cost of net investment as a component of the labour-value of commodities. Commensurability is thereby restored.

Pasinetti understands that, in certain circumstances, classical labour-values under-count the total labour costs of production. He therefore constructs a more general measure of labour cost appropriate to the more general economic setting.

5.3 A complete generalisation of Marx's transformation problem

Pasinetti now switches to an institutional stage of investigation where capitalists, as owners of firms, receive profit income. Capitalists reallocate their capital between sectors seeking the highest returns until a general, uniform profit-rate prevails across all sectors of the economy; i.e.,

$$\mathbf{p} = \mathbf{p}\mathbf{A} + \mathbf{p}\mathbf{A}\pi + \mathbf{l}w. \tag{5.7}$$

Marx ([1894] 1971, ch. 9) called these "prices of production". Production-prices (5.7), in contrast to natural prices (5.5), impose a single price structure on the integrated economy as a whole. Also, at this institutional stage, the meaning of 'profit' alters. Profit, in the context of capitalist property relations, is not merely a structural variable, determined by technology and growth requirements, i.e. $\mathbf{p}_i\mathbf{A}(g + r_i)$, but is now a distributional variable received in proportion to the money-capital invested in means of production within each sector of production, i.e. $\mathbf{pA}\pi$.

Pasinetti demonstrates that the "pure labour theory of value" breaks down in the institutional circumstances of capitalism. Pasinetti writes equation (5.7) in the equivalent form,

$$\mathbf{p} = \mathbf{pA} + \mathbf{pA}(g + r_i) + \mathbf{pA}(\pi - g - r_i) + \mathbf{l}w, \tag{5.8}$$

where $g + r_i$ is the growth rate of demand for any consumption good we care to choose (here we have chosen the ith commodity). For convenience define the matrix $\mathbf{M}_i = \mathbf{A}(\mathbf{I} - \mathbf{A}(1 + g + r_i))^{-1}$. We can therefore write production-price equation (5.7) in n different, but equivalent, forms,

$$\mathbf{p} = \mathbf{v}_i^\star(\mathbf{I} - \mathbf{M}_i(\pi - g - r_i))^{-1} w, \qquad i = 1, 2, \ldots, n.^7 \tag{5.9}$$

Consider, for a moment, the special, or accidental case, in which the general profit-rate equals the growth rate of demand for commodity i; that is, $\pi = g + r_i$. Equation (5.9) then collapses to $\mathbf{p} = \mathbf{v}_i^\star w$ and production-prices are proportional to the vertically hyper-integrated labour coefficients of hyper-subsystem i. But in general, $\pi \neq g + r_i$ for any i, and therefore production-prices are not proportional

[7] Equation (5.9) is identical to equation (4.5) in Pasinetti (1988), and we derive it from equation (5.8):

$$\begin{aligned}
\mathbf{p} &= \mathbf{pA} + \mathbf{pA}(g + r_i) + \mathbf{pA}(\pi - g - r_i) + \mathbf{l}w \\
\mathbf{p}(\mathbf{I} - \mathbf{A}(1 + g + r_i)) &= \mathbf{l}w + \mathbf{pA}(\pi - g - r_i) \\
\mathbf{p} &= \mathbf{l}(\mathbf{I} - \mathbf{A}(1 + g + r_i))^{-1}w + \mathbf{pA}(\mathbf{I} - \mathbf{A}(1 + g + r_i))^{-1}(\pi - g - r_i) \\
&= \mathbf{v}_i^\star w + \mathbf{pM}_i(\pi - g - r_i).
\end{aligned}$$

to any hyper-integrated labour coefficient of any hyper-subsystem.

In fact, production-prices vary independently of the vertically hyper-integrated labour coefficients because prices are a function of a global distributional variable, π, whereas the hyper-integrated labour coefficients are not. Production-prices therefore cannot be reduced to labour costs, whether measured in terms of classical labour-values or Pasinetti's hyper-integrated coefficients. Pasinetti (1988, p. 131) notes, therefore, that equation (5.9) "can also be regarded as providing a complete generalisation of Marx's 'transformation problem'" to the case of a non-uniformly growing economy.

Pasinetti concludes, in an earlier work, that this "analysis amounts to a demonstration that a theory of value in terms of pure labour can never reflect the price structure that emerges from the operation of the market in a capitalist economy, simply because the market is an institutional mechanism that makes proportionality to physical quantities of labour impossible to realise" (Pasinetti, 1981, p. 153). Pasinetti therefore restricts the "pure labour theory of value" to the pre-institutional stage of investigation, where it "has to be taken as providing a *logical frame of reference* – a *conceptual construction* which defines a series, actually a family of series, of ideal *natural* prices, which possess an extraordinarily high number of remarkable, analytical, and *normative*, properties" (my emphasis) (Pasinetti, 1988, p. 132).

In the next section I further generalise Pasinetti's vertically integrated approach. I define non-natural, or 'institutional' subsystems, the "vertically super-integrated subsystems", which correspond to the reproduction conditions of the specific institutional setup of capitalism. I prove that production-prices are proportional to a more general measure of labour cost, the "vertically super-integrated labour coefficients". In consequence, the labour theory of value, suitably generalised, equally applies to the "operation of the market in a capitalist economy".

5.4 A general solution to the transformation problem

We first consider the special case of a steady-state economy before generalising to Pasinetti's growth model.

5.4.1 A special case: the steady-state economy

Production-prices are a function of the distribution of nominal income between profits and wages. In order to define the "vertically super-integrated subsystems" we require the corresponding physical data that specifies the distribution of real income. Assume, therefore, that workers receive the real wage, $\mathbf{w} = [w_i]$, and capitalists receive the consumption bundle, $\mathbf{c} = [c_i]$, such that the net product $\mathbf{n} = \mathbf{w} + \mathbf{c}$.

In conditions of zero growth, i.e. $g = 0$ and $r_i = 0$ for all i, Pasinetti's quantity equation (5.4) reduces to $\mathbf{q} = \mathbf{q}\mathbf{A}^{\mathrm{T}} + \mathbf{n}$, which we expand as

$$\mathbf{q} = \mathbf{q}\mathbf{A}^{\mathrm{T}} + \mathbf{w} + \mathbf{c}. \tag{5.10}$$

We analyse the following special-case, steady-state economy:

Definition 8. *A "steady-state economy with production-prices" produces quantities,* $\mathbf{q} = \mathbf{q}\mathbf{A}^{T} + \mathbf{w} + \mathbf{c}$*, at prices,* $\mathbf{p} = \mathbf{p}\mathbf{A}(1 + \pi) + \mathbf{l}w$*, where workers and capitalists spend what they earn,* $\mathbf{p}\mathbf{w}^{T} = \mathbf{l}\mathbf{q}^{T}w$ *and* $\mathbf{p}\mathbf{c}^{T} = \mathbf{p}\mathbf{A}\mathbf{q}^{T}\pi$.

In this economy the net product is produced, distributed and consumed within the period of production. Over multiple periods the economy self-replaces with a constant composition and scale.[8]

The production and distribution of the net product are necessarily related. For example, the quantity of commodity i consumed by worker households per unit of wage income is $w_i/\mathbf{l}\mathbf{q}^{\mathrm{T}}w$. The income received by worker households, per unit output in sector j, is $l_j w$. Hence, consumption coefficient $w_{i,j} = w_i l_j/\mathbf{l}\mathbf{q}^{\mathrm{T}}$ denotes the quantity of commodity i distributed to worker households per unit output of j. Define

$$\mathbf{W} = \frac{1}{\mathbf{l}\mathbf{q}^{\mathrm{T}}}\mathbf{w}^{\mathrm{T}}\mathbf{l} = [w_{i,j}],$$

as a matrix of worker consumption coefficients. $\mathbf{W}$ compactly describes the physical flow rate of consumption goods to worker households per unit outputs.

[8]Pasinetti's economy, in conditions of zero growth, reduces to a closed Leontief system with final demand equal to the consumption of workers and capitalists; see Pasinetti (1977, pp. 60–61).

Production-price equation (5.7) implies that profit is proportional to the money-capital 'tied up' in circulating capital, i.e. $\mathbf{pAq}^{\mathrm{T}}\pi$. The quantity of commodity i consumed by capitalist households per unit of profit income is therefore $c_i/\mathbf{pAq}^{\mathrm{T}}\pi$. The profit income received by capitalist households, per unit output in sector j, is $\mathbf{pA}^{(j)}\pi$. Hence, consumption coefficient $c_{i,j} = c_i\mathbf{pA}^{(j)}/\mathbf{pAq}^{\mathrm{T}}$ denotes the quantity of commodity i distributed to capitalist households per unit output of j. Define

$$\mathbf{C} = \frac{1}{\mathbf{pAq}^{\mathrm{T}}}\mathbf{c}^{\mathrm{T}}\mathbf{pA} = [c_{i,j}], \tag{5.11}$$

as a matrix of capitalist consumption coefficients. $\mathbf{C}$ compactly describes the physical flow rate of consumption goods to capitalist households per unit outputs.

Matrices $\mathbf{W}$ and $\mathbf{C}$ specify the relative quantities of commodities produced for consumption by workers and capitalists per unit of total output. Both matrices specify relative material flows of commodities; for example, the unit of measurement of each quantity $w_{i,j}$ or $c_{i,j}$ is identical to the unit of measurement of the corresponding element $a_{i,j}$ in the technique $\mathbf{A}$. In consequence, matrices $\mathbf{A}$, $\mathbf{W}$ and $\mathbf{C}$ are all 'physical' input-output matrices that denote the flow of goods between sectors of production and households.[9]

Recall that a subsystem is a "self-replacing system" that replaces used-up means of production and produces a final output. A Sraffian subsystem, for example, is the direct and indirect production that produces a single component of the *net product* as final output, where the net product consists of consumption goods. All consumption goods, in a Sraffian subsystem, are final outputs or 'surplus', and therefore not replaced by the subsystem.

A given economic system, however, can be decomposed into alternative kinds of subsystems. Define a "vertically super-integrated subsystem" as the direct, indirect and "super-indirect" production that produces a single component of the *real wage* as final output, where "super-indirect" refers to the production of capitalist consumption goods. Only the real wage of workers, in a super-integrated subsystem,

[9]The price terms in equation (5.11) are a property of the order of exposition. They cancel out yielding physical flow rates.

is a final output or 'surplus'. In consequence, a super-integrated subsystem replaces the real income of capitalists. More formally, the total output of the ith vertically super-integrated subsystem is

$$\hat{\mathbf{q}}_i = \hat{\mathbf{q}}_i \mathbf{A}^{\mathrm{T}} + \hat{\mathbf{q}}_i \mathbf{C}^{\mathrm{T}} + \mathbf{w}_i,$$

where $\mathbf{w}_i$ is a zero vector except for the ith component that equals w_i, which is the wage demand for commodity i. A super-integrated subsystem additionally vertically integrates the production of capitalist consumption goods. The capitalist consumption matrix, $\mathbf{C}$, therefore appears as a real cost of production. The total output of the steady-state economy is then the composition of the vertically super-integrated subsystems, i.e. $\mathbf{q} = \sum \hat{\mathbf{q}}_i$.

Sraffa's and Pasinetti's natural subsystems are defined by technological and accumulation conditions alone. In contrast, the super-integrated subsystems are also defined by social and institutional conditions. A super-integrated subsystem captures the institutional fact that production, in a capitalist system, materially reproduces a capitalist class at a given level of real income.

A "vertically super-integrated labour coefficient", denoted $\hat{v}_i$, is the total labour supplied to the ith super-integrated subsystem when it produces a unit component of the real wage as final output, i.e. when $w_i = 1$. For clarity, we now calculate this quantity step-by-step.

Consider the production of 1 unit of commodity i in super-integrated subsystem i. How much labour does this production require? It requires l_i units of direct labour, $\mathbf{l}\mathbf{A}^{(i)}$ units of indirect labour, and $\mathbf{l}\mathbf{C}^{(i)}$ units of super-indirect labour, giving a total of $\mathbf{l}(\mathbf{A}^{(i)} + \mathbf{C}^{(i)})$ units of labour operating in parallel to produce the output, replace used-up means of production and replace capitalist consumption goods, respectively. Define $\tilde{\mathbf{A}} = \mathbf{A} + \mathbf{C}$ as the technique augmented by capitalist consumption. Matrix $\tilde{\mathbf{A}}$ compactly represents the commodities used-up during the production of each commodity-type *including* the commodities consumed by capitalists. The sum of direct, indirect and super-indirect labour is then $l_i + \mathbf{l}\hat{\mathbf{A}}^{(i)}$.

However, the indirect and super-indirect production itself uses-up means of production and consumption goods, specifically the bundle $\tilde{\mathbf{A}}\tilde{\mathbf{A}}^{(i)}$, which is contemporaneously replaced by the supply of additional labour, $\mathbf{l}\tilde{\mathbf{A}}\hat{\mathbf{A}}^{(i)}$. To count all the direct, indirect and super-indirect labour we must continue the sum; that is,

$$\begin{aligned} \hat{v}_i &= l_i + \mathbf{l}\tilde{\mathbf{A}}^{(i)} + \mathbf{l}\tilde{\mathbf{A}}\tilde{\mathbf{A}}^{(i)} + \mathbf{l}\tilde{\mathbf{A}}^2\tilde{\mathbf{A}}^{(i)} + \ldots \\ &= l_i + \mathbf{l}\left(\sum_{n=0}^{\infty}\tilde{\mathbf{A}}^n\right)\tilde{\mathbf{A}}^{(i)}. \end{aligned}$$

This sum represents the total labour supplied to the ith super-integrated subsystem when it produces 1 unit as final output.

The vector $\tilde{\mathbf{v}}$ of super-integrated coefficients is therefore $\tilde{\mathbf{v}} = \mathbf{l} + \mathbf{l}\left(\sum_{n=0}^{\infty}\tilde{\mathbf{A}}^n\right)\tilde{\mathbf{A}} = \mathbf{l}\sum_{n=0}^{\infty}\tilde{\mathbf{A}}^n$. Assuming that capitalist consumption is feasible, given the technology, then matrix $\tilde{\mathbf{A}}$ is productive, and we may replace the infinite series with the Leontief inverse, $\mathbf{l}\sum_{n=0}^{\infty}\tilde{\mathbf{A}}^n = \mathbf{l}(\mathbf{I} - \tilde{\mathbf{A}})^{-1}$; in consequence:

Definition 9. *The "vertically super-integrated labour coefficients", $\tilde{\mathbf{v}}$, in a steady-state economy with production-prices, are*

$$\begin{aligned} \tilde{\mathbf{v}} &= \mathbf{l} + \tilde{\mathbf{v}}\tilde{\mathbf{A}} \\ &= \mathbf{l} + \tilde{\mathbf{v}}\mathbf{A} + \tilde{\mathbf{v}}\mathbf{C}, \end{aligned} \tag{5.12}$$

which is the sum of direct, indirect and super-indirect labour costs.

The definition of the super-integrated coefficients does not provide or rely upon any theory of income distribution or profit. However, in order to calculate the super-integrated coefficients the distribution of real income must be a given datum, in the same manner that, in order to calculate production-prices, the distribution of nominal income must be a given datum. Conjectural variation of either the real or nominal distribution of income then affects both the super-integrated coefficients and production-prices.

The super-integrated labour coefficients, although more complex than classical labour-values, nonetheless directly relate, in a straightforward manner, to the labour supplied during the production period.

For example, Pasinetti (1980, p. 21) classifies the total labour supplied in two ways: as (i) the sum of direct labour supplied to each sector of production, $\sum l_i q_i = \mathbf{l}\mathbf{q}^{\mathrm{T}}$, or (ii) the sum of direct and indirect labour supplied to each Sraffian subsystem, $\sum v_i n_i = \mathbf{v}\mathbf{n}^{\mathrm{T}}$. The classifications are quantitatively equal, that is $\mathbf{l}\mathbf{q}^{\mathrm{T}} = \mathbf{v}\mathbf{n}^{\mathrm{T}}$, because the Sraffian subsystems collectively produce the net product as final output and exhaust the total supplied labour:

Proposition 10. *The total labour supplied equals the classical labour-value of the net product,* $\mathbf{l}\mathbf{q}^T = \mathbf{v}\mathbf{n}^T$.

Proof. From (5.10), $\mathbf{q} = \mathbf{q}\mathbf{A}^{\mathrm{T}} + \mathbf{n} = \mathbf{n}(\mathbf{I} - \mathbf{A}^{\mathrm{T}})^{-1}$. Hence $\mathbf{l}\mathbf{q}^{\mathrm{T}} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{n}^{\mathrm{T}} = \mathbf{v}\mathbf{n}^{\mathrm{T}}$. $\square$

The super-integrated subsystems provide another partition of the economy. The total labour supplied can also be classified as (iii) the sum of direct, indirect and super-indirect labour supplied to each super-integrated sector, $\sum \hat{v}_i w_i = \hat{\mathbf{v}}\mathbf{w}^{\mathrm{T}}$. Again, this classification is quantitatively equal to the total labour supplied, that is $\mathbf{l}\mathbf{q}^{\mathrm{T}} = \hat{\mathbf{v}}\mathbf{w}^{\mathrm{T}}$, because the super-integrated subsystems collectively produce the real wage as final output and exhaust the total supplied labour:

Proposition 11. *The total labour supplied equals the super-integrated labour-value of the real wage,* $\hat{\mathbf{v}}\mathbf{w}^T = \mathbf{l}\mathbf{q}^T$.

Proof. From (5.11), $\mathbf{C}\mathbf{q}^{\mathrm{T}} = (1/\mathbf{p}\mathbf{A}\mathbf{q}^{\mathrm{T}})\mathbf{c}^{\mathrm{T}}\mathbf{p}\mathbf{A}\mathbf{q}^{\mathrm{T}} = \mathbf{c}^{\mathrm{T}}$. Substitute into (5.10) to yield, $\mathbf{q} = \mathbf{q}\mathbf{A}^{\mathrm{T}} + \mathbf{q}\mathbf{C}^{\mathrm{T}} + \mathbf{w} = \mathbf{q}\hat{\mathbf{A}} + \mathbf{w} = \mathbf{w}(\mathbf{I} - \hat{\mathbf{A}})^{-1}$. Hence $\mathbf{l}\mathbf{q}^{\mathrm{T}} = (\mathbf{I} - \hat{\mathbf{A}})^{-1}\mathbf{w}^{\mathrm{T}} = \hat{\mathbf{v}}\mathbf{w}^{\mathrm{T}}$. $\square$

Now we've defined the super-integrated coefficients we can relate them to production-prices.

Theorem 4. *The production-prices of a steady-state economy are proportional to the super-integrated labour coefficients,* $\mathbf{p} = \tilde{\mathbf{v}}w$.

Proof. Since capitalists spend what they earn, $\mathbf{p}\mathbf{A}\mathbf{q}^{\mathrm{T}}\pi = \mathbf{p}\mathbf{c}^{\mathrm{T}}$. Substitute for π into price equation (5.7): $\mathbf{p} = \mathbf{p}\mathbf{A}(1 + \frac{\mathbf{p}\mathbf{c}^{\mathrm{T}}}{\mathbf{p}\mathbf{A}\mathbf{q}^{\mathrm{T}}}) + \mathbf{l}w = \mathbf{p}\mathbf{A} + \frac{\mathbf{p}\mathbf{c}^{\mathrm{T}}}{\mathbf{p}\mathbf{A}\mathbf{q}^{\mathrm{T}}}\mathbf{p}\mathbf{A} + \mathbf{l}w = \mathbf{p}(\mathbf{A} + \frac{1}{\mathbf{p}\mathbf{A}\mathbf{q}^{\mathrm{T}}}\mathbf{c}^{\mathrm{T}}\mathbf{p}\mathbf{A}) + \mathbf{l}w = \mathbf{p}\mathbf{A} + \mathbf{p}\mathbf{C} + \mathbf{l}w = \mathbf{p}\tilde{\mathbf{A}} + \mathbf{l}w = \mathbf{l}(\mathbf{I} - \tilde{\mathbf{A}})^{-1}w = \tilde{\mathbf{v}}w$. $\square$

Production-prices equal the total wage bill of each super-integrated subsystem, i.e. the wages of the direct, indirect and super-indirect labour supplied to produce unit commodities. The more general definition of labour costs replicates the result, established by Adam Smith for an "early and rude state" of society, that 'labour embodied' equals 'labour commanded'.

The 'physical' configuration of the steady-state economy, specifically the prevailing technique and distribution of real income, determine both the structure of the vertically super-integrated labour coefficients and the structure of production-prices. Recall that Marx's transformation problem arises because production-prices vary with the distribution of income but classical labour-values do not. The super-integrated coefficients, in contrast, also vary with the distribution of income because they vertically integrate over the production of the real income of capitalists. In consequence, production-prices and labour costs, suitably measured, are necessarily dual to each other and "two sides of the same coin".

Next I generalise this result to Pasinetti's growth model. The generalisation does not require any new arguments or ideas. However, the super-integrated coefficients, in the case of non-uniform growth, include both hyper and super-indirect labour.

5.4.2 The general case: Pasinetti's non-uniform growth model

In the more general circumstances of non-uniform growth the final demand is variable. The net product is therefore a function of time, i.e. $\mathbf{n}(t) = \mathbf{w}(t) + \mathbf{c}(t)$. Assume an initial distribution of real income, $\mathbf{w}(0)$ and $\mathbf{c}(0)$. The trajectory of final demand, from equation (5.3), is then $\mathbf{n}(t) = [w_i(0)\mathrm{e}^{(g+r_i)t}] + [c_i(0)\mathrm{e}^{(g+r_i)t}]$. The vectors $\mathbf{w}$ and $\mathbf{c}$ now implicitly refer to time-varying consumption bundles that, following Pasinetti, drive the growth of the economy.[10]

For notational convenience define the "non-uniform capital investment vector", $\mathbf{g} = \sum_{i=1}^{n} r_i \mathbf{q}_i \mathbf{A}^{\mathrm{T}} = [g_i]$, where $\mathbf{q}_i = [q_{i,j}]$ is the total output of hyper-subsystem i as defined by equation (5.1), and each g_i is the quantity of commodity i produced

[10] Of course, in the context of an actual capitalist economy, rather than Pasinetti's system, growth is not driven by exogenous real demand. My goal here is to explore the full implications of Pasinetti's imposition of a 'capitalist' price structure on his model rather than develop a realistic growth model of capitalism.

as additional means of production, in the economy as a whole, in order to meet the total non-uniformly growing demand.[11] Let $\mathbf{\Gamma} = \mathbf{diag}(\mathbf{g})\ \mathbf{diag}(\mathbf{q})^{-1} = [\lambda_{i,j}]$ be a diagonal "non-uniform capital investment matrix", where each element on the diagonal, $\lambda_{i,i} = g_i/q_i$, is the quantity of i produced as additional means of production, per unit output, to meet the total non-uniformly growing demand (and $\lambda_{i,j} = 0$ for $i \neq j$). Rewrite Pasinetti's quantity equation (5.4) in the equivalent form,

$$\mathbf{q} = \mathbf{q}\mathbf{A}^{\mathrm{T}}(1+g) + \mathbf{q}\mathbf{\Gamma} + \mathbf{w} + \mathbf{c}. \tag{5.13}$$

The total profit income remains $\mathbf{p}\mathbf{A}\mathbf{q}^{\mathrm{T}}\pi$ as in the simpler case of a steady-state economy but now a fraction of profit is invested in additional means of production to satisfy increased demand: $\mathbf{p}\mathbf{A}\mathbf{q}^{\mathrm{T}}g$ is invested to satisfy the increase in demand due to population growth and $\mathbf{p}\mathbf{\Gamma}\mathbf{q}^{\mathrm{T}}$ is invested to satisfy the non-uniform change in demand. The residual profit that remains for capitalists to spend on personal consumption is therefore $Y = \mathbf{p}(\mathbf{A}(\pi - g) - \mathbf{\Gamma})\mathbf{q}^{\mathrm{T}}$.

The full specification of Pasinetti's non-uniformly growing economy with production-prices is therefore:

Definition 10. *A "non-uniformly growing economy with production-prices" produces quantities,* $\mathbf{q} = \mathbf{q}\mathbf{A}^{T}(1+g) + \mathbf{q}\mathbf{\Gamma} + \mathbf{w} + \mathbf{c}$*, at prices,* $\mathbf{p} = \mathbf{p}\mathbf{A}(1+\pi) + \mathbf{l}w$*, where workers and capitalists spend what they earn,* $\mathbf{p}\mathbf{w}^{T} = \mathbf{l}\mathbf{q}^{T}w$ *and* $\mathbf{p}\mathbf{c}^{T} = \mathbf{p}(\mathbf{A}(\pi - g) - \mathbf{\Gamma})\mathbf{q}^{T}$.

The production and distribution of the net product are, once again, necessarily related. The matrix of worker consumption coefficients, **W**, is unchanged from the steady-state case. However, the matrix of capitalist consumption coefficients, **C**, differs because capitalists invest a fraction of their profit income in additional means of production. The quantity of commodity i consumed by capitalists per unit of residual profit income is now c_i/Y. The residual profit received, per unit output in sector j, is $\mathbf{p}(\mathbf{A}^{(j)}(\pi - g) - \mathbf{\Gamma}^{(j)})$. Hence consumption coefficient $c_{i,j} = c_i\mathbf{p}(\mathbf{A}^{(j)}(\pi - g) - \mathbf{\Gamma}^{(j)})/Y$ denotes the quantity of commodity i distributed to capitalists per unit

[11] Vector $\mathbf{g}$ features in Pasinetti's equation (5.4) that defines the total output of the integrated economy.

output of j. Define

$$\mathbf{C} = \frac{1}{\mathbf{p}(\mathbf{A}(\pi - g) - \mathbf{\Gamma})\mathbf{q}^{\mathrm{T}}} \mathbf{c}^{\mathrm{T}} \mathbf{p}(\mathbf{A}(\pi - g) - \mathbf{\Gamma}) = [c_{i,j}] \tag{5.14}$$

as the matrix of capitalist consumption coefficients. (Note that, when $g = 0$ and $r_i = 0$ for all i, this definition of $\mathbf{C}$ reduces to the definition for the steady-state economy).

Pasinetti's hyper-integrated subsystems include the direct, indirect and hyper-indirect production that produces a single component of the net product as final output. A vertically super-integrated subsystem, in the context of non-uniform growth, is the direct, indirect, hyper and super-indirect production that produces a single component of the real wage as final output. The total output of the ith vertically-super integrated subsystem is

$$\hat{\mathbf{q}}_i = \hat{\mathbf{q}}_i \mathbf{A}^{\mathrm{T}} + \hat{\mathbf{q}}_i \mathbf{A}^{\mathrm{T}} g + \hat{\mathbf{q}}_i \mathbf{\Gamma} + \hat{\mathbf{q}}_i \mathbf{C}^{\mathrm{T}} + \mathbf{w}_i,$$

where $\mathbf{w}_i$ is defined as before and $\mathbf{q} = \sum \hat{\mathbf{q}}_i$.

The net investment in a hyper-integrated subsystem, from equation (5.1), is $\mathbf{q}_i \mathbf{A}^{\mathrm{T}}(g + r_i)$, which is independent of the cross-demand effects of the non-uniform growth of the other hyper-integrated subsystems (i.e., the r_j for all $j \neq i$). In contrast, the net investment in a super-integrated subsystem, $\hat{\mathbf{q}}_i \mathbf{A}^{\mathrm{T}} g + \hat{\mathbf{q}}_i \mathbf{\Gamma}$, includes cross-demand effects.

The technique augmented by hyper and super-indirect real costs of production is then

$$\hat{\mathbf{A}} = \mathbf{A} + \mathbf{A}g + \mathbf{\Gamma} + \mathbf{C}.$$

Matrix $\hat{\mathbf{A}}$ compactly represents the commodities used-up during the production of each commodity-type including the production of net investment goods and capitalist consumption goods.

Definition 11. *The "vertically super-integrated labour coefficients", $\hat{\mathbf{v}}$, in a non-*

uniformly growing economy with production-prices, are

$$\begin{aligned}\hat{\mathbf{v}} &= \mathbf{l} + \hat{\mathbf{v}}\hat{\mathbf{A}} \\ &= \mathbf{l} + \hat{\mathbf{v}}\mathbf{A} + \hat{\mathbf{v}}(\mathbf{A}g + \mathbf{\Gamma}) + \hat{\mathbf{v}}\mathbf{C}, \end{aligned} \tag{5.15}$$

which is the sum of direct, indirect, hyper and super-indirect labour costs.[12]

The hyper and super-integrated coefficients are identical in circumstances of zero non-uniform growth and zero capitalist consumption. And both the hyper and super-integrated coefficients reduce to the classical definition of labour-value in circumstances of zero growth and zero capitalist consumption (i.e., Smith's "early and rude state"). As before the super-integrated subsystems collectively produce the real wage as final output and exhaust the total supplied labour; in consequence, $\mathbf{l}\mathbf{q}^{\mathrm{T}} = \hat{\mathbf{v}}\mathbf{w}^{\mathrm{T}}$.

We now state the main result of this chapter:

Theorem 5. *The production-prices of a non-uniformly growing economy are proportional to the super-integrated labour coefficients,* $\mathbf{p} = \hat{\mathbf{v}}w$.

Proof. From (5.14), $\mathbf{pC} = (1/Y)\mathbf{pc}^{\mathrm{T}}\mathbf{p}(\mathbf{A}(\pi - g) - \mathbf{\Gamma})$. Hence, $\mathbf{p}(\mathbf{A}(\pi - g) - \mathbf{\Gamma}) = (Y/\mathbf{pc}^{\mathrm{T}})\mathbf{pC}$. Write price equation (5.7) in the equivalent form, $\mathbf{p} = \mathbf{pA} + \mathbf{p}(\mathbf{A}g + \mathbf{\Gamma}) + \mathbf{p}(\mathbf{A}(\pi - g) - \mathbf{\Gamma}) + \mathbf{l}w$, and then substitute to yield $\mathbf{p} = \mathbf{pA} + \mathbf{p}(\mathbf{A}g + \mathbf{\Gamma}) + (Y/\mathbf{pc}^{\mathrm{T}})\mathbf{pC} + \mathbf{l}w$. Since capitalists spend what they earn, $\mathbf{pc}^{\mathrm{T}} = Y$. Hence, $\mathbf{p} = \mathbf{pA} + \mathbf{p}(\mathbf{A}g + \mathbf{\Gamma}) + \mathbf{pC} + \mathbf{l}w = \mathbf{p}\hat{\mathbf{A}} + \mathbf{l}w = \mathbf{l}(\mathbf{I} - \hat{\mathbf{A}})^{-1}w = \hat{\mathbf{v}}w$. □

Production-prices, in Pasinetti's non-uniformly growing economy, equal the total wage bill of each super-integrated subsystem, i.e. the wages of the direct, indirect, hyper and super-indirect labour supplied to reproduce unit commodities. Once again, a more general definition of labour costs replicates Adam Smith's result that 'labour embodied' equals 'labour commanded'.[13]

Theorem 5, it should be emphasised, undermines the logical basis for any claim that a labour theory of value is incoherent because production-prices and labour-

[12] Note that definition 11 reduces to definition 9 in conditions of zero growth.
[13] Note that theorem 5 reduces to theorem 4 in conditions of zero growth.

values are quantitatively incommensurate in linear production models (e.g., Samuelson (1971); Lippi (1979); Steedman (1981)).

5.4.3 Technical and social cost structures

Marx's transformation problem, and Pasinetti's generalisation, reduce to mismatches between production-prices and labour costs. Production-prices include institutional or social costs, specifically a profit-rate that includes the income of a capitalist class. In contrast, classical labour-values, and Pasinetti's hyper-integrated labour coefficients, are purely technical costs of production, and therefore ignore the real cost of producing capitalist income. Transformation problems necessarily arise when we contravene Pasinetti's separation thesis and compare a nominal cost structure that belongs to an institutional stage of analysis with a real cost structure that belongs to a natural, or pre-institutional, stage of analysis. A commensurate relationship cannot obtain between cost structures defined by incommensurate accounting conventions.

Pasinetti recognises the need to extend the classical theory in order to explain the structure of natural price systems. He constructs more general measures of labour cost that take into account additional features of the circumstances of production, such as the labour cost of net capital investment. Despite this important conceptual advance, Pasinetti nonetheless believes, in virtue of the transformation problems, that a labour theory of value "can never reflect the price structure that emerges from the operation of the market in a capitalist economy" (Pasinetti, 1981, p. 153).

Theorems 4 and 5, by generalising the vertically integrated approach to encompass social and institutional conditions, demonstrate the contrary. Production-prices, in both steady-state and non-uniformly growing economies, are proportional to physical quantities of labour, the vertically super-integrated labour coefficients, which include the additional labour supplied to produce the real income of capitalists. The super-integrated labour coefficients capture the real cost structure that "emerges from the operation of the market in a capitalist economy" (Pasinetti, 1981, p. 153). The transformation problems therefore dissolve once we observe Pasinetti's separation thesis and compare the nominal and real cost structures that manifest at the same, institutional stage of analysis.

5.5 Conclusion

Pasinetti's separation thesis, and his generalisation of the vertically integrated approach, are powerful analytic devices. However, Pasinetti's specific proposal to restrict the labour theory of value to a normative role, a kind of "logical frame of reference", is unnecessary. Pasinetti's theoretical innovations instead point in the opposite direction and toward a full generalisation of the classical labour theory and its reinstatement as the foundational theory of value for economic analysis within the "production paradigm" (Pasinetti, 1986). The more general labour theory spans both the natural and institutional stages of analysis and therefore can address normative issues in the critique of political economy, and factual issues in the analysis of specific economic systems.

The more general theory, sketched here in an initial and preliminary manner, admits both technical and social measures of labour cost and applies both kinds of measures in the appropriate contexts. For example, in this more general framework, classical labour-values apply to distribution-independent questions about an economy, such as measuring the technical productivity of labour (e.g., Flaschel (2010, part 1)) or the surplus-labour supplied by workers (e.g., Marx ([1867] 1954)); whereas the super-integrated labour coefficients apply to distribution-dependent questions, such as the relationship between relative prices and the actual labour time supplied to produce commodities; i.e., issues in the theory of value.

The post-Sraffian separation of the classical surplus approach to income distribution from its labour theory of value does not constitute a sophisticated rejection of naive 'substance' theories of value but indicates a failure to resolve the classical contradictions, such as Marx's transformation problem. The separation ultimately derives from the classical error of comparing technical with social cost structures (see Chapter 2 and Wright (2014a)). The post-Sraffian reconstruction of classical economics therefore dispenses with an essential aim of a theory of economic value, which is to explain what the unit of account might measure or refer to. Theorems 4 and 5, which demonstrate that production-prices are proportional to physical quantities of labour, start to put the pieces back together again.

Chapter 6

Substance or field? A note on Mirowski

Most critics reject the classical labour theory based on the transformation problem. Phillip Mirowksi, however, offers a novel critique of Marx's value theory. In this chapter I critically examine Mirowski's thesis.

6.1 The "swan song" of classical substance-based theories of value?

Mirowski's *More Heat Than Light* (1989) traces the history of theories of economic value. Mirowski critically examines the deep connections between modern economic theory and the physical sciences, especially with regard to conservation principles. Marx is accorded a special place in Mirowski's history.

Mirowski (1989, pp. 174–185) claims that "Marx simultaneously argued for two contradictory versions of the labour theory of value: the first of which we shall call the crystallised-labour or substance approach; the second is called the real-cost or virtual approach."

The "substance approach" is the proposition that labour is "embodied" or "crystallised" in its product at the moment of its production. In consequence, every commodity is a carrier of an invariable amount of an underlying labour substance. In contrast, the real-cost approach is the proposition that a commodity "can only be said to possess a labour value in relation to the contemporary configuration of production. Although its physical complexion or its past history might persist unaltered, its real-cost labour value would be subjected to change by technological alterations anywhere in the economy" (Mirowski, 1989, p. 181).

Mirowski claims the "real-cost approach" is "in direct contradiction to the crystallised approach", repeating an earlier argument by Cohen (1981) who also emphasised this dichotomy. Mirowski reasons as follows:

> "A clear example of the real-cost labour theory is provided by Marx's discussion of the effects of a harvest failure upon the existing stocks of cotton harvested in the previous year. In this passage he insists that a harvest failure would instantaneously revalue the embodied labour value of the cotton inventories in an upward direction, under the reasoning that the 'socially necessary' amount of labour-time to produce a bale had risen. This discussion stands in stark contrast to what would happen in a regime of crystallised values: There the cotton inventories would undergo no revaluation, even though the newly harvested cotton would." (Mirowski, 1989, p. 181).

Mirowski notes that a "chief characteristic" of a substance theory of value is "the external residence of value in the commodity" (Mirowski, 1989, p. 399), i.e. labour is a substance localised in the physical body of the commodity. Clearly a change of labour productivity in cotton production cannot alter the amount of the labour substance already embodied in existing cotton inventories, unless we admit the existence of a mysterious kind of 'action at a distance'. Mirowski wonders why Marx would "commit this blunder" since it means his theory suffers from a "crippling problem" of simultaneously maintaining a contradictory crystallised-labour and real-cost approach to labour-values. Mirowski concludes, therefore, that Marx's work represents the terminus or "swan song" of classical substance-based theories of value.

Is Mirowski correct in this assessment?

6.2 Marx's "social substance"

Mirowski unfortunately misreads Marx's concept of substance. Marx, for example, explicitly contrasts his concept of substance to a physical concept; he writes,

> "the value of commodities is the very opposite of the coarse materiality of their substance, not an atom of matter enters into its composition. Turn and examine a single commodity, by itself, as we will, yet in so far as it remains an object of value, it seems impossible to grasp it. If, however, we bear in mind that the value of commodities *has a purely social reality*, and that they acquire this reality only in so far as they are *expressions or embodiments* of one identical *social substance*, viz., human labour, it follows as a matter of course, that value can only manifest itself in the *social relation* of commodity to commodity" (Marx, [1867] 1954) (emphasis added).

For Marx the labour substance "has a purely social reality" and therefore cannot physically reside in commodities. (And, we might add, how could a 'labour substance' be physically present in a commodity?) Arthur (2005) notes that all English translations of Marx's use of *Darstellung* in Capital "are defective in offering 'embodiment' as the translation". Arthur instead suggests that the phrase "labour is 'presented there' in the value of the product" better captures the intended meaning.

Marx, in the above quoted passage, is inviting us to consider that labour-value is an objective property of a commodity that supervenes upon a social practice, specifically a system of generalised commodity production. For example, in the appendix on the "value form" in the first edition of Capital, Marx writes, "The fact that products of labour – such useful things as coat, linen, wheat, iron, etc. – are *values, definite magnitudes of value* and in general *commodities*, are properties which naturally pertain to them only *in our practical interrelations* and not by nature like, for example, the property of being heavy or being warming or nourishing" (Marx, 1994b). Let's try to unpack this collection of subtle but important ideas.

In Marx's theory the "social substance" is abstract labour, which is the expenditure of the labour-power of workers considered as a homogeneous mass of productive capacity (the labour "that forms the substance of value is homogeneous labour, expenditure of one uniform labour-power" (Marx, [1867] 1954)). Marx's notion of a "labour substance" that "congeals" and gets "embodied" in a commodity is *equiv-*

alent to the concept that every commodity has an objective cost measured in terms of labour-time. For example, under the theoretical assumption of equal exchange, Marx ([1867] 1954, p. 59) writes, "The equations, 20 yards of linen = 1 coat, or 20 yards of linen are worth one coat, implies that the same quantity of value-substance (congealed labour) is embodied in both; that the two commodities have each cost the same amount of labour of the same quantity of labour-time". Marx then immediately adds, "But the labour-time necessary for the production of 20 yards of linen or 1 coat varies with every change in the productiveness of weaving or tailoring".[1]

In what sense can labour-value be "crystallised", "embodied" or "expressed" in the body of a commodity if the amount of the value-substance is sensitive to a change in the productivity of labour?

6.3 Labour-value is a field property

An object with mass has a weight in virtue of its relations to the gravity field in which it is embedded. Weight cannot be found 'in' an object, no matter how closely we examine it; nonetheless weight is a measurable property of an object necessary to explain its motion. Although weight is a property of an individual mass its 'weightiness' derives from the context in which the mass is placed. Change the surrounding gravity field, for example by transporting the object to the moon, and the very same mass has a different weight. Let's call this kind of property a 'field property'.

Marx ([1867] 1954, p. 62–63) explicitly draws an analogy with weight to illustrate how labour-value supervenes upon a social practice. The analogy also helps to illustrate the nature of the 'embodiment' of the value-substance. A commodity with use-value acquires a labour-value in virtue of its relations to a system of generalised commodity production in which it is embedded. Labour-values cannot be found 'in' commodities; nonetheless labour-values are measurable properties of commodities

[1]Marx repeatedly states that labour-values are not determined by historical labour costs. For example, he writes "...the value of a commodity is determined not by the quantity of labour actually objectified in it, but by the quantity of living labour necessary to produce it" Marx ([1867] 1954). And "... the value of commodities is determined not by the labour-time originally taken by their production, but rather by the labour-time that their reproduction takes, and this steadily decreases as the social productivity of labour develops" Marx ([1894] 1971).

necessary to explain the 'motion' of their prices. If the technical conditions of production should change, for example due to a change in labour productivity, then labour-values also change. A labour-value, therefore, is also a field property: it is a property of a commodity derived from the economic context in which it is placed.

Linear production theory can help illustrate the field nature of labour-values. The economic context is partially defined by a discrete field that includes the technology, represented by input-output matrix $\mathbf{A}$, and the labour vector, $\mathbf{l}$. The labour-values of commodities are determined by the field; the formula $\mathbf{v} = \mathbf{l}(\mathbf{I} - \mathbf{A})^{-1}$ makes this relationship precise and computable. If the 'field' should change, such as a change in the productivity of labour (from $\mathbf{l}$ to $\mathbf{l}'$) then labour-values change (from $\mathbf{v}$ to $\mathbf{v}'$). The labour-values of any existing inventories are immediately 're-evaluated' since it now costs a different amount of total labour-time to produce that collection of commodities.

In other words, the labour-value of a commodity is *defined* in terms of a technology 'field'. In consequence, a causal agent is simply not required to perform the consequent 're-evaluation' since the change in the labour-value of inventories is a conceptual, not a causal, necessity.

A change in the productivity of labour also modifies the attractor of the economy. An attractor predicts the motion of a system but (normally) is not explicitly represented within the system. So although the 're-evaluation' of labour-values is immediate it only empirically manifests over time in the 'motion' of commodities, such as the movement of market prices, which begin to converge to the new set of labour-values.

Does a mass 'have' a weight? We say it does, even though 'weight' is a field property and, on deeper reflection, is a *relation* between a mass, a gravity field and the laws of Newtonian mechanics. The same is true for labour-value: it is a relation between a use-value, the productivity of labour, and the dynamic laws of motion of capitalist competition, i.e. the "law of value" (Marx, [1867] 1954). In this restricted sense the value-substance is 'crystallised', 'embodied' or 'expressed' in the body of a commodity, and therefore we say that a commodity 'is' or 'has' a labour-value.

We can therefore understand why Marx ([1867] 1954) writes of the "phantomlike objectivity" of the value-substance. A labour-value is a property of a material structure (such as a linen coat) that has physical extent and spatial location. Commodities, therefore, are "mere congealations of human labour" (Marx, [1867] 1954). But labour-value is not a physical property localised within the body of a commodity, like wine poured into a bottle.

Labour-values, therefore, will not respect Mirowski's commonsense notions of physical causality (for the same conclusion, from the perspective of Dialectical Materialism, see Brown (2008)). A change in the amount of value-substance embodied in an existing commodity due to a change in the conditions of production no more requires 'action at a distance' than does the change in status of a married person to a divorced person due to a legal act that happens to occur many hundreds of miles away. Labour-values are an emergent property of a social practice and therefore have a 'social' not a 'physical' reality. The supposed contradiction that Mirowski identifies, between Marx's talk of crystallised-labour and real-costs, is due to Mirowski's misinterpretation of Marx's concept of substance.

6.4 Integration over a field

Mirowski (1989, p. 177) recognises that Marx's theory has an explanatory structure analogous to field theories in the physical sciences. But his physical interpretation of the value-substance prevents him from understanding both Marx's theory and modern formalisations of it. For example, the Leontief inverse in the standard equation for classical labour-values, as we have seen, can be expanded as a infinite series; that is,

$$\begin{aligned} \mathbf{v} &= \mathbf{l}(\mathbf{I}-\mathbf{A})^{-1} \\ &= \mathbf{l}\sum_{n=0}^{\infty}\mathbf{A}^n \\ &= \mathbf{l}(\mathbf{I}+\mathbf{A}+\mathbf{A}^2+\mathbf{A}^3+\dots). \end{aligned} \tag{6.1}$$

It is common in the Marxian literature to interpret each term in the infinite series as representing production that occurred at a particular 'date'. The infinite series then represents a 'process' that occurs in time that, in the limit, reduces all commodities to labour alone (see Chapter 4).

Mirowski is correct to insist that the 'dated' expansion must be interpreted as a instantaneous property of the technique; that is, labour-values are functions of the prevailing technology 'field'. Indeed, Marx consistently used current, not historical, labour costs in his theoretical work (see Moseley (2010) for a survey of the textual evidence). But Mirowski states that "contrary to many modern Marxist writers, this is definitely not the crystallised-labour approach, except under the most counterfactual of circumstances that there has been no change in the entire history of capitalism with regards to the means of production" (Mirowski, 1989, p. 182). This follows since, according to Mirowski's understanding of crystallised-labour the labour is "poured" (Mirowski, 1989, p. 183) into commodities and conserved in their bodies through time, much like a container stores an amount of liquid. Mirowski concludes that equation (6.1) is therefore "simply false" under a substance interpretation because past labour costs must have differed to those prevailing today.

But, as we have established, in Marx's theory the value-substance is not a physical substance stored in the body of a commodity. In fact, the reduction to 'dated' labour is a counterfactual interpretation of the meaning of current day real costs of production. Again, a field analogy can help: we can define the electrostatic potential energy of a charged particle in a field as the work that must be done to move it from an infinite distance away to its present location in the field. Physicists have used this definition to elucidate the meaning of potential energy. But the definition does not imply in any way that the particle was in fact moved through an infinite distance (and, we might add, how could a particle move an infinite distance?).

Labour-values and potential energy are similar in this respect: both are instantaneous properties of 'objects' in a 'field' that have mathematical representations in terms of integrals or sums over fields. The reduction to 'dated' labour does not imply a real process that occurs in historical time. For example, Sraffa, who perhaps first introduced the dated interpretation of the series reduction, is always careful to place 'date' in scare quotes.

Mirowski's argument bears a family resemblance to that of Bose (1980) who argues that abstract labour cannot be the substance of value since the reduction of commodities to labour-costs can never eliminate a commodity residue. No matter how far we go back 'in time' we always find labour combined with commodity inputs. As Keen (2001, p. 289) remarks, if non-labour inputs were entirely eliminated then some commodities would be produced with zero commodity inputs, "or in other words, by magic". Keen considers Bose's logic to be "impeccable" and therefore concludes, with Bose, that economic value cannot be reduced to labour-time. Bose certainly presents an impeccably literal interpretation of the series representation of labour-value accounting.

The infinite series expansion is also a method to compute the amount of coexisting labour supplied to reproduce a commodity (see Chapter 2). This is an instantaneous property of a commodity that denotes the current quantity of labour supplied to the vertically integrated sector that produces it. Here, the concept of historical time or dates simply do not feature.

Rather than being "simply false", as Mirowski suggests, equation (6.1) is a well-defined measure of the total amount of labour 'embodied' in a commodity. The measure has been operationalised in empirical studies, both in the Marxian literature and also in the guise of employment multipliers in the Leontief-inspired input-output literature (e.g., see ten Raa (2005)).

6.5 Substance and field

Mirowski misinterprets Marx's principle of the conservation of value as a claim that a physical value-substance is transported around the economy stored in the body of commodities (Mirowski, 1989, p. 143). So the value-substance, once embodied, cannot subsequently change due to technical revolutions. Since Marx states that labour-values can change then Mirowski (1989, p. 183) only sees contradiction:

> In the crystallised-labour approach, the value substance is necessarily conserved in exchange, with Marx adding the further stricture that value is conserved in the transition between productive input and the output. The value accounts are clear and straightforward, not the least

> because they conform to the previous pattern of classical political economy. When it comes to the real-cost approach, all of the above principles are violated in one or another trans-temporal phenomenon; and Marx was not at all forthcoming about what he intended to put in their place. If we let the mathematical formalism dictate what is conserved, then [the reduction to dated labour expression (6.1)] dictates that it should be the technology that is conserved, for that plays the role of the field in the formalism; but as Marxian economics, this is nonsense.

Mirowksi's contradictions and anomalies vanish once we recognise the difference between local conservation of labour-value and global field changes. For example, in an economy in a steady-state equilibrium, such as those studied in Chapters 2 and 4, both classical and super-integrated labour costs are fixed and the value-substance is conserved in its 'journey' from its source in living labour, via multiple productive transformations, until destroyed in the sink of consumption.

Now consider an exogenous shock that modifies either of the 'field' variables $\mathbf{A}$ or $\mathbf{l}$. Since labour-values are a field concept then, as a conceptual necessity, they change. In consequence, the labour-value of commodities, and therefore the labour-value of stored inventories, and also the total value-substance flowing in the economy, all immediately alter. The technical innovation causes a change in the global field and therefore a global, and discontinuous, re-evaluation of labour-values. However, in this new regime, the value-substance continues to be locally conserved in exchange.

We can push the field analogy further: the magnitude of a flux passing through a surface depends on the surrounding field. If the field changes then so does the flux. But physicists do not therefore reject local continuity equations. They instead develop more complex models, which retain a kernel of local continuity, within the context of time-varying fields. In many respects, Marx's approach in Volume 1 of *Capital* to conservation and non-conservation follows this pattern: he initially assumes the (local) conservation of value only to later introduce a special causal agent, human labour-power, which breaks conservation and produces relative surplus-value.

Marx, like Smith and Ricardo, is well aware that technical change occurs all the time. But to understand the dynamics of a complex system we first need to abstract to keep some elements fixed in order to analyse dynamics that occur at different time scales (also see Foley (2008) for a discussion of the distinctive methodological approach of classical political economy). The assumption of local conservation of a value-substance should be understood in this context.

Mirowski claims that the value theories of Quesnay, Smith, Ricardo and Marx are all "manifestations of a single class of value theory" (Mirowski, 1989, p. 143). But the concept of a physical substance belongs to the Physiocrats, not Marx, who in contrast explicitly emphasises that value is a 'social', and not a 'physical', substance. Mirowski's thesis therefore presents an inadequate understanding of the content and intent of Marx's theory. A better analysis of the relationship between Marx and his precursors, including the Physiocrats, is given by Marx himself, in his posthumous *Theories of Surplus Value* (Marx, 2000).

Mirowski forces Marx to choose between a prosaic substance *or* "nascent" field theory of value in order to avoid a contradiction. But Marx, if we are prepared to read his text carefully, presents a remarkably sophisticated and consistent substance *and* field theory of value,aspects of which can be precisely formulated in the modern language of field theory. In the next chapter I examine how the classical theory of gravitation of market to natural prices can be formalised in terms of dynamical systems theory.

Chapter 7

The general law of value

My formal analyses of the classical theory, in Chapters 2 to 5, employed linear production theory to examine steady state or growing economies in natural price equilibrium. These models implicitly assume that the dynamics of capitalist economies cause market prices to converge toward natural prices. This final chapter of the thesis drops this assumption and applies the formal tools of dynamic systems theory to examine the dynamics of capitalist competition.

The chapter is divided into three parts. First, I discuss various methodological preliminaries; second, I present the formal model and analyse its dynamics and equilibrium point; and third, I discuss the implications for our understanding of the classical law of value.

7.1 Methodological preliminaries

The coordination of millions of independent production activities in a large-scale market economy is neither perfect nor equitable but nonetheless "one should be far more surprised by the existing degree of coordination than by the elements of disorder" (Boggio, 1995). The classical authors developed a theoretical framework in which this surprising fact could be understood.

7.1.1 The classical mechanism of gravitation toward natural prices

The classical authors, such as Smith and Marx, explained that economic coordination is the unintended consequence of the self-interested decisions of economic actors engaged in competition (e.g., Smith's "invisible hand" and Marx's "law of value"). Capitalists, who seek the best returns on their investments, withdraw capital from unprofitable sectors and reallocate it to profitable sectors. The injection (resp. withdrawal) of capital increases (resp. decreases) the supply of product to the market, which functions to match supply to demand, and therefore tends to reduce (resp. increase) market prices and sector profits. In essence the scramble for profit eliminates arbitrage opportunities until supply equals effective demand, a general or uniform profit-rate prevails across the whole economy, and capitalists lack any incentive to reallocate their capital. The classical claim that the market prices of reproducible commodities gravitate toward or around their natural prices (e.g., Smith ([1776] 1994), Book 1, Chapter VII) or "prices of production" (Marx, [1894] 1971) depends on this kind of account of the dynamics of capitalist competition.

For example, Ricardo's statement that natural prices are relatively stable prices robust to "accidental and temporary deviations" (Ricardo, [1817] 1996) between supply and demand, which manifest when quantities supplied equal quantities demanded, only makes sense in the context of the homoeostatic properties of capitalist competition. For example, Marx ([1894] 1971, pg. 366) writes,

> "the general rate of profit is never anything more than a tendency, a movement to equalise specific rates of profit. The competition between capitalists – which is itself this movement toward equilibrium – consists here of their gradually withdrawing capital from spheres in which profit is for an appreciable length of time below average, and gradually investing capital into spheres in which profit is above average."

On this view, market prices are short-term, out-of-equilibrium prices that arise from imbalances between supply and demand whereas natural prices are long-term, equilibrium prices that derive from the objective conditions of production. For example, Ricardo ([1817] 1996) writes,

> "It is the cost of production which must ultimately regulate the price of commodities, and not, as has often been said, the proportion between supply and demand: the proportion between supply and demand may,

> indeed, for a time, affect the market value of a commodity, until it is supplied in greater or less abundance, according as the demand may have increased or diminished; but this effect will only be of temporary duration."

This vision of the homoeostatic kernel of capitalist competition is more-or-less shared by all the classical economists. However, they did not develop formal dynamic models. Marx perhaps went furthest by embarking on a close and extensive study of the calculus (Marx, 1983; Alcouffe and Wells, 2009), since he believed that differential equations held the promise of "determining the main laws of capitalist crisis" (Marx, Letter to Engels, May 31, 1873, quoted by Kol'man and Yanovskaya (1983)). Yet Marx's formal models remained small-scale numerical examples of simultaneous equations (e.g., his Volume 2 reproduction schemes (Marx, 1974); see Trigg (2006) for a modern elaboration) or numerical examples of two-step iteration (e.g., his Volume 3 discussion of the transformation of values to prices of production; see Shaikh (1977) for a modern elaboration). Marx did not apply the differential calculus to develop dynamic models of capitalist competition.

The theory of gravitation, as promulgated by the classical authors, therefore remains an informal theory. And without a formal, causal analysis we cannot be confident that the classical account constitutes a logically coherent explanation of some of the aspects of economic coordination that we empirically observe in actual capitalist economies.

This uncertainty undermines related parts of classical economic theory. For example, the classical theory of value is fundamentally based on a claimed relationship between natural prices, which market prices supposedly gravitate toward, and objective costs-of-production. For example, Smith, Ricardo and Marx all claim that, in the absence of profit on capital and rent on land, then market prices gravitate to natural prices proportional to labour-values (e.g., see Chapter 2 and Wright (2008)). Marx further argues that, under capitalist conditions with profit on capital, then prices tend to gravitate toward profit-equalising prices of production that, although not proportional to, are nonetheless constrained by, and conservatively related to, classical labour values (see section 2.3). If the classical mechanism of gravitation cannot or does not exist then such statements would seem irrelevant to our understanding of economic reality. A formal demonstration of the causal claims of the classical theory of gravitation is therefore a necessary precondition, or premise, of the classical theory of value.

7.1.2 Mechanisms and laws

The classical account of gravitation attempts to identify a mechanism of capitalism (competitive reallocation of capital in search of returns) that generates a lawful regularity (the tendency of market prices to gravitate toward natural prices at which point supply equals demand). The classical authors would not expect that empirical prices, at any particular time of observation, would actually correspond to natural prices. How, then, should we understand the law-like claims of the classical theory?

Roy Bhaskar, in *A Realist Theory of Science* (1997), introduces important distinctions between the real (the domain of causal agents, or mechanisms, that exist), the actual (events that take place in virtue of the action of mechanisms) and the empirical (events observed or sensed by human beings). Empirical reality, according to Bhaskar, is jointly determined by the resultant effect of the complex interactions of multiple mechanisms. The main purpose of a scientific experiment, in the physical sciences, is to prevent, constrain or control for the action of some subset of mechanisms in order to empirically observe the action of a specific mechanism of interest in isolation. Scientists intervene in reality in order to allow a hidden or underlying mechanism to exclusively cause the empirical data they collect. For example, scientists designed experiments and built specialised apparatus to hold other interfering factors constant (such as pressure etc.) in order to identify a law of thermal expansion. On this view, scientific laws are not reducible to constant conjunctions of events, but instead denote the actions of enduring mechanisms, which may not always manifest their behaviour in empirically straightforward or detectable ways.

An economy is a complex system constituted by multiple mechanisms. However, in the social sciences we normally lack the causal powers to intervene and hold factors constant. So we are forced to take a more indirect route, and imagine doing so, by adopting counterfactual assumptions that perform a theoretical 'experiment'. For example, Marx in volume 1 of *Capital*, counterfactually assumes that prices are proportional to labour-values and, in volume 3, especially during his discussion of the transformation, he assumes that prices converge to a stable set of natural prices, which implies that other disturbing factors, such as technological change, are either absent or constant.

The causal consequences of a counterfactual assumption, in contrast to an experimental intervention, cannot be immediately verified by experience. So the route from theory to verification is more indirect and error prone. Nonetheless we can, in principle, construct more complete models, from combinations of fundamental and simpler mechanisms identified via counterfactual assumptions, to finally test against empirical reality. For example, Marx (1993b, pp. 100–101) writes, in the context of discussing "the method of Political Economy" that:

> "if I were to begin with the population, this would be a chaotic conception of the whole, and I would then, by means of further determination, move analytically towards ever more simple concepts, from the imagined concrete towards ever thinner abstractions until I had arrived at the simplest determinations. From there the journey would have to be retraced until I had finally arrived at the population again, but this time not as the chaotic conception of a whole, but as a rich totality of many determinations and relations".

In other words, postulate simpler mechanisms, understand how they work in isolation, and then combine them to understand how they interact. At this point, it may be possible to verify the final theory against data.

The classical analysis of gravitation more or less consciously shares the same methodological basis. The classical authors understood that many factors, not least turbulent and ceaseless technical change, continually alter the natural price equilibrium before the economy has time to converge. However, in order to make theoretical progress, and to attempt to understand the homoeostatic kernel of capitalist competition, we must adopt counterfactual assumptions. For example, in this chapter, I assume a fixed technology throughout, which controls for the interfering effects of the mechanisms of technological progress. In addition, I assume constant returns to scale, which controls for the changes of technique that necessarily arise from changes in the scale of production.

An economy is never in a state of natural price equilibrium. However, this empirical state-of-affairs, in itself, does not refute the classical theory of gravitation. For example, we can prevent the mercury column of a thermometer from reaching thermal equilibrium by quickly and repeatedly submerging it in liquids at different temperatures. In this situation, the thermometer always fails to measure the temperature of its surroundings. But this empirical phenomenon does not refute the law of thermal equilibrium. It simply indicates that another mechanism (in this case our intervention) has prevented it from fully manifesting. Although equilibrium is never attained the thermometer's reading, at all times, converges to the temperature of its surrounding liquid.

The law-like claims of the classical theory should be understood in a similar manner: in economic reality market prices do not realise their natural price equilibrium; and, furthermore, the conditions that define that equilibrium also change. Nonetheless, the classical theory claims that the empirical trajectory of market prices is partially controlled by the mechanism of capitalist competition, and therefore a component of the change in empirical prices is explained by the convergence of market prices toward the current natural price equilibrium.

Marx ([1894] 1971, Ch. 48) remarks that "all science would be superfluous if the outward appearance and the essence of things directly coincided". In this chapter I attempt to identify the essence of a mechanism. This is necessary, but not sufficient, step towards confronting empirical reality.

7.1.3 Cross-dual adjustment

The development of nonlinear dynamic models of economic processes is relatively neglected in all areas of economics, including Post Keynesian analysis. Modern analyses of the classical theory of value unfortunately are no different in this respect and have been mainly conducted in terms of equilibrium models that implicitly assume gravitation has operated to completion. However, as Keen (1998, p. 86) remarks, "if our understanding of capitalism is to be extended beyond that developed by Sraffa and Kalecki, both Sraffians and Kaleckians must work to develop an analysis which acknowledges the dynamical, multisectoral, behavioural foundations of capitalism". This remark also applies to modern Marxian theory.

Since the 1950's a small number of economists have applied dynamical systems theory in order to analyse the classical theory of gravitation. Morishima (e.g., 1990, p. 84) characterises the classical adjustment process as "cross-dual". The process is "dual" because adjustment includes simultaneous changes in both prices and quantities, and "cross" because imbalances between quantities supplied and demanded entail price changes, and imbalances between costs and revenues entail quantity changes (see Figure 7.1).

The term "cross-dual" also serves to demarcate the classical process of gravitation from neoclassical tâtonnement that occurs 'out of time' with pure price adjustment and infinitely fast, or instantaneous, quantity adjustment (e.g., Varian (1992, pg. 398) and Tuinstra (2001), and also see Flaschel, Franke, and Semmler (1997, ch. 2), Flaschel (2010) and Salvadori and Signorino (2013) for discussion of the differences). In the overwhelming majority of cases, neoclassical 'dynamic' general equilibrium analysis stipulates instantaneous adjustment to equilibrium states and therefore remains "barren and irrelevant as an apparatus of thought to deal with the manner of operation of economic forces" (Kaldor, 1972) since it assumes away the very adjustment processes that actually constitute their content.[1]

A key question asked by modern studies is whether formal dynamical models of cross-dual adjustment converge toward a stable equilibrium (see Steedman (1984) for an early survey and also more recently Flaschel (2010)). The main obstacle to this kind of work is the analytical intractability of large-scale systems of nonlinear differential equations. A rich mathematical theory of dynamics exists but it does not provide fully general, and automatic, methods for solving and analysing such

[1]An interesting exception is the work of Mas-Collel (1986) who formalises Walras' description of cross-dual dynamics for a production economy. Much of modern macroeconomic theory posits 'representative agents' that solve optimisation problems with infeasible computational and information resources. Wright (2009) offers an alternative approach.

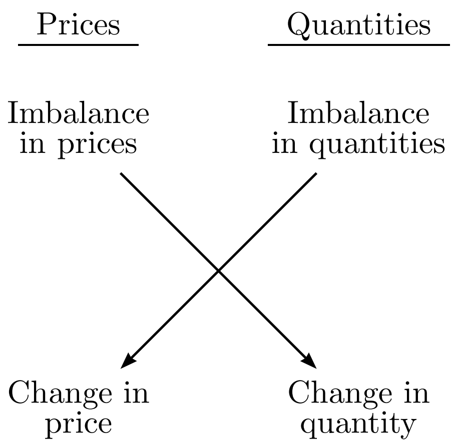


Figure 7.1: Classical cross-dual adjustment

systems, especially their stability properties.

The analytical results in the literature are mixed, ranging from instability (i.e., lack of convergence) to stability (i.e., bounded orbits around natural prices) to asymptotic stability (i.e., convergence to natural prices). The mixed stability results reflect the variety of models developed under the rubric of cross-dual dynamics (e.g., see the collection edited by Semmler (1985)). For example, Dupertuis and Sinha (2008) claim that, in the context of zero growth, a 'centre of gravitation' cannot be an attractor for market prices for "all such possible [price and quantity adjustment] mechanisms"; however, their argument does not quantify over all possible adjustment mechanisms. In contrast, Flaschel (2010, ch. 15) proves that a cross-dual model of an economy on a balanced growth path – with constant returns to scale, joint-production and a constant composition of demand – is stable. If the adjustment rules are modified so that capital reallocation takes into account the rate of change of profit (rather than simply the size of profit) then the model is globally asymptotically stable. Bellino and Serrano (2011) summarise that the cross-dual models formulated in the 1980's and 90's basically demonstrate that pure cross-dual models are "intrinsically unstable" unless various modifications are introduced.

This chapter introduces a new variant of pure cross-dual dynamics that demonstrates that the classical theory of gravitation provides a successful and logically coherent explanation of the homoeostatic kernel of capitalist competition. The analysis is further evidence in favour of "Garegnani's (1990) and Serrano's (2011) more positive view that the classical principle of competition through capital mobility is enough to ensure gravitation under quite general conditions concerning technology and effectual demands" (Bellino and Serrano, 2011).

7.2 A nonlinear dynamic model of classical macrodynamics

The $n \in \mathbb{Z}^+$ sectors consist of competing firms that specialise in the production of the same commodity type. As in previous chapters, the technique is a non-negative $n \times n$ input-output matrix, $\mathbf{A} = [a_{i,j}]$. Each $a_{i,j} \geq 0$ is the quantity of commodity i directly required to output 1 unit of commodity j. Assume that matrix $\mathbf{A}$ is fully connected, $\mathbf{I} - \mathbf{A}$ is of full rank and there exists a vector $\mathbf{x}^{\mathrm{T}} \in \mathbb{R}^n_+$ such that $\mathbf{x}^{\mathrm{T}} > \mathbf{A}\mathbf{x}^{\mathrm{T}}$; that is, the technique is productive. The direct labour coefficients are a $1 \times n$ vector, $\mathbf{l} = [l_i]$. Each $l_i > 0$ is the quantity of labour directly required to output 1 unit of commodity i. Assume constant returns to scale; $\mathbf{A}$ and $\mathbf{l}$ are therefore fixed throughout.

Each commodity type has a single market price denoted by the $1 \times n$ vector $\mathbf{p}(t) = [p_i(t)]$. The scale of output in each sector is the $1 \times n$ vector $\mathbf{q}(t) = [q_i(t)]$. (For notational convenience I often omit explicit time parameters).

The constant L denotes the size of the available labour force (which, at any time, may not be fully employed) and the constant M denotes the total nominal value of the stock of fiat money that circulates in exchange. These are the only non-reproducible, fixed resources. Note that, in this model, money is a first-class object in the sense that M denotes a quantity of nominal value in the material form of a stock of physical assets (e.g., paper or digital money) that is exchanged.

7.2.1 Worker households

Worker households earn wage income by selling their labour to firms. They spend a fraction of their money wealth on the real wage.

Workers' propensity to consume

The instantaneous stock of money held by worker households (their aggregate 'bank balances') is $m_w(t)$. Workers have a constant propensity to consume the fraction, $\alpha_w \in (0,1]$, of this sum in the goods market[2]. The aggregate expenditure of worker households is $\alpha_w m_w$.

The real wage

For simplicity assume the aggregate real wage is always sufficient to ensure the reproduction of the available labour force, L. The composition of the real wage is constant but its scale varies (i.e, all goods are perfect complements in the aggregate). The $1 \times n$ real wage vector, $\mathbf{w}(t) = [w_i(t)]$, has a constant composition defined by the $1 \times n$ wage composition vector $\underline{\mathbf{w}} = [\underline{w_i}]$, and a variable scale; that is $\mathbf{w} = k\underline{\mathbf{w}}$ for some scale factor $k > 0$.

The fraction $\alpha_w m_w / \mathbf{p}\underline{\mathbf{w}}^{\mathrm{T}}$ denotes the number of real wage bundles of composition $\underline{\mathbf{w}}$ that can be purchased at money prices $\mathbf{p}$. The real wage is therefore

$$\mathbf{w} = \frac{\alpha_w m_w}{\mathbf{p}\underline{\mathbf{w}}^{\mathrm{T}}} \underline{\mathbf{w}}^{\mathrm{T}},$$

where $k = \alpha_w m_w / \mathbf{p}\underline{\mathbf{w}}^{\mathrm{T}}$ is the variable scale factor. Composition vector $\underline{\mathbf{w}}$ defines a ray in commodity space that the real wage traverses. Given a constant aggregate expenditure lower (resp. higher) prices imply higher (resp. lower) real consumption.

Workers' money stocks

The level of employment, at any time, is $\mathbf{l}\mathbf{q}^{\mathrm{T}}$. The stock of workers' money, or 'savings', m_w, increases due to an inflow of wage income, $\mathbf{l}\mathbf{q}^{\mathrm{T}} w(t)$, where $w(t)$ is the money wage rate, and decreases due to an outflow of expenditures, which is the fraction $\alpha_w m_w$ spent on the real wage. The rate of change of money stocks is there-

[2]This definition improves upon the more familiar Keynesian propensity to consume, which is normally defined as the ratio of consumption to income in the context of static equilibrium models (Keynes, [1936] 1997, Book 3, ch. 8). In this dynamic model the stock of money held by workers fluctuates according to the difference between the flow rates of expenditure and income. Hence, 'propensity to consume' is a function of workers' 'bank balances' not their current income. (In pure equilibrium models, of course, such distinctions collapse).

fore the difference between income and expenditure,

$$\frac{\mathrm{d}m_w}{\mathrm{d}t} = \mathbf{l}\mathbf{q}^{\mathrm{T}} w - \alpha_w m_w. \tag{7.1}$$

Hence $\frac{\mathrm{d}m_w}{\mathrm{d}t} > 0$ indicates 'saving' and $\frac{\mathrm{d}m_w}{\mathrm{d}t} < 0$ indicates 'dis-saving'.

The labour market

Marx ([1847] 2008, p. 5), for example, states that the wage rate fluctuates with the supply and demand of labour: "the same general laws which regulate the price of commodities in general, naturally regulate wages, or the price of labour-power. Wages will now rise, now fall, according to the relation of supply and demand, according as competition shapes itself between the buyers of labour-power, the capitalists, and the sellers of labour-power, the workers".

Assume, therefore, that the wage rate, w, given a fixed working population L, varies with the demand for labour. So an increase (resp. decrease) in the level of employment, $\mathbf{l}\frac{\mathrm{d}\mathbf{q}^{\mathrm{T}}}{\mathrm{d}t} > 0$ (resp. < 0) causes a relative wage increase (resp. decrease); that is $\frac{1}{w}\frac{\mathrm{d}w}{\mathrm{d}t} \propto \mathbf{l}\frac{\mathrm{d}\mathbf{q}^{\mathrm{T}}}{\mathrm{d}t}$. In addition, as the level of employment rises, and the labour market tightens, the wage rises until, in the limit, it approaches ∞ at the hypothetical maximum of full employment; that is, $\frac{1}{w}\frac{\mathrm{d}w}{\mathrm{d}t} \propto \frac{1}{L - \mathbf{l}\mathbf{q}^{\mathrm{T}}}$ (i.e., no extra labour resources can be hired at any price). Combining these two factors we get

$$\frac{\mathrm{d}w}{\mathrm{d}t} = \eta_w \mathbf{l}\frac{\mathrm{d}\mathbf{q}^{\mathrm{T}}}{\mathrm{d}t}\frac{1}{L - \mathbf{l}\mathbf{q}^{\mathrm{T}}} w, \tag{7.2}$$

where $\eta_w > 0$ is a constant elasticity of the wage rate with respect to unemployment. We adopt, therefore, a Phillips-like (1958) description of the labour market such that the change in the wage rate depends both on the level of employment and the rate of change of employment.

7.2.2 Capitalist households

Capitalist households earn interest income, in their role as lenders of money-capital, and receive profit-of-enterprise, in their role as owners of firms. They spend a fraction of their money wealth on consumption goods.

Capitalists' propensity to consume

The instantaneous stock of money held by capitalist households is $m_c(t)$. Capitalists spend a constant fraction, $\alpha_c \in (0,1]$, of this sum. The aggregate expenditure of capitalist households is $\alpha_c m_c$.

Capitalists' consumption

Aggregate capitalist consumption is specified in a similar manner to worker consumption. The fraction $\alpha_c m_c / \mathbf{p}\underline{\mathbf{c}}^{\mathrm{T}}$ denotes the number of bundles of composition $\underline{\mathbf{c}}$ purchased at prices $\mathbf{p}$. Capitalist consumption is therefore

$$\mathbf{c}(t) = \frac{\alpha_c m_c}{\mathbf{p}\underline{\mathbf{c}}^{\mathrm{T}}}\underline{\mathbf{c}}.$$

Interest income

Marx, in Volume 3 of *Capital*, outlines an abstract specification of the economic relations between capitalists and firms. He splits the capitalist class into two functional roles: finance capitalists or "money-capitalists" who lend money at interest to fund production, and industrial capitalists who, as owners and managers of firms, borrow money to expand production in order to gain "profit of enterprise".[3] Total profit therefore breaks down into two different kinds of profit income: interest and profit of enterprise (or simply 'profit').

Firms finance their production from a wide variety of funding sources, such as internal profits, short-term overdrafts, loans of different duration with fixed and variable rates of interest, and longer-term sources, such as bonds and equity. For simplicity I ignore this complexity. Instead, assume that firms in sector i finance their costs of production, $\kappa_i(t) = (\mathbf{p}\mathbf{A}^{(i)} + l_i w)q_i$ (comprising the cost of input goods and labour given the current level of output) by borrowing money-capital from finance capitalists. Finance capitalists receive interest payments on the money-capital

[3]This functional division encompasses situations where the same individual performs both roles either in a single firm or over multiple firms; for example Marx ([1894] 1971, pg. 373) writes that "the capitalist operating on his own capital, like the one operating on borrowed capital, divides the gross profit into interest due to himself as owner, as his own lender, and into profit of enterprise due to him as to an active capitalist performing his function".

currently 'tied-up' in production on a continuous ('daily') basis, at a varying, instantaneous interest-rate $r(t)$. The interest income received by money-capitalists from sector i is therefore, $\psi_i(t) = \kappa_i r$. The aggregate interest income of capitalists is then

$$\sum_{i=1}^{n} \psi_i = \sum_{i=1}^{n} \kappa_i r = (\mathbf{pA} + \mathbf{l}w)\mathbf{q}^{\mathrm{T}} r.$$

The volume of money-capital supplied, $\sum \kappa_i$, is the outstanding principal currently 'tied up' in production. Note that the phrase 'supply of money-capital' does not refer to an occurrent supply of money but to the provision of loan services, i.e. the maintenance of a creditor-debtor relationship between finance and industrial capitalists. Assume, for simplicity, that the supply of money-capital does not incur direct labour costs, such as the labour of managing and servicing loans.

Industrial capitalists continually revise their borrowing requirements as economic conditions change, thereby altering the aggregate level of borrowing. An increase in the level of borrowing, $\frac{\mathrm{d}\kappa_i}{\mathrm{d}t} > 0$, denotes a new supply of fiat money from finance capitalists to firms in sector i; and a decrease in the level of borrowing, $\frac{\mathrm{d}\kappa_i}{\mathrm{d}t} < 0$, denotes a repayment of principal from firms to finance capitalists.[4]

The total money-capital supplied to production is therefore independent of the stock of money in circulation; for example, Marx ([1894] 1971, pg. 510) writes that "*Prima facie* loan capital always exists in the form of money, later as a claim to money, since the money in which it originally exists is now in the hands of the borrower in actual money-form. For the lender it has been transformed into a claim to money, into a title of ownership. The same mass of actual money can, therefore, represent very different masses of money-capital".

Profit-of-enterprise

Industrial capitalists, as owners of firms, receive profit-of-enterprise, which is the residual income that remains once all firm costs are paid from revenue.

[4] In this model, therefore, the stock of fiat money is an exogenous constant but the volume of outstanding loans to industrial capitalists is an endogenous variable. Fractional reserve banking is absent; hence the granting of a loan is an actual transfer of fiat money that creates new debt but does not create new commercial bank money.

A firm's costs include its costs of production (capital and labour inputs), κ_i, plus the interest payments to service its outstanding debt to money-capitalists, $\kappa_i r$. The total costs of production in sector i, including the cost of borrowing, are therefore $\kappa_i(1+r)$.[5]

The classical process of gravitation is partly an explanation of how markets may clear and therefore we cannot simply assume market clearing at all points in time, as is standard in many economic models. In general the demand for commodity i does not equal its supply.

The real demand for a commodity has two components: demand from other sectors and demand from households. The demand from sectors, $\mathbf{A}_{(i)}\mathbf{q}^{\mathrm{T}}$, is a function of the technique and the current scale of production. The demand from capitalist households is the ith component of capitalist consumption, c_i; and the demand from worker households is the ith component of the real wage, w_i. The total demand for commodity i is then $d_i = \mathbf{A}_{(i)}\mathbf{q}^{\mathrm{T}} + w_i + c_i$. The total revenue of sector i is then the price of its sold output, $p_i d_i$.

We can now construct a profit function. The current profit (or loss) in sector i is the difference between total revenue and total cost; that is

$$\pi_i(t) = p_i d_i - \kappa_i(1+r). \tag{7.3}$$

Capitalists' money stocks

The instantaneous stock of capitalist money wealth, or 'savings', m_c, consist of the aggregate money holdings of finance and industrial capitalists. The stock of money is augmented by an inflow of profit – consisting of total interest income, $\sum_{i=1}^{n} \psi_i$, and total entrepreneurial profits (or losses), $\sum_{i=1}^{n} \pi_i$ – and reduced by an outflow of consumption spending, which is the fraction of savings, $\alpha_c m_c$ spent on consumption

[5] Vickers (1987) analyses the capital structure of firms, in particular the partial financing of production by debt capital. He defines "money capital requirement coefficients" as the amount of money-capital required to finance a unit of 'factor capacity'. In an economy with pure circulating capital and production entirely financed by borrowing then unit costs of production κ_i/q_i are also "money capital requirement coefficients", measured in units of nominal debt per unit output, and denote the amount of money-capital currently required to finance unit output of commodity i.

bundle **c**. The change in money stock is the sum of income minus expenditure; that is,

$$\frac{\mathrm{d}m_c}{\mathrm{d}t} = \sum_{i=1}^{n} \psi_i + \sum_{i=1}^{n} \pi_i - \alpha_c m_c, \tag{7.4}$$

Marx writes that the interest rate is "assumed to be given beforehand, before the process of production begins, hence before its result, the gross profit, is achieved" (Marx, [1894] 1971, pg. 373). Interest payments are an *ex ante* cost of production whereas profit (or loss) is an *ex post* residual. In consequence, profit-of-enterprise, $\sum_{i=1}^{n} \pi_i$, in contrast to interest income, varies in sign and therefore represents either a profit inflow (from firms to industrial capitalists) or a loss-covering outflow (from industrial capitalists to firms).

The interest-rate

Marx, in Volume 3 of *Capital*, adopts a loanable funds theory of the rate of interest.[6] Assume that the stock of loanable funds is the total money stock held by capitalists.[7] Finance capitalists raise the cost of borrowing when their funds decrease because industrial capitalists tend to outbid each other when competing to buy the reduced supply of loans; conversely, finance capitalists lower the cost of borrowing when their funds increase because they tend to underbid each other when competing to sell the increased supply of loans to industrial capitalists. The relative change in the interest-rate is therefore negatively proportional to the relative change in the quantity of loanable funds; that is,

$$\frac{1}{r}\frac{\mathrm{d}r}{\mathrm{d}t} = -\eta_c \frac{1}{m_c}\frac{\mathrm{d}m_c}{\mathrm{d}t}, \tag{7.5}$$

where $\eta_c > 0$ is a constant elasticity of the interest rate with respect to the stock of loanable funds. Equation (7.5) has a cross-dual form: a change in the quantity

[6] "As concerns the perpetually fluctuating market rate of interest, however, it exists at any moment as a fixed magnitude, just as the market-price of commodities, because in the money-market all loanable capital continually faces functioning capital as an aggregate mass, so that the relation between the supply of loanable capital on one side, and the demand for it on the other, decides the market level of interest at any given time" (Marx, [1894] 1971, pg. 366).

[7] Hence, workers savings do not contribute to the stock of loanable funds.

of loanable funds causes a corresponding change in the price of money-capital. The interest rate therefore varies with the scarcity (or abundance) of the total stock of loanable funds.

Note, however, that the stock of loanable funds may turn over multiple times to support very different quantities of outstanding debt. Hence any level of demand for loans, at the given interest rate, may in principle be supplied.

7.2.3 Firms

Firms buy inputs and hire-in labour to produce output that is sold in the market. They strategically adjust the prices they charge and the quantities they produce in response to market conditions.

Inventories

The supply of commodities in general does not equal the demand. In consequence, each sector of production stores a stock of unsold inventories, denoted $s_i(t)$. A mismatch in supply and demand translates into a change in the size of inventories. For example, underproduction relative to demand causes inventories to shrink, whereas overproduction causes inventories to grow. The rate of change of inventories is therefore equal to the excess supply; that is,

$$\frac{\mathrm{d}s_i}{\mathrm{d}t} = q_i - d_i. \tag{7.6}$$

For simplicity assume that commodities are imperishable so unsold inventories are stored indefinitely.[8]

Adjustment of market prices

A sector's overall price and quantity adjustment is the aggregate of the adjustments of the individual firms that comprise it. Firms raise prices when inventories shrink since buyers outbid each other to obtain the scarce product, whereas firms lower

[8] A more general model would allow inventories to be destroyed according to a per sector decay rate. Then the inventory held by service sectors could be interpreted as short-term excess capacity, for example due to the ability of service providers to store intermediate products and work with greater intensity.

prices when inventories grow since firms underbid each other to sell to scarce buyers. The sector as a whole, therefore, adjusts the relative price of its commodity in proportion to excess demand, that is $\frac{1}{p_i}\frac{\mathrm{d}p_i}{\mathrm{d}t} \propto -\frac{\mathrm{d}s_i}{\mathrm{d}t}$. This has a cross-dual form: a quantity imbalance, represented by the change in inventory size, translates into a price adjustment.

Assume that the change in price approaches positive ∞ as inventory approaches zero and the commodity is completely scarce, that is $\frac{1}{p_i}\frac{\mathrm{d}p_i}{\mathrm{d}t} \propto \frac{1}{s_i}$. Combining these two factors we get the price adjustment equation

$$\frac{\mathrm{d}p_i}{\mathrm{d}t} = -\eta_i \frac{\mathrm{d}s_i}{\mathrm{d}t}\frac{p_i}{s_i}, \tag{7.7}$$

where $\eta_i > 0$ is a constant elasticity of price with respect to excess supply. Sectors with small (resp. large) inventories tend to adjust prices relatively quickly (resp. slowly). For simplicity assume that firms do not reduce prices to dump inventory on the market but instead maintain an inventory buffer to manage unpredictable variance in excess demand.

Adjustment of output

Industrial capitalists, as owners and managers of firms, adjust their production plans based on profit and loss. A firm that returns a profit (resp. loss) borrows more (resp. less) money in the market for loanable funds in order to increase (resp. decrease) supply with the expectation of earning greater profit (resp. reducing losses).

Industrial capitalists, as a whole, own a portfolio of firms grouped into sectors that, at any time, make different profits or losses. The profit-rate in sector i,

$$\frac{\pi_i}{\kappa_i(1+r)},$$

is the ratio of profit to production costs, including the cost of money-capital.

The profit-rate is the expected increase of profit-of-enterprise from 1 unit of additional investment of money-capital in sector i, *ceteris paribus*. Capitalists aim to maximise their profit by differentially injecting or withdrawing money investments based on these profit-rate signals. The relative change in the scale of production is therefore proportional to the profit rate, that is $\frac{1}{q_i}\frac{\mathrm{d}q_i}{\mathrm{d}t} \propto \frac{\pi_i}{\kappa_i(1+r)}$. This has a cross-dual form: a price imbalance, represented by the profit rate, translates into a quantity adjustment. In consequence, define the quantity adjustment equation

$$\frac{1}{q_i}\frac{\mathrm{d}q_i}{\mathrm{d}t} = \eta_{n+i}\frac{\pi_i}{\kappa_i(1+r)}, \tag{7.8}$$

where $\eta_{n+i} > 0$ is a constant elasticity of supply with respect to profit. Sectors with a high (resp. low) profit-rate increase (resp. reduce) their borrowing in order to increase (resp. decrease) the supply of goods to the market.

We can also interpret quantity adjustment equation (7.8) in terms of the return on investment,

$$r_i(t) = \frac{p_i d_i - \kappa_i}{\kappa_i},$$

which is the expected return from 1 unit of additional investment of money in sector i prior to its distribution (as interest income or profit-of-enterprise). An equivalent expression for quantity adjustment is then

$$\begin{aligned}\frac{\mathrm{d}q_i}{\mathrm{d}t} &= \eta_{n+i}\frac{q_i}{1+r}(r_i - r) \\ &\propto (r_i - r).\end{aligned} \tag{7.9}$$

Hence, industrial capitalists expand production if the return on investment is greater than the cost of borrowing. The demand for credit from industrial capitalists therefore varies with sectoral returns.

This completes the description of the model in terms of worker and capitalist households and the sectors of production. Next we compose the parts into a single system of equations.

7.2.4 A classical macrodynamic system

First, we simplify the model, and gain further insight by noting the following relationships.

Market prices and scarcity

Solve price adjustment equation (7.7) to obtain market prices as a function of inventory levels; that is,

$$p_i = k_i \frac{1}{s_i^{\eta_i}} \tag{7.10}$$

for all i, where $k_i = p_i(0)s_i(0)^{\eta_i}$. A high market price indicates low inventory. An obvious and natural interpretation of this relationship is that market prices indicate or measure the relative scarcity (or abundance) of a commodity.

Solve wage adjustment equation (7.2) to obtain the wage rate as a function of the level of employment,

$$w(t) = k_w \frac{1}{(L - \mathbf{lq}^{\mathrm{T}})^{\eta_w}}, \tag{7.11}$$

where

$$k_w = w(0)(L - \mathbf{lq}(0)^{\mathrm{T}})^{\eta_w}$$

is a positive constant. The wage rate therefore indicates the scarcity of unemployed labour available for hire.

Solve interest rate adjustment equation (7.5) to obtain the interest rate as a function of the quantity of loanable funds,

$$r(t) = k_r \frac{1}{m_c^{\eta_c}}, \tag{7.12}$$

where $k_r = r(0)m_c(0)^{\eta_c}$. The interest rate is also a market price that indicates scarcity, in this case the scarcity of loanable funds.

These 'scarcity equations' allow us reduce inventories, the wage rate, and the interest rate to functions of prices, quantities and capitalist savings.

Conservation of the money stock

The aggregate expenditure, $\alpha_w m_w + \alpha_c m_c$, varies depending on the distribution of savings between workers and capitalists. Since firms do not hold stocks of money then aggregate expenditure returns to households as income – in the form of wages, interest or profit. The sum of savings, $m_w + m_c$, is therefore always equal to the fixed stock of money.

Lemma 1. *Aggregate savings are constant and equal the total money stock,*

$$\begin{aligned} m_w(t) + m_c(t) &= m_w(0) + m_c(0) \\ &= M. \end{aligned}$$

Proof. Sum equations (7.3) to get

$$\sum_{i=1}^{n} \pi_i = \alpha_w m_w + \alpha_c m_c - \mathbf{lq}^{\mathrm{T}} w - (\mathbf{pA} + \mathbf{l}w)\mathbf{q}^{\mathrm{T}} r. \tag{7.13}$$

Sum equations (7.1) and (7.4) to get

$$\begin{aligned} \frac{\mathrm{d}m_w}{\mathrm{d}t} + \frac{\mathrm{d}m_c}{\mathrm{d}t} =& - \alpha_w m_w - \alpha_c m_c + \\ & \mathbf{lq}^{\mathrm{T}} w + (\mathbf{pA} + \mathbf{l}w)\mathbf{q}^{\mathrm{T}} r + \sum_{i=1}^{n} \pi_i. \end{aligned} \tag{7.14}$$

Substitute (7.13) into (7.14) to get $\frac{\mathrm{d}m_w}{\mathrm{d}t} + \frac{\mathrm{d}m_c}{\mathrm{d}t} = 0$. Hence $m_w(t) + m_c(t) = k$, where k is a constant of integration. At $t = 0$ we have $k = m_w(0) + m_c(0)$. □

The conservation of money in exchange implies a direct trade-off between workers and capitalists over ownership of the stock of money wealth in the economy. What one class gains the other must lose. So although aggregate expenditure always returns as income it nonetheless transfers from one class to another during its circulation and distribution. In general, the aggregate expenditure does not equal the total wage and interest income. The difference is profit-of-enterprise.[9] Lemma 1 allows

[9]Simplify further and assume zero interest income, i.e. $r_0 = 0$. Then (7.13) is $\sum_{i=1}^{n} \pi_i = \alpha_w m_w + \alpha_c m_c - \mathbf{lq}^{\mathrm{T}} w$. So total profit of enterprise is positive if total spending exceeds the total wage bill. If workers spend what they earn then total profit is realised entirely by capitalist consumption ('In point of fact, paradoxical as it may seem at the first glance, the capitalist class itself casts into circulation the money that serves towards the realisation of the surplus-value contained in its commodities' (Marx (1974, Ch. 17) and see also Trigg (2002b)). In such circumstances sector-level losses represent transfers within the capitalist class.

This model supports Keen's point that a fixed stock of base money turns over multiple times to support variable income flows in excess of that stock. The so-called 'paradox of monetary profit' in us to reduce capitalist savings to a function of worker savings, i.e. $m_c = M - m_w$.

A system of nonlinear, ordinary differential equations

We can now reduce the phenomenological model to a system of prices, quantities and the distribution of income.

Definition 12. *The classical macrodynamic system is a* $(2n+1)$*-dimensional system of nonlinear, ordinary differential equations in prices,* $\mathbf{p}(t)$*, quantities,* $\mathbf{q}(t)$*, and workers' savings,* $m_w(t)$*:*

$$\begin{aligned}
\frac{dp_i}{dt} &= -\eta_i k_i p_i^{1-\eta_i}\left(q_i - \mathbf{A}_{(i)}\mathbf{q}^T - \frac{\alpha_c(M-m_w)}{\mathbf{p}\underline{\mathbf{c}}^T}\underline{c}_i - \frac{\alpha_w m_w}{\mathbf{p}\underline{\mathbf{w}}^T}\underline{w}_i\right) && (7.15)\\
\frac{dq_i}{dt} &= \eta_{n+i}\left(\frac{p_i(\mathbf{A}_{(i)}\mathbf{q}^T + \frac{\alpha_c(M-m_w)}{\mathbf{p}\underline{\mathbf{c}}^T}\underline{c}_i + \frac{\alpha_w m_w}{\mathbf{p}\underline{\mathbf{w}}^T}\underline{w}_i)}{(\mathbf{p}\mathbf{A}^{(i)} + l_i\frac{k_w}{(L-\mathbf{l}\mathbf{q}^T)^{\eta_w}})(1+\frac{k_r}{(M-m_w)^{\eta_c}})} - q_i\right) && (7.16)\\
\frac{dm_w}{dt} &= \eta_w k_w \mathbf{l}\frac{d\mathbf{q}^T}{dt}\frac{1}{(L-\mathbf{l}\mathbf{q}^T)^{\eta_w+1}}, && (7.17)
\end{aligned}$$

with $3n+3$ *initial conditions (*$\mathbf{p}(0)$*,* $\mathbf{q}(0)$*,* $\mathbf{s}(0)$*,* $m_w(0)$*,* $w(0)$*,* $r(0)$*),* $2n+2$ *elasticity parameters (*$\boldsymbol{\eta} = [\eta_i]$*,* η_w*,* η_c*), a given stock of money* M *and available labour force* L*.*

7.2.5 Out-of-equilibrium trajectories

Systems of nonlinear differential equations yield closed-form solutions only in special cases. The macrodynamic system is no different. I therefore analyse the trajectories by numerical simulation. The interactive numerical simulation is available for download – see Wright (2014b) and the appendix for details.

An example trajectory

What kinds of dynamics does this model generate? The dynamics are very rich and can only be fully appreciated by experimenting with the parameters of the numerical simulation. However, the following example of a small, 3-sector economy is indicative.

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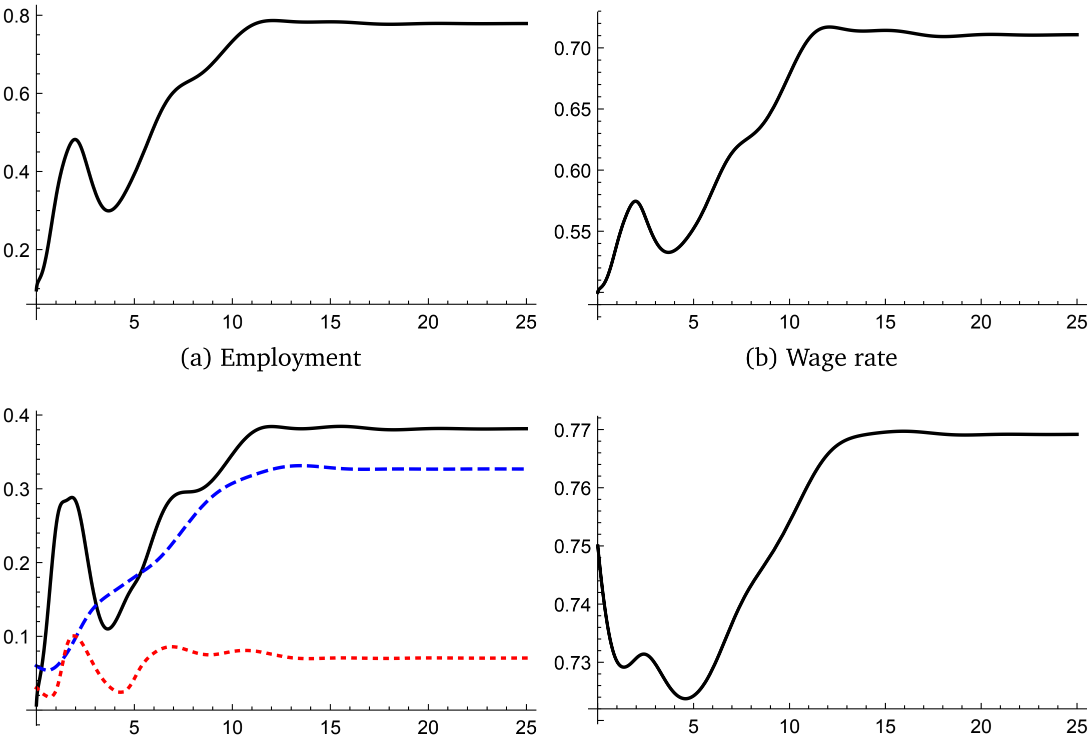
the Circuitist approach Graziani (2003) disappears once sufficient attention is paid to the dynamic relationships between stocks and flows (Keen, 2010).



(a) Employment

(b) Wage rate

(c) Sectoral employment. (Corn: black, iron: dashed, sugar: dots).

(d) Aggregate expenditure

Figure 7.2: Classical gravitation: aggregate expenditure, the labour market and the division of labour.

Our example economy produces corn, iron and sugar, with parameters

$$\mathbf{A} = \begin{bmatrix} 0.2 & 0 & 0.4 \\ 0.2 & 0.8 & 0 \\ 0 & 0 & 0.1 \end{bmatrix},$$

$\mathbf{l} = [0.7, 0.6, 0.3]$, $\mathbf{w} = [0.6, 0, 0.2]$ (workers consume corn and sugar but not iron), $\mathbf{c} = [0.2, 0, 0.4]$ (capitalists proportionally consume more sugar than corn compared to workers), $\mathbf{p}(0) = [1, 0.8, 0.5]$, $\mathbf{q}(0) = [0.01, 0.1, 0.1]$ (the initial supply of corn is relatively low), $\mathbf{s}(0) = [0.01, 0.1, 0.25]$ (the initial stock of corn is relatively low), $w(0) = 0.5$, $r(0) = 0.03$, $m_w(0) = m_c(0) = 0.5$ (worker and capitalist savings are initially equal and the total money stock in the economy is $M = 1$), $\alpha_w = 0.8$ and

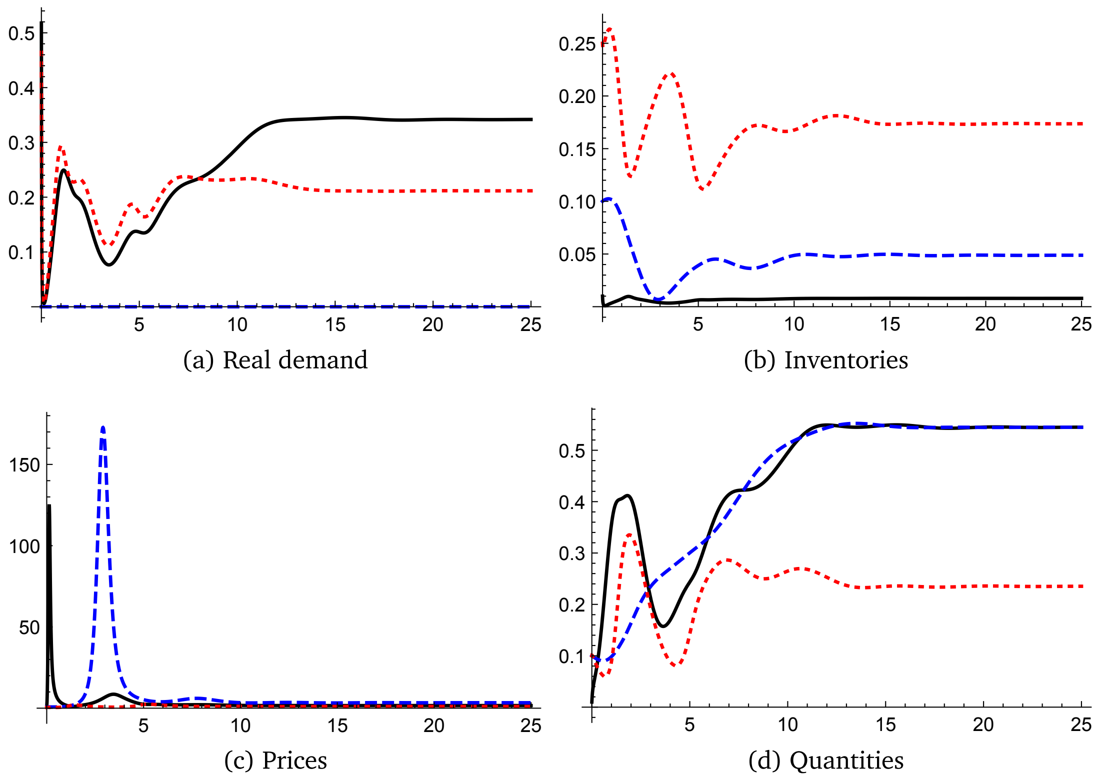
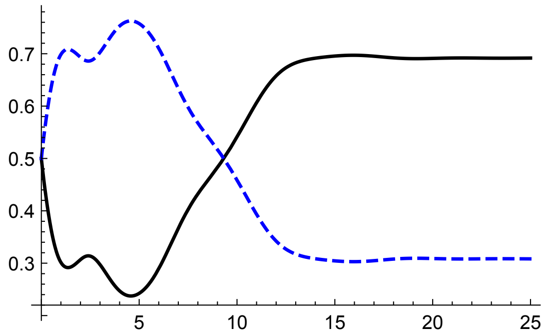


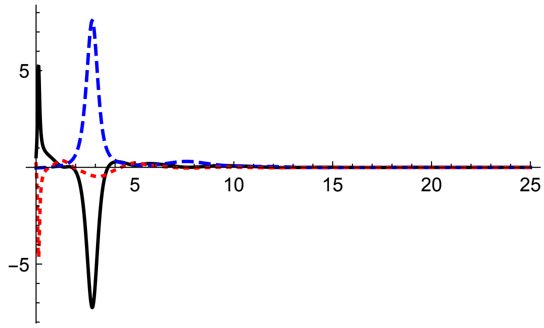
Figure 7.3: Classical gravitation: real demand, quantities supplied and market prices. (Corn: black, iron: dashed, sugar: dots).

$\alpha_c = 0.7$ (workers have a higher propensity to consume), $L = 1$, the price elasticities are $\eta_1 = \eta_2 = \eta_3 = 2$, the quantity elasticities are $\eta_4 = \eta_5 = \eta_6 = 1$, the wage is relatively inelastic, $\eta_w = 0.25$, and the interest rate relatively elastic, $\eta_c = 2$. These parameters generate an economy that follows a growth trajectory until it reaches a self-replacing equilibrium where prices, quantities and the distribution of income are constant over time. Figures 7.2, 7.3, 7.4 and 7.5 graph the trajectories, which we'll now analyse.

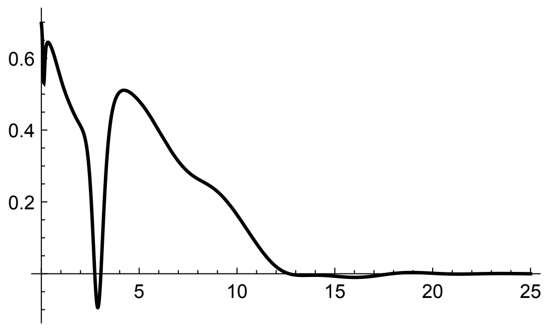
The scale of real demand from households depends on aggregate expenditure and the current price structure. In this example the employment level rises (see Figure 7.2a) because in general real demand outstrips the capacity of the economy to supply commodities in the required amounts (e.g., Figure 7.3b graphs the inventory stock, which initially depletes to satisfy the excess demand). More workers are hired



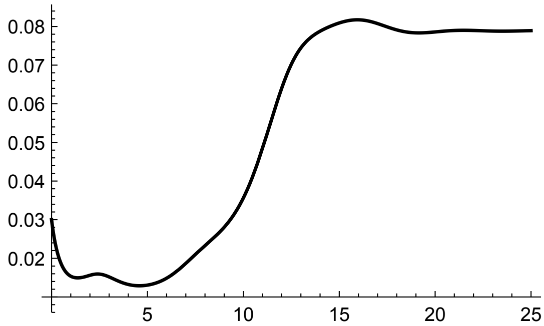
(a) Worker (black) and capitalist (dashed) savings



(b) Profit of enterprise. (Corn: black, iron: dashed, sugar: dots).



(c) Total profit of enterprise



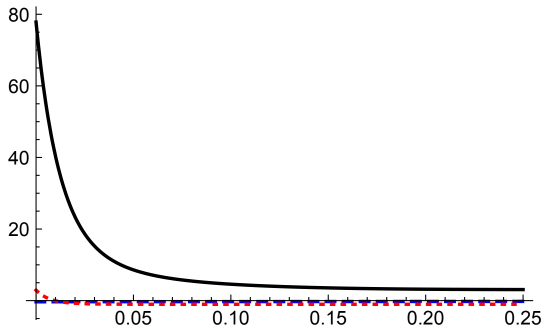
(d) Rate of interest

Figure 7.4: Classical gravitation: money wealth, profits and the interest-rate.

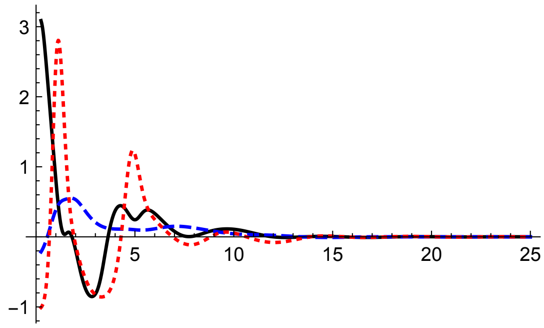
to meet the demand, which causes a corresponding increase in the wage rate, shown in Figure 7.2b. The division of labour adapts (see Figure 7.2c) until in equilibrium the scale and composition of the net product equals real demand, at which point inventory stocks stabilise.

The aggregate expenditure initially falls until at $t \approx 5$ it steadily climbs to its maximum (see Figure 7.2d). The two regimes correspond to a transfer of money wealth to workers who have a higher propensity to consume. Aggregate expenditure returns as income, either in the form of wages, interest or profit. The stock of money is conserved and therefore trajectory of capitalist savings exactly mirrors worker savings (see Figure 7.4a).

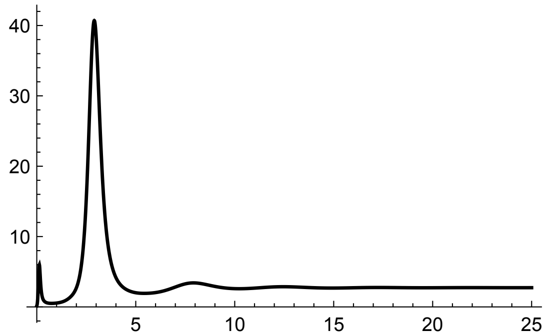
Real consumption hits two lows prior to $t \approx 5$ (see Figure 7.3a) caused by relatively low total household spending and two dramatic price spikes (see Figure 7.3c)



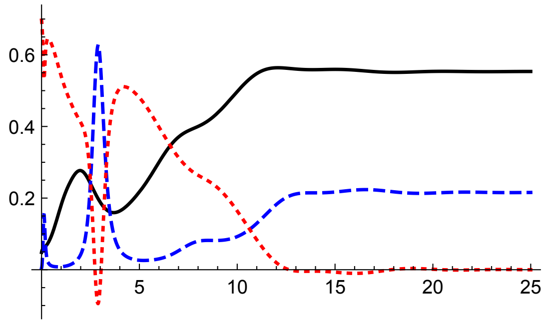
(a) Rate of return minus the interest rate in period $0 \leq t \leq 0.25$ (the initial return in the corn sector is very high). (Corn: black, iron: dashed, sugar: dots).



(b) Rate of return minus the interest rate in period $0.25 \leq t \leq 25$ (the final net return is zero). (Corn: black, iron: dashed, sugar: dots).



(c) Money-capital 'tied up' in production



(d) The distribution of income. (Wages: black, interest: dashed, profit: dots).

Figure 7.5: Classical gravitation: Net rates-of-return, money-capital supplied and the distribution of income.

that function to ration temporarily scarce commodities (corn and iron). But supply adjusts, consumer inflation dissipates, and after $t \approx 5$ real consumption steadily rises. Expansion of output is profit-led. Figure 7.4c plots the total profit of enterprise. In general, total profit is either positive during gravitation or close to zero near equilibrium. The exception is a short period at $t \approx 3$ where losses in the corn and sugar sectors outweigh profit in the iron sector (see Figure 7.4b).

Firms sell inventory to satisfy excess demand. Low inventory causes price spikes. Price spikes tend to raise sectoral profits (compare the price spikes, graphed in Figure 7.3c, with profit of enterprise, graphed in Figure 7.4b). Industrial capitalists can therefore gain a higher return than the cost of borrowing and invest in production (see Figures 7.5a and 7.5b, especially the initial high return in the corn sector). The new funds are used to increase the scale of production (see Figure 7.3d, especially the initial high growth in the corn sector). Figure 7.4b plots total profits per sector, which initially exhibit wide fluctuations, indicating differential returns on money invested, until settling to a uniform zero profit rate at equilibrium, at which point activity levels are stable (Figure 7.3d).

Figure 7.4d graphs the interest rate, which fluctuates with the total stock of loanable funds. Figure 7.5c graphs total loans advanced, which is sensitive to the price structure and the scale of production. Total interest income is a function of the volume of lending and the interest rate. For instance, the high price of iron at $t \approx 3$ increases costs of production and therefore the volume of borrowing, which results in more interest income for rentiers (Figure 7.5d). In the same period profit of enterprise falls (Figure 7.5d). Why is this? A dramatic spike in interest income, given the level of aggregate expenditure, implies less income in the form of profit and wages. Industrial capitalists are subject to a cost-push 'profit squeeze', which at root derives from the relative scarcity of real-capital, specifically iron. The high costs of production throttle growth (see Figure 7.3d at $t \approx 3$). But this contraction is temporary. Labour and real-capital is reallocated to iron production, which increases supply, lowering its price and therefore costs of production in general. The volume of lending falls and profits-of-enterprise recover.

This single example is indicative but does not exhaust the range of dynamics the model can generate.

7.2.6 The equilibrium steady-state

Numerical simulations indicate that the macrodynamic system converges to a locally asymptotically stable, equilibrium steady-state.[10] This section analyses some

[10] I have been unable to formally prove stability for this model apart for the single-sector special-case (available on request from the author) and a restricted multi-sector special-case (see Wright (2011)). The multi-sector proof required the fairly advanced technique of vector Lyapunov functions. However, by varying the functional forms of the cross-dual adjustment equations, and extensive numerical simulation, I am convinced that a general theorem regarding the stability of cross-dual dynamics is waiting to be proved. Such a theorem would be very useful since it would provide a solid theoretical foundation for a research programme of 'nonlinear dynamic production theory'. For those interested

of the properties of the natural price equilibrium. I restrict the analysis to $\mathscr{D} = \{[\mathbf{p}(t), \mathbf{q}(t), m_w(t)] \in \overline{\mathbb{R}}_+^{2n+1} : \mathbf{l}\mathbf{q}^{\mathrm{T}}(t) < L\}$, i.e. economically relevant equilibria.

Zero profit-of-enterprise

Output quantities adjusts according to profit rate differentials. By definition the output is constant in equilibrium. In consequence, the equilibrium profit-of-enterprise is uniformly zero and capitalists have no incentive to reallocate capital.

Lemma 2. *Profit-of-enterprise is uniformly zero in equilibrium, $\pi_i = 0$ for all i.*

Proof. Substitute $\frac{\mathrm{d}q_i}{\mathrm{d}t} = 0$ into quantity adjustment equation (7.8) to get $\pi_i = 0$ for all i. □

Profit-of-enterprise, in a fully competitive system, is a disequilibrium phenomenon that derives from temporary over and under-supply relative to demand. Profit represents an arbitrage opportunity that attracts (and repels) capital investment. But the scramble for profit has the unintended consequence of reducing imbalances between supply and demand, which ultimately eliminates arbitrage opportunities and causes profits to fall.

The zero profit condition is equivalent to the equality of the return on investment and the interest-rate.

Lemma 3. *The equilibrium rate of return in all sectors equals the equilibrium interest-rate, $r_i^* = r^*$ for all i.*

Proof. Substitute $\frac{\mathrm{d}q_i}{\mathrm{d}t} = 0$ into quantity adjustment equation (7.9) to get $r_i^* = r^*$ for all i. □

The equilibrium price structure does not provide an incentive for industrial capitalists to alter their production plans. In reality, of course, other sources of anticipated reward or loss, not included in this model, motivate capitalists to continually change the scale of production.

in tackling this challenge I suspect that dissapativity theory, from the control theory literature, might be a good starting point.

Kalecki's aphorism

Money conservation identity (7.13) implies that the aggregate expenditure returns either in the form of wage income, interest or profit. Since profit-of-enterprise is zero in equilibrium then total income consists of wages and interest.

Lemma 4. *Equilibrium total income is total wages and interest income, which equals the aggregate expenditure:*

$$\mathbf{l}\mathbf{q}^{*T}w^* + (\mathbf{p}^*\mathbf{A} + \mathbf{l}w^*)\mathbf{q}^{*T}r^* = \alpha_w m_w^* + \alpha_c m_c^*.$$

Proof. By Lemma 2, $\pi_i = 0$ for all i. Substitute the zero profit condition into equation (7.13). □

The aggregate expenditure, outside of equilibrium, transfers from one class to another during its circulation (i.e., the distribution of nominal income fluctuates). Equilibrium is simpler: no transfers occur and both workers and capitalists earn what they spend.

Lemma 5. *In equilibrium workers earn what they spend,*

$$\mathbf{l}\mathbf{q}^{*T}w^* = \alpha_w m_w^*$$

Proof. Set $\frac{\mathrm{d}m_w}{\mathrm{d}t} = 0$ in equation (7.1). □

Lemma 6. *In equilibrium capitalists earn what they spend,*

$$(\mathbf{p}^*\mathbf{A} + \mathbf{l}w^*)\mathbf{q}^{*T}r^* = \alpha_c m_c^* \tag{7.18}$$

Proof. Set $\frac{\mathrm{d}m_c}{\mathrm{d}t} = 0$ in equation (7.4) and use the zero profit condition of Lemma 2 to yield the conclusion. □

These properties are an instance of Kalecki's aphorism (Kalecki, 1954, Ch. 3) that capitalists earn what they spend while workers spend what they earn (Trigg, 2006, Ch. 3). Equilibrium profit is entirely composed of capitalist consumption ("In point of fact, paradoxical as it may seem at the first glance, the capitalist class itself casts into circulation the money that serves towards the realisation of the surplus-value contained in its commodities" (Marx, 1974, Ch. 17) and see also Trigg (2002b)).

A positive interest-rate

Interest income, unlike profit-of-enterprise, is not an out-of-equilibrium phenomena but rather a structural feature of production financed by money-capital. In consequence, the interest-rate is positive in equilibrium.

Lemma 7. *In equilibrium, positive capitalist savings, $m_c^* > 0$, imply a positive interest-rate, $r^* > 0$.*

Proof. Equation (7.12) and $m_c^* > 0$ implies $r^* > 0$. □

Unemployment

Lemma 8. *A positive equilibrium wage, $w^* > 0$, implies positive unemployment, $0 < \mathbf{lq}^{*T} < L$.*

Proof. Equation (7.11) implies, $L - \mathbf{lq}^{*\mathrm{T}} = (k_w(\frac{1}{w^*}))^{1/\eta_w}$. If $w^* > 0$ then the RHS of this equation is positive; and hence $L - \mathbf{lq}^{*\mathrm{T}} > 0$. □

Classical macrodynamics therefore generate a typically Keynesian result: market adjustment, including market-driven wage-rates at any level of elasticity and the pooling and lending of loanable funds does not, by itself, guarantee full employment. In equilibrium labour is efficiently allocated across the different sectors of production but the economy does not, in general, operate at full capacity. This is a key macroeconomic coordination problem that is not solved by decentralised, profit-drive resource allocation and market competition. A pure capitalist system does not generate incentives for capitalists to coordinate their plans in order to operate the economy at full capacity.

Production prices

The natural price equilibrium of this nonlinear dynamic model is structurally equivalent to the system of production prices defined in linear production theory:

Proposition 12. *Equilibrium prices in terms of the equilibrium wage, w^*, and interest rate, r^*, are*

$$\mathbf{p}^* = (\mathbf{p}^*\mathbf{A} + \mathbf{l}w^*)(1 + r^*), \tag{7.19}$$

where

$$\begin{aligned} w^* &= k_w \frac{1}{(L - \mathbf{l}\mathbf{q}^{*T})^{\eta_w}} \\ r^* &= k_r \frac{1}{(M - m_w^*)^{\eta_c}}. \end{aligned} \tag{7.20}$$

Proof. Apply the zero profit condition of Lemma 2 to profit function 7.3 to obtain $p_i^* d_i^* = \kappa_i (1 + r^*)$ for all i. Inventory adjustment equation (7.6), with $\frac{\mathrm{d}s_i}{\mathrm{d}t} = 0$, implies $q_i = d_i$. Hence, $p_i^* q_i^* = \kappa_i (1 + r^*)$. Expand, simplify, and write in vector form. Expressions 7.11 and 7.12 define the equilibrium wage and interest rates. □

Equation (7.19) is identical to the standard equation for profit-equalising prices of production with wages paid *ex ante* (i.e., advanced by capitalists) (e.g., see Pasinetti (1977); Kurz and Salvadori (1995); Abraham-Frois and Berrebi (1997).)

In equilibrium costs and revenues are in balance in all sectors and the same uniform rate of profit prevails. This profit however consists entirely of interest income. Profit-of-enterprise, in this deterministic model, is a disequilibrium phenomenon and therefore almost always non-uniform.[11]

Output

Proposition 13. *Equilibrium quantities in terms of the equilibrium net product,* $\mathbf{n}^* = \mathbf{w}^* + \mathbf{c}^*$, *are*

$$\mathbf{q}^* = \mathbf{n}^*(\mathbf{I} - \mathbf{A}^T)^{-1} \tag{7.21}$$

where

$$\mathbf{w}^* = \frac{\alpha_w m_w^*}{\mathbf{p}^* \underline{\mathbf{w}}^T} \underline{\mathbf{w}}$$

[11]The uniform 'profit rate' in equilibrium interpretations of linear production theory should therefore be normally considered an *ex ante* interest-rate that prevails in money-capital markets, and therefore a cost of production, rather than an *ex post* residual.

is the equilibrium real wage and

$$\mathbf{c}^* = \frac{\alpha_c (M - m_w^*)}{\mathbf{p}^* \underline{\mathbf{c}}^T} \underline{\mathbf{c}}$$

is the equilibrium real consumption of capitalists.

Proof. Set $\frac{\mathrm{d}s_i}{\mathrm{d}t} = 0$ in equation (7.6) to get $q_i = d_i$ for all i. Expand, simplify and write in vector form to yield the conclusion. □

Equation (7.21) can be written as $\mathbf{q}^* = \mathbf{q}^* \mathbf{A}^{\mathrm{T}} + \mathbf{n}^*$. Interpret this equation as stating that the equilibrium scale of production consists of the collection of commodities used-up as means of production, $\mathbf{q}^* \mathbf{A}^{\mathrm{T}}$ (the circulating real capital), and the net product, $\mathbf{n}^*$, which is final consumption. The equilibrium activity levels in the economy are therefore determined by the technique and the composition and scale of final consumption. Final consumption, however, is itself determined by aggregate expenditure and the prevailing price structure. The real and monetary aspects of the economy are interdependent.

Equation (7.21) is also structurally equivalent to the standard linear production equation of an economy with circulating capital (e.g., see the discussion of the open Leontief system in Pasinetti (1977, Ch. 4)).

A system of nonlinear simultaneous equations

The equilibrium position – that is equilibrium prices, quantities and the distribution of income – is defined by a system of nonlinear simultaneous equations.

Proposition 14. *The* $2n + 1$ *unknowns – equilibrium absolute prices,* $\mathbf{p}^*$*, quantities,* $\mathbf{q}^*$*, and the distribution of the money stock, represented by workers' savings* m_w^* *– are* *jointly determined by the following* $2n+1$ *system of nonlinear simultaneous equations,*

$$\mathbf{p}^* = \left(\mathbf{p}^*\mathbf{A} + (\frac{\alpha_w m_w^*}{\mathbf{l}\mathbf{q}^{*T}})\mathbf{l}\right)(1 + k_r \frac{1}{(M - m_w^*)^{\eta_c}}) \tag{7.22}$$

$$\mathbf{q}^* = \mathbf{q}^*\mathbf{A}^T + \frac{\alpha_w m_w^*}{\mathbf{p}^*\underline{\mathbf{w}}^T}\underline{\mathbf{w}} + \frac{\alpha_c(M - m_w^*)}{\mathbf{p}^*\underline{\mathbf{c}}^T}\underline{\mathbf{c}} \tag{7.23}$$

$$m_w^* = \frac{k_w}{\alpha_w}\frac{\mathbf{l}\mathbf{q}^{*T}}{(L - \mathbf{l}\mathbf{q}^{*T})^{\eta_w}}. \tag{7.24}$$

Proof. Equilibrium prices are given by Proposition 12. By Lemma 5 the equilibrium wage rate, w^*, can be replaced by $\alpha_w m_w/\mathbf{l}\mathbf{q}^{*\mathrm{T}}$. The equilibrium interest rate, r^*, can be replaced by a function of equilibrium worker savings (equation (7.20)). These replacements yield equation (7.22). Equation (7.23) is given directly by Proposition 13. Lemma 5 gives equilibrium worker savings as $m_w^* = \frac{1}{\alpha_w}\mathbf{l}\mathbf{q}^{*\mathrm{T}}w^*$. Use equation (7.11) to replace the equilibrium wage rate by a function of the equilibrium level of employment to yield equation (7.24). □

Equation system (7.22,7.23,7.24) implicitly defines the 'long-period' position of the economy that would empirically manifest on condition that a subset of the economy's parameters (e.g., technique, propensities to consume, elasticities of distribution, η_w and η_c, composition of demand etc.) remain constant during gravitation. At any arbitrary time $t = 0$ we can measure the out-of-equilibrium initial conditions (e.g., $w(0)$, $\mathbf{q}(0)$, $r(0)$, $m_c(0)$) and then solve the equation system to predict the eventual steady-state.

The equilibrium is entirely independent of initial prices, $\mathbf{p}(0)$, initial inventories, $\mathbf{s}(0)$, the initial distribution of money wealth, $m_w(0)$ and $m_c(0)$, and price and quantity adjustment elasticities, η_i for $i \in [1, 2n]$. As might be expected, out-of-equilibrium scarcity prices of reproducible commodities turn out to be irrelevant not only to the determination of equilibrium prices but any aspect of the long-period position. So stable economies differentiated only by their market prices, initial stocks of inventory and sectoral elasticities all converge to the same economic state. Scarcity prices are transient phenomena that quickly dissipate as production is reorganised to meet final demand – they affect the path the economy takes to equilibrium but not the equilibrium itself.

Linear production theory is embedded as a special-case in the equilibrium of this macrodynamic model. The price and quantity systems, in linear production theory, are coupled via exogenous constraints (e.g., see the discussion of linking the nominal and real distribution of income in Chapters 2, 4 and 5). In contrast, in this macrodynamic model, the price and quantity systems are endogenously coupled via a third equation that links the equilibrium distribution of income to the level of employment (equation (7.24)).

Macrodynamic analysis therefore generalises linear production theory and overcomes some of its inherent limitations. For example, natural prices, as defined by linear production theory, have two degrees-of-freedom, an arbitrary *numéraire* and the distribution of income (see chapter 4). In contrast, in this macrodynamic model, the (absolute) equilibrium price level and the distribution of income are jointly determined. A fully specified, self-replacing, equilibrium state therefore endogenously emerges from the dynamics of gravitation.

The analysis of the model presented in this chapter is incomplete. For example, equation system (7.22,7.23,7.24) itself constitutes a self-contained, simple model that will support a comparative statics analysis of stable, long-period positions, which could answer many interesting counterfactual questions, such as how a change in the distribution of income affects the level of employment, or how the profit or surplus-value caused by an exogenous labour-saving technical change is ultimately distributed as profits and wages etc. In this chapter I instead focus on some of the implications for the classical theory of value and distribution.

7.2.7 A remark on the theory of income distribution

The distribution of income is the key variable that links the price and quantity systems. What factors determine the equilibrium distribution of income?

Corollary 2. *Equilibrium workers' savings, m_w^*, are implicitly defined by*

$$m_w^* = \frac{1}{\alpha_w} \frac{1}{1+r^*} \mathbf{l}(\mathbf{I}-\mathbf{A})^{-1} \left(\frac{\alpha_w m_w^*}{\mathbf{d}\underline{\mathbf{w}}^T} \underline{\mathbf{w}}^T + \frac{\alpha_c m_c^*}{\mathbf{d}\underline{\mathbf{c}}^T} \underline{\mathbf{c}}^T \right),$$

where $\mathbf{d} = \mathbf{l}[\mathbf{I} - \mathbf{A}(1 + r^*)]^{-1}$*. Hence,* m_w^* *is a function of constants* $\mathbf{A}$*,* $\mathbf{l}$*,* $\underline{\mathbf{w}}$*,* $\underline{\mathbf{c}}$*,* η_c*,* α_w*,* α_c *and a subset of the initial conditions,* $m_w(0)$*,* $m_c(0)$ *and* $r(0)$*.*

Proof. Proposition 12 implies

$$\mathbf{p}^* = \mathbf{l}(\mathbf{I} - \mathbf{A}(1 + r^*))^{-1} w^*(1 + r^*) = \mathbf{d}w^*(1 + r^*). \tag{7.25}$$

Proposition 13 implies

$$\mathbf{q}^* = \left(\frac{\alpha_w m_w^*}{\mathbf{p}^* \underline{\mathbf{w}}^{\mathrm{T}}} \underline{\mathbf{w}} + \frac{\alpha_c m_c^*}{\mathbf{p}^* \underline{\mathbf{c}}^{\mathrm{T}}} \underline{\mathbf{c}} \right) (\mathbf{I} - \mathbf{A}^{\mathrm{T}})^{-1}. \tag{7.26}$$

Substitute (7.25) into (7.26) and pre-multiply both sides by direct labour coefficients $\mathbf{l}$ to yield the scalar equation,

$$\mathbf{l}\mathbf{q}^{*\mathrm{T}} w^* = \frac{1}{1 + r^*} \mathbf{l}(\mathbf{I} - \mathbf{A})^{-1} \left(\frac{\alpha_w m_w^*}{\mathbf{d}\underline{\mathbf{w}}^{\mathrm{T}}} \underline{\mathbf{w}}^{\mathrm{T}} + \frac{\alpha_c m_c^*}{\mathbf{d}\underline{\mathbf{c}}^{\mathrm{T}}} \underline{\mathbf{c}}^{\mathrm{T}} \right).$$

From Lemma 5, $\mathbf{l}\mathbf{q}^{*\mathrm{T}} w^* = \alpha_w m_w$. □

Corollary 2 implies that – for a given technique, $[\mathbf{A}, \mathbf{l}]$, and composition of demand, $\underline{\mathbf{w}}$ and $\underline{\mathbf{c}}$ – the equilibrium class distribution of savings, represented by m_w^*, and therefore the equilibrium income distribution and aggregate expenditure, are entirely independent of market prices, the scale of production and the dynamics of the labour market. Income shares are instead determined by a set of nominal factors, which we shall call 'monetary factors', specifically propensities to consume, the initial distribution of savings, and the interest rate 'policy', represented by elasticity η_c. So convergent economies with the same monetary factors all converge to the same nominal income distribution. Income shares are therefore insensitive to a wide range of economic disturbances and parameters, such as market prices, activity levels and labour market conditions. This result is suggestive since the relative stability of income shares is a notable feature of actual capitalist economies (Foley and Michl, 1999, Ch. 2).

The total wage bill, or equivalently the aggregate expenditure of workers, is fixed by monetary factors. The conditions in the labour market – for instance wage elasticity η_w – then determine the wage rate and level of employment consistent with this level of expenditure.[12]

In consequence, key economic variables – such as the distribution of income and the level of output and employment – are political and not mere technical outcomes, since they crucially depend on how the different economic classes react to their changing economic circumstances. For instance, income shares are primarily determined by how money-capitalists manage the interest-rate in response to fluctuations in their money wealth. This model is therefore consistent with Sraffa's suggestion that price equation (4.1) be closed by "the level of the money rates of interest" (Sraffa, 1960, p. 39).

This model, although classical in inspiration, shares features with Post Keynesian economic analysis (Rogers, 1989, Ch. 7). For example, money is not neutral but has real effects; the interest rate is a conventional variable that lacks a 'natural' rate; the supply of credit is endogenous and not constrained by the 'money supply'; and the long-period equilibrium is determined by the principle of 'effective demand', such that there is a "limit to the profitable expansion of output" (Chick, 1983, p. 71) before full employment is reached. Many authors have noted that Keynes' vision of a capitalist economy as a 'monetary production economy' is along many dimensions consistent with Marx's analysis, e.g. Reati (2000); Hein (2009); Rogers (1989); Smithin (2009).

7.3 The substance of value

The classical macrodynamic system, described in this chapter, is a multisector, nonlinear, dynamic model of the classical process of gravitation of market prices to natural prices. Numerical simulations indicate that the natural price equilibrium is locally asymptotically stable. We may conclude, therefore, that the classical theory of grav-

[12] This conclusion, if robust, deserves further analysis since it may have important consequences for the political economy of trade unionism. Trade unionists assume that their economic interventions not only benefit their membership but the working class as a whole. However, if workers are subject to a zero-sum economic game, with parameters controlled by capital, then trade unionism merely raises the wages of one section of the working class at the expense of another.

itation is a successful and logically coherent explanation of the homoeostatic kernel of capitalist competition.

A necessary precondition of the classical theory of value – the claim that, under appropriate assumptions, the market prices of reproducible commodities gravitate toward their natural prices – is therefore formally verified. What are the implications of this result for the classical theory of value?

7.3.1 Marx's law of value

Marx inherited the concept of a "law of value" from his classical forebears, especially Smith and Ricardo. The law of value is the claim that market prices gravitate towards natural prices proportional to labour-values.[13]

Smith ([1776] 1994, Ch. 6), for example, proposed that natural prices, in the absence of profit and rent, correspond to labour-values; for example, he writes that in an "early and rude state of society which precedes both the accumulation of stock [capital] and the appropriation of land" then the "quantities of labour necessary for acquiring different objects seems to be the only circumstance which can afford any rule for exchanging them for one another". The rule of the exchange of equivalents in terms of labour effort emerges from the self-interested decisions of economic actors who meet as equals in the marketplace.

Marx ([1894] 1971, p. 178) elaborates on this point and equates the law of value with the classical mechanism of gravitation. He states some counterfactual conditions that allow it to fully manifest:

> "For prices at which commodities are exchanged to approximately correspond to their values, nothing more is necessary than 1) for the exchange of the various commodities to cease being purely accidental or only occasional; 2) so far as direct exchange of commodities is concerned, for these commodities to be produced on both sides in approximately sufficient quantities to meet mutual requirements, something

[13] Other classical authors, such as John Stuart Mill, equate the law of value to the gravitation of market prices to natural prices, where natural prices are determined by costs of production that are not entirely reducible to labour costs (Mill, 1909).

> learned from mutual experience in trading and therefore a natural outgrowth of continued trading; and 3) so far as selling is concerned, for no natural or artificial monopoly to enable either of the contracting sides to sell commodities above their value or to compel them to undersell. By accidental monopoly we mean a monopoly which a buyer or seller acquires through an accidental state of supply or demand.
>
> "The assumption that the commodities of the various spheres of production are sold at their value merely implies, of course, that their value is the centre of gravity around which their prices fluctuate, and their continual rises and drops tend to equalise."

Marx states that the law acts to "maintain the social equilibrium of production amidst its accidental fluctuations" and asserts itself as a "blind law of Nature" (Marx, [1894] 1971, Ch. 51) independent of the consciousness of the economic actors. According to Marx, although individual economic actors may differ in their subjective evaluations of the worth or utility of commodities, market prices are nevertheless regulated by labour-values in virtue of the law of value, which is an objective economic law that emerges as the unintended consequence of generalised commodity production. The law of value is a theory of economic coordination of social labour time via out-of-equilibrium mismatches between the labour-embodied in, and labour-commanded by, commodities (see especially Rubin (1973) and Pilling (1986)).[14] It is worth emphasising, therefore, that Marx incorporates the homoeostatic properties of the "invisible hand" of the market (Smith, [1776] 1994, Book IV, Ch. 2) in his economic theory, notwithstanding his emphasis and focus on the specific kinds of economic crises that capitalist economies necessarily exhibit. The law of value, in an important sense, is identical to the classical mechanism of gravitation.

[14]For instance, Marx (1992, Ch. 1, Sec. 1), in a polemic with Proudhon, writes, "If M. Proudhon admits that the value of products is determined by labour time, he should equally admit that it is the fluctuating movement alone that in a society founded on individual exchanges makes labour the measure of value. There is no ready-made constituted 'proportional relation', but only a constituting movement".

However, Smith, Ricardo and Marx all restrict the applicability of the law of value in various ways. Smith restricts the law to pre-civilised times. Ricardo explains that the law is not the only cause of natural prices. Marx, in contrast to both Smith and Ricardo, maintains that the law equally applies to capitalist societies and also is the ultimate regulator of natural prices, albeit modified by capitalist property relations. For example, Engels (1971) summarises Marx's unfinished views on the manifestation of the law of value in different economic formations as follows:

> "In a word: the Marxian law of value holds generally, as far as economic laws are valid at all, for the whole period of simple commodity production – that is, up to the time when the latter suffers a modification through the appearance of the capitalist form of production. Up to that time, prices gravitate towards the values fixed according to the Marxian law and oscillate around those values, so that the more fully simple commodity production develops, the more the average prices over long periods uninterrupted by external violent disturbances coincide with values within a negligible margin."

The whole period of simple commodity production, according to Engels (1971), dates from a "time before all written history" and "prevailed during a period of from five to seven thousand years" until the advent of capitalist production and the domination of production by capitalist enterprises and the formation of a general profit-rate. The arrival of capitalism on the historical scene modifies how the law of value manifests. Marx's theory of the transformation proposes that natural prices, in capitalist circumstances, are no longer proportional to labour-values, but are conservative transforms of them (see Chapter 2).

The classical macrodynamic model of this chapter includes simple commodity production as a special case, which we can study at the Pasinettian natural stage of investigation (and thereby avoid endorsing Engel's historical claims).

By setting the initial interest-rate to $r(0) = 0$ then production is no longer financed by money-capital. Also, by setting capitalists' money wealth to zero, $m_c(0) = 0$, then the total money wealth in the economy is entirely owned by workers.

These parameter settings effectively remove the capitalist class from the model and create a classless economy. The allocation of resources is driven by profit signals as before but now production is financed by workers' savings (which, in a closed economy, are constant). Workers receive wages plus 'profit-of-enterprise', which we can interpret as a profit share or social dividend. In consequence, all the individuals in the economy receive the same kind of income. The social roles of finance capitalist, who earns interest on their money-capital, and industrial capitalist, who earns profit from their ownership of the firm, no longer exist. Instead, we have a single social role of 'worker' or 'firm member' who earns wages from their labour and profit either from their membership of the firm or society as a whole. In this special case, market prices gravitate towards natural prices proportional to classical labour-values (i.e., Proposition 1 of Chapter 2 applies in equilibrium):

Theorem 6. *The natural prices of simple commodity production are proportional to classical labour-values,*

$$\mathbf{p}^{\star} = \mathbf{v}w.$$

Proof. Given $r(0) = 0$ then $k_r = 0$ and equilibrium price equation, (7.22), reduces to $\mathbf{p}^{\star} = \mathbf{p}^{\star}\mathbf{A} + \mathbf{l}(\alpha_w m_w^{\star}/\mathbf{l}\mathbf{q}^{\star\mathrm{T}})$. Lemma 5 implies $\mathbf{p}^{\star} = \mathbf{p}^{\star}\mathbf{A} + \mathbf{l}w^{\star} = \mathbf{l}(\mathbf{I}-\mathbf{A})^{-1}w^{\star} = \mathbf{v}w^{\star}$, from definition 1 (in Chapter 2). □

Conjecture 1. *Marx's law of value: in simple commodity production, market prices gravitate toward natural prices proportional to classical labour-values,*

$$\lim_{t\to\infty} \mathbf{p}(t) = \mathbf{v}w^{\star}.$$

Proof. (Incomplete.) Theorem 6 establishes the relationship between natural prices and classical labour-values in equilibrium. A formal derivation of the local asymptotic stability of the equilibrium is required to complete the proof. (I prove the local asymptotic stability of simple commodity production, with a simplified profit function, in Wright (2011); unfortunately, this proof does not seem to generalise). □

This result reproduces the intuition behind Adam Smith's beaver and deer example (Smith, [1776] 1994, Book 1, Ch. VII) (see also Wright (2008) and Cogliano

(2013, Ch. 2)). In the special case of simple commodity production, the profit motive, and competition between firms, drives the economy toward prices proportional to labour-values.

Simple commodity production lacks any inter-class distributional conflict and therefore classical labour-values, which measure strictly technical costs of production, predict the equilibrium price structure. The economic relations are especially transparent in these institutionally simple, or natural, conditions.

7.3.2 The general law of value

In capitalist conditions, in contrast, the existence of inter-class distributional conflict entails that the technical conditions of production alone, that is classical labour-values, cannot predict the equilibrium price structure, whether that equilibrium has a single, general profit-rate or multiple, non-uniform profit-rates.[15] The classical macrodynamic system makes it particularly clear that the class struggle over the distribution of the surplus is an ineradicable joint cause of the gravitation of market to natural prices. In consequence market prices are not solely regulated by classical labour-values. Institutions, in particular the distributional rules instantiated by capitalist property relations, also matter. Contra Marx, the notion that natural prices are fully or completely determined by classical labour-values, or that classical labour-values function as their ultimate and sole regulator, is false. Chapter 2 already presented this conclusion and explained that such notions commit a category-mistake. This chapter generalises that conclusion and demonstrates how it applies in the context of the convergence of multiple, non-uniform profit-rates to a single, general profit-rate.

At this point an important distinction between the *cause* of a relative price structure and the *semantic content* of that structure is worth stating. Marx's law of value, when applied to simple commodity production, seeks to establish two claims: first, the causal claim that natural prices are solely determined, and therefore fully ex-

[15] Discussion of the transformation problem, for example, is normally conducted in the special case of a uniform profit-rate, consistent with Marx's presentation in Volume 3 of *Capital*. However, apart from a small number of special cases, the transformation problem also manifests in more complex models with a distribution of profit-rates.

plained, by real costs of production measured in labour time; and, second, the semantic claim that "labour is the substance, and the immanent measure of value" (Marx, [1867] 1954, p. 503) in the sense that monetary phenomena refer to, express, or measure labour time in virtue of the lawful relation that obtains between them.

The causal claim – that labour costs are the sole cause or regulator of natural prices – does not generalise beyond simple commodity production because natural prices, in general, are jointly determined by multiple factors not reducible to the technical conditions of production. For example, the equilibrium defined by the set of nonlinear simultaneous equations, (7.22,7.23,7.24), consists of multiple causal factors that jointly determine equilibrium production-prices, super-integrated labour-values, the scale of production and the distribution of nominal and real income.

However, the semantic claim – that classical gravitation establishes a lawful relationship between natural prices and labour costs – does generalise beyond simple commodity production because natural prices, in general, are proportional to total labour costs. The classical macrodynamic system, in conjunction with a more general labour theory of value, demonstrates that the market prices of reproducible commodities, even in the presence of capitalist profit, ultimately represent physical labour costs. The gravitation of market prices to natural prices is simultaneously a process by which the nominal and real cost structures of the economy grope toward a state of mutual consistency:

Theorem 7. *The natural prices of the classical macrodynamic system are proportional to super-integrated labour-values,*

$$\mathbf{p}^{\star} = \tilde{\mathbf{v}}^{\star} w.$$

Proof. From Lemma 6, $r^{\star} = \alpha_c m_c^{\star} / \mathbf{m}^{\star}\mathbf{q}^{\star\mathrm{T}}$, where $\mathbf{m}^{\star} = \mathbf{p}^{\star}\mathbf{A} + \mathbf{l}w^{\star}$. From Proposition 12, $\mathbf{p}^{\star} = \mathbf{m}^{\star} + \mathbf{m}^{\star} r^{\star} = \mathbf{m}^{\star} + \mathbf{m}^{\star}(\alpha_c m_c^{\star}/\mathbf{m}^{\star}\mathbf{q}^{\star\mathrm{T}}) = \mathbf{p}^{\star}\mathbf{A} + (\alpha_c m_c^{\star}/\mathbf{m}^{\star}\mathbf{q}^{\star\mathrm{T}})\mathbf{m}^{\star} + \mathbf{l}w^{\star}$. Multiply equation (13) by $\mathbf{p}^{\star}$ to yield, $\mathbf{p}\mathbf{c}^{\star\mathrm{T}} = \alpha_c m_c^{\star}$. Hence we can write equilibrium prices as, $\mathbf{p}^{\star} = \mathbf{p}^{\star}\mathbf{A} + (\mathbf{p}^{\star}\mathbf{c}^{\star\mathrm{T}}/\mathbf{m}^{\star}\mathbf{q}^{\star\mathrm{T}})\mathbf{m}^{\star} + \mathbf{l}w^{\star} = \mathbf{p}^{\star}(\mathbf{A} + (\mathbf{c}^{\star\mathrm{T}}\mathbf{m}^{\star}/\mathbf{m}^{\star}\mathbf{q}^{\star\mathrm{T}})) + \mathbf{l}w^{\star} = \mathbf{p}^{\star}(\mathbf{A} + \mathbf{C}^{\star}) +$

$\mathbf{l}w^{\star}$, where $\mathbf{C}^{\star} = (1/\mathbf{m}^{\star}\mathbf{q}^{\star\mathrm{T}})\mathbf{c}^{\star\mathrm{T}}\mathbf{m}^{\star}$ is the equilibrium capitalist consumption matrix as defined in Chapter 2. Let $\tilde{\mathbf{A}}^{\star} = \mathbf{A} + \mathbf{C}^{\star}$, which is the technique augmented by equilibrium capitalist consumption. Hence $\mathbf{p}^{\star} = \mathbf{p}^{\star}\tilde{\mathbf{A}}^{\star} + \mathbf{l}w^{\star} = \mathbf{l}(\mathbf{I} - \tilde{\mathbf{A}}^{\star})^{-1}w^{\star} = \tilde{\mathbf{v}}w^{\star}$, by the definition of super-integrated labour-values (see definition 3). □

The attractor of the classical macrodynamic system is an economic state in which prices are proportional to total labour costs. In equilibrium, the real and nominal cost structures of the economy are mutually consistent, or dual to each other.

Total labour costs measure the current coexisting labour supplied to produce a commodity, including the labour supplied to produce the the real income of capitalists. In consequence, super-integrated labour-values vary during convergence to equilibrium because the real distribution of income varies; that is,

$$\tilde{\mathbf{v}}(t) = \tilde{\mathbf{v}}(t)\,(\mathbf{A} + \mathbf{C}(t)) + \mathbf{l},$$

where $\mathbf{C}(t) = \bar{\mathbf{c}}^{\mathrm{T}}(t)\mathbf{m}(t) = \frac{1}{\mathbf{m}(t)\mathbf{q}^{\mathrm{T}}(t)} \frac{\alpha_c m_c(t)}{\mathbf{p}(t)\underline{\mathbf{c}}^{\mathrm{T}}} \underline{\mathbf{c}}^{\mathrm{T}}\mathbf{m}(t)$.[16] Figure 7.6 plots the out-of-equilibrium trajectories of market prices and super-integrated labour-values in each sector of production during convergence to equilibrium for the 3-sector economy studied in section 7.2.5.

Numerical simulations imply the following conjecture:

Conjecture 2. *The general law of value: in the classical macrodynamic system, market prices gravitate toward natural prices proportional to super-integrated labour-values,*

$$\lim_{t\to\infty} \mathbf{p}(t) = \tilde{\mathbf{v}}^{\star}w^{\star}.$$

Proof. (Incomplete.) Theorem 7 establishes the relationship between natural prices and super-integrated labour-values in equilibrium. A formal derivation of the local asymptotic stability of the equilibrium is required to complete the proof. □

Ricardo stated that once we have "possession of the knowledge of the law which regulates the exchangeable-value of commodities, we should be only one step from

[16]This expression is merely the time-parameterised version of the definition given in section 2.7, Chapter 2.

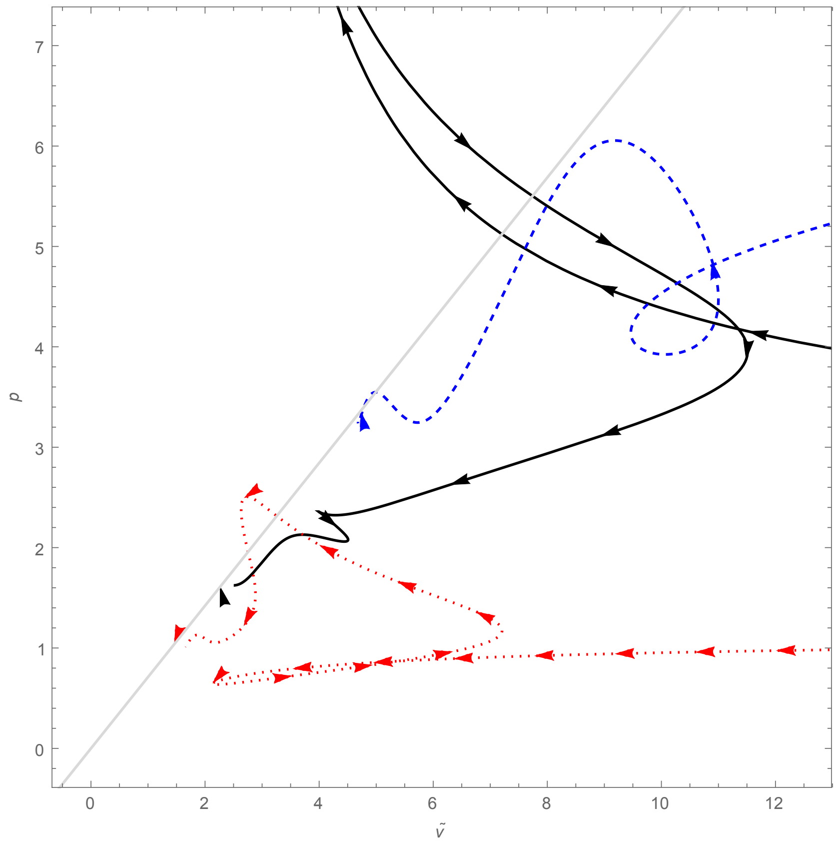


Figure 7.6: The general law of value: a 3-sector example of classical gravitation as the convergence of prices and super-integrated labour-values. The ray through the origin is $p = \tilde{v}w^{\star}$, where p is the price-axis, $\tilde{v}$ is the value-axis and $w^{\star}$ is the equilibrium wage-rate. The three trajectories plot the $(p_i(t), \tilde{v}_i(t))$ curves of the corn (black), iron (dashed) and sugar (dots) sectors, where $p_i(t)$ is the market price and $\tilde{v}_i(t)$ is the super-integrated labour-value of commodity i. All trajectories converge to equilibrium points on the ray, at which point prices are proportional to super-integrated labour-values, $\mathbf{p}^{\star} = \tilde{\mathbf{v}}^{\star} w^{\star}$.

the discovery of the measure of absolute value" (Ricardo, 2005b, 315). The classical macrodynamic system is the 'law' that regulates the exchange-value of reproducible commodities. The general law of value is then the next step, which establishes the proposition that labour time is a measure of absolute value.

Marx's opening chapters of *Capital* go further and seek to establish a semantic relationship between monetary phenomena and labour time in virtue of the causal regularities induced by the social practice of generalised commodity production. The classical category-mistake derailed this project (as explained in Chapters 2 and 3). The general law of value, which avoids the category-mistake, re-establishes the semantic relationship.

An analogy with a more prosaic kind of measuring device is helpful here. We say that a mercury thermometer measures temperature because the height of its mercury column bears a lawful relationship to the ambient temperature. For example, when we place a mercury thermometer in boiling water the mercury expands, due to the law of thermal expansion, and if the thermometer reaches a state of thermal equilibrium with its environment, then at this point the mercury's height is a veridical measure of the temperature. A necessary condition for the ascription of semantic content is a causal law that connects a representation and its referent. In this example, we state that the height of the mercury column represents temperature in virtue of the law of thermal expansion that connects them.

By analogy, we state that prices measure labour costs because they bear a lawful relationship to them. For example, when an economy suffers an exogenous shock, such as a change in the technical conditions of production or a change in the distribution of income, then the prices of commodities change, due to the law of value, and if the economy reaches a state of economic equilibrium, then at this point the structure of natural prices is a veridical measure of the structure of real costs of production measured in labour time.

In both cases we state that 'some X represents some Y' because X is lawfully related to Y. The general law of value therefore reconnects the form of value with the substance of value, and supplies a necessary condition for Marx's claim that "labour is the substance, and the immanent measure of value" (Marx, [1867] 1954, p. 503).

7.4 Conclusion

The classical theory of value and distribution is predicated on an informal theory of gravitation of market prices to natural prices. The dynamic model presented in this chapter formalises that theory and demonstrates that it successfully explains the homoeostatic kernel of capitalist competition.

The reallocation of capital in response to profit signals reorganises the division of labour until supply equals demand. Simultaneously, market prices converge to production-prices proportional to real costs measured in labour time. The causal regularities of capitalist competition therefore instantiate a “law of value” between prices and labour-values.

Ricardo wished to find a non-market measure of absolute value and proposed that labour costs might fulfil that role. Marx, in contrast, aimed to formulate the economic laws that explain why exchange-value represents labour time, just as the physical law of thermal expansion explains why a thermometer represents temperature. The classical macrodynamic system, formulated in this chapter, contributes to Marx’s project by establishing the foundational proposition that production-prices represent labour costs in virtue of the general law of value that connects them.

Chapter 8

Conclusion

In this concluding chapter I summarise the contributions of this thesis and then point to directions for further work.

8.1 Contributions

In Section 1.3 I stated the detailed objectives of the thesis. Here I explain how the thesis addresses those objectives.

I tackle the first objective – to evaluate Ricardo and Marx's version of the classical labour theory of value – in Chapter 2, where I reproduce Ricardo's problem of an invariable measure, and Marx's problem of the transformation, in the formal language of linear production theory.

Chapter 2 introduces a new concept, central to the contributions of this thesis, which is a total labour cost (definition 2, Chapter 2), where the modifier 'total' denotes the inclusion of *all* the coexisting labour supplied to produce a commodity in the sense that it *only* excludes the reproduction cost of labour. Throughout the thesis I repeatedly apply this definition to a variety of production models of capitalist economies. For example, the definition yields a new measure of the "difficulty of production" of commodities, which I call the super-integrated labour-values (e.g., definition 3 in Chapter 2, definition 5 in Chapter 4 and definition 11 in Chapter 5), which, in the context of simple models of a capitalist economy, additionally includes the super-indirect labour supplied to produce the real income of capitalists. I explain that classical labour-values exclude this labour as a real cost of production and therefore do not measure the actual labour supplied to reproduce commodities in the institutional circumstances of a capitalist economy.

I note that classical labour costs, and total labour costs, happen to be identical in the special case of "simple commodity production" (i.e., production in the absence of specifically capitalist property relations) and I note this also happens to be the case where the classical labour theory of value 'works'. The reason the classical theory 'works', in this special case, is that total nominal costs (i.e., the natural price system) are compared with total labour costs: apples are compared with apples. The classical theory then breaks down, once we introduce capitalist property relations, because the classical definition of labour cost no longer satisfies the definition of a total labour cost. The classical theory, therefore, commits a category-mistake when it compares total nominal costs with *partial* labour costs – and then expects a commensurate relationship to obtain between them: apples are compared with oranges. Chapter 2 proposes a novel reading of the history and development of the classical labour theory of value in terms of an unidentified and recurring category-mistake in the sense of Gilbert Ryle. Category-mistakes are precisely the kind of hidden conceptual errors that cause longstanding and insoluble theoretical difficulties, and therefore my reading also proposes to explain why the problems of the classical theory have persisted for so long without resolution.

My next objective was to decide whether the problems of the classical theory are insurmountable or merely a property of the particular kind of theory proposed by the classical authors. The argument of Chapter 2 implies that that the problems are indeed surmountable, once the category-mistake is identified, since they derive from a logical error at the stage of theory formulation. In consequence, once we realise how to avoid the category-mistake, the theoretical problems of the classical theory – such as the problem of an invariable measure and the problem of the transformation – may be surpassed and transcended.

The key result of Chapter 2 is a theorem that proves that natural prices are identical to the wages of the total coexisting labour supplied to produce commodities. Natural prices and real costs of production measured in labour time are 'two sides of the same coin' (see Theorem 1, Chapter 2). This contribution is novel in the sense that it acknowledges and accepts the standard and longstanding criticisms of the classical labour theory of value, yet simultaneously demonstrates, contrary to the standard view, that a more general version of the classical labour theory of value – which includes both classical and total measures of labour cost – avoids the standard criticisms and constitutes a logically coherent theory of economic value.

Chapter 2 presents the main contribution of this thesis, which is the outline of a general labour theory of value. The theory is general in the sense that it includes the classical theory as a special case.

The classical theory of value conflates distribution-dependent and distribution-independent topics, where the term distribution refers to the division of the net product between workers and capitalists. The more general theory, in contrast, separates these concerns. For example, the general theory applies classical labour-values to distribution-independent topics, such as the technical productivity of labour or Marx's theory of surplus-value, and applies super-integrated labour-values to distribution-dependent topics, such as the relationship between natural prices and real costs of production. The more general theory therefore has the theoretical capacity to answer a wider range of questions.

A further objective was to determine to what extent the problems and limitations of the classical theory affect the fundamental logical structure of Marx's economic theory as a whole. In Chapter 3 I propose a critical and novel reading of the fundamental logical structure of Marx's economic theory. I argue that Marx's attempt to construct a unified theory of value and exploitation commits a logical fallacy. First, Marx critiques the cost logic engendered by capitalist property relations by arguing that the surplus-labour supplied to capitalists is an unnecessary cost of production; and he applies classical labour cost accounting, which excludes surplus-labour as a real cost of production, in order to provide a quantitative basis for his critique. Second, Marx also attempts to explain the cost logic engendered by capitalist property relations, i.e. production-prices that include surplus-value as a necessary cost of production, in terms of labour costs; and here, again, he applies classical labour cost accounting to provide the quantitative basis for this explanation. Marx therefore attempts to explain production-prices, which embody the cost logic of capitalism (factual costs expressed in nominal terms), in terms of classical labour-values, which are independent from, and in Marx's hands, explicitly reject that cost logic (counterfactual costs expressed in real terms). But a factual cost structure cannot be explained in terms of a counterfactual cost structure. I present a novel argument that this fundamental logical contradiction not only manifests in Marx's theory of the transformation but also manifests in Marx's theory of the irrational nature of money-capital. I conclude, therefore, that the problems of the classical labour theory of value affect the fundamental logical structure of the entirety of Marx's *Capital*.

A further objective was to decide whether the difficulties that Marx encounters when formulating a unified theory of value and exploitation are insurmountable, or, again, merely a property of the particular kind of labour theory of value he employs. In Chapter 3, I explain that the more general labour theory of value, proposed in Chapter 2, avoids the difficulties of Marx's theory while preserving his normative claims regarding the exploitative nature of capitalism, and therefore strengthens rather than refutes Marx's analysis of capitalist production.

A further objective was to understand and evaluate the contributions of Sraffa and Pasinetti in relation to the value question.

Chapter 4 explains that Sraffa's reconstruction of classical economics includes a solution to Ricardo's problem of an invariable measure of value restricted to the special case of changes in the distribution of income. Sraffa achieves this result by constructing his standard commodity, which has the special property that its price is independent of the price fluctuations that accompany a change in the distribution of income. Sraffa then takes the step of reducing the standard commodity to the "variable quantity of labour" it commands in the market, and thereby partially reconstructs the classical theorists' attempt to measure exchange-value in terms of a homogeneous value substance, such as labour time. Sraffa's somewhat cryptic, and as he himself suggests "curious", answer to the value question is that natural prices refer to, or denote, the labour commanded by this special bundle of commodities.

Chapter 4 proposes a critical and novel reading of the theoretical meaning of Sraffa's claim. I demonstrate that Sraffa's "variable quantity of labour" is an indirect or proxy reference to the total labour costs defined in Chapter 2 (Theorem 3, Chapter 4). Sraffa does not recognise that his reduction is not merely a Smithian, labour-commanded measure but is also a labour-embodied measure, in the sense that it implicitly denotes a physical real cost of production, namely the super-integrated labour-value of the standard commodity. I conclude, therefore, that although Sraffa's reconstruction of the classical theory of value overcomes some of its problems it nonetheless remains incomplete. The more general theory, as explained in Chapters 2 and 4, provides a more complete solution to Ricardo's problem by identifying an invariable measure of value that is invariant to both changes in technique and changes in the distribution of income.

Chapter 5 engages with a paper by Pasinetti that explicitly addresses Marx's transformation problem. Pasinetti adopts Marx's position that production-prices, of necessity, cannot be proportional to classical labour-values in virtue of the distorting effects of capitalist property relations. Pasinetti generalises Marx's transformation problem to an economy that exhibits non-uniform growth. He therefore rejects Marx's further proposal that the transformation is conservative. Pasinetti, unlike Marx, restricts the labour theory to a normative role, suitable for the analysis of natural or pre-institutional economic systems, yet incapable of functioning as a descriptive theory of exchange-value in actually existing economies.

Pasinetti demonstrates that the classical concept of a labour-value must be generalised to include non-technical costs of production, such as the labour supplied to replace net investment goods, in order to understand the cost structures of more general economic systems, such as those that exhibit non-uniform growth. In Chapter 5 I argue that Pasinetti's demonstration of the need to generalise the classical concept of labour cost, even in the absence of capitalist property relations, is a key contribution to the development of the classical theory of value. Pasinetti demonstrates that we gain new analytical insights by decomposing an economy into different *kinds* of vertically integrated subsystems, and that the different measures of labour-value, such as the classical measure or Pasinetti's hyper-integrated coefficients, correspond to the coexisting labour supplied to these different kinds of subsystems.

My contribution, in Chapter 5, further generalises Pasinetti's vertically integrated approach to encompass real cost structures induced by the institutional conditions of production. I explain that a super-integrated labour cost measures the coexisting labour supplied to a self-replacing, super-integrated subsystem that produces the real wage as final output and replaces means of production, net investment goods and the real income of capitalists. I present a solution to Pasinetti's "general transformation problem" by proving that the production-prices of Pasinetti's non-uniformly growing economy are proportional to super-integrated labour-values (Theorem 5, Chapter 5). I therefore conclude that Pasinetti's restriction of the labour theory of value to a normative role is unwarranted.

Chapters 4 and 5 offer a critical evaluation of Sraffa and Pasinetti's contributions to the classical theory of value. In both cases I identify a positive contribution – such as Sraffa's partial solution to Ricardo's problem of an invariable measure and Pasinetti's generalisation of the classical concept of a labour-value – but also an incompleteness or limitation caused by their failure to identify, and avoid, the classical category mistake. In both cases I argue that a more general labour theory, proposed in this thesis, extends the work of these authors and supplies the basis for a more complete, and satisfactory, answer to the value question.

Another objective of this thesis was to understand why competing conceptions of the meaning of a labour-value have arisen in the classical tradition, specifically the historical versus replacement cost conceptions, and then decide upon their status. Chapter 6 engages with Mirowski's historical account of the development of the concept of economic value. Mirowksi argues that the competing conceptions first arise in *Capital* where Marx attempts to synthesise a substance theory, inherited from the Physiocrats and corresponding to historical real costs, with a "nascent" field theory, corresponding to current replacement costs. Marx, in consequence, simultaneously holds two contradictory versions of the labour theory of value. My contribution in Chapter 6 argues that this contradiction is merely apparent, and largely due to a misreading of Marx's text. I provide a new argument for the view that Marx held a logically consistent substance and field theory of value by analogy with field properties from the physical sciences; and I note that the substance and field theories collapse to the same theory in the special case of equilibrium. Chapter 6 therefore contributes to avoiding the conceptual confusions that sometimes arise from the (seemingly) competing conceptions of the nature of a labour-value.

A further objective of this thesis was to translate the informal classical theory of competition into a formal, multisector dynamic model. Chapter 7 presents a novel cross-dual model of capitalist macrodynamics. I demonstrate, via numerical simulation, that the classical authors' intuitions are correct, and that the scramble for profit, where capitalists reallocate their money-capital seeking the highest returns, is indeed a mechanism that causes market prices to converge towards stable natural prices, and quantities supplied to converge towards stable levels that meet effective demand. My results provide additional and new evidence that the classical theory of gravitation is a successful and logically coherent explanation of the homoeostatic kernel of capitalist competition. My results here confirm a necessary premise upon which the classical labour theory of value rests.

Marx's theory of value claims that prices represent labour costs in virtue of the causal regularities of generalised commodity production. A further aim, therefore, was to evaluate Marx's claim within the formal framework of the dynamic model. A further contribution of Chapter 7 is my demonstration that the classical process of gravitation is simultaneously a process by which the nominal and real cost structures of the economy converge to a state of mutual consistency; that is, market prices converge to natural prices proportional to super-integrated labour-values. I propose a novel reconstruction of Marx's version of the classical law of value (see Theorems 6 and 7 and conjectures 1 and 2 in Chapter 7) and I argue that this reconstruction satisfies a necessary condition, missing in the classical theory, for the value-theoretic claim that labour is the substance of value.

As stated in the introduction, the primary aim of this thesis is to analyse and critique the classical labour theory of value in order to decide to what extent it does, or does not, answer the value question. My contribution to this larger objective is the following conclusion: the classical theory does not provide a wholly satisfactory answer to the value question in virtue of its logical problems; however, a generalisation of the classical theory, which avoids the category-mistake and includes both classical and total measures of labour cost, avoids these problems. The general law of value, presented in Chapter 7, is a mechanism that causes market prices and objective costs of production, measured in labour time, to grope towards a state of mutual consistency, or proportionality, at which point the total labour of society is allocated to different sectors of production according to effective demand. The value question asks, *What does the unit of account, e.g. £1, represent or measure?* In Chapter 7 I argue that a necessary condition for the ascription of semantic content to a symbolic representation, such as the unit of account, is the existence of a lawful relation between the symbol and its purported content. The general law of value establishes such a lawful relation, and therefore provides a causal basis, and logical justification, for answering "labour" to the value question.

In the next section, on further work, I discuss whether the general law of value provides a sufficient condition for the ascription of this semantic content.

8.2 Further work

Every chapter of this thesis can be generalised and extended in one way or another. In what follows I suggest units of further work, in each paragraph, that could form the basis of a focused research effort.

The argument of chapter 2 should be generalised to models of joint production, and models that include fixed capital that depreciates. Many of the negative results for the classical labour theory of value, which manifest in models of joint production, can be entirely re-evaluated from the perspective of total labour-values.

The concept of total labour-value needs to be applied, and tested against, general social accounting matrices that include more of the institutional structure of capitalist economies (e.g., banking and finance, a government sector, imports and exports etc.) This requires a deeper analysis of Richard Stone's system of national accounts (e.g., see Flaschel (2010)) and also some of the Leontief-inspired input-output literature (e.g., ten Raa (2005)), and relating this literature to Pasinetti's approach of generalising Sraffa's concept of a vertically-integrated subsystem to include more 'indirect' production.[1] The eventual aim, from the point-of-view of developing the theory of value, should be to understand the semantic content of prices in more gen-

[1] For example, Garbellini (2010) discusses the theoretical prerequisites for relating Pasinetti's structural economic dynamics to empirical analysis; and Garbellini and Wirkierman (2014) apply vertical hyper-integration to an input-output accounting framework to measure empirical productivity trends.

eral economic situations, including the prices of more exotic commodities, such as financial instruments.

In Chapter 2 I explain that a category-mistake generates the more well-known problems of the classical theory of value. And in Chapter 3 I examine another, less well known, manifestation in Marx's theory of money-capital. I think it may be profitable to identify further manifestations of the mistake in Smith, Ricardo and Marx, and also in the wider classical literature. For example, I think a critical understanding of the classical distinction of productive and unproductive labour might benefit from this perspective.

Chapter 3 is a brief and compressed argument that the essential logical structure of Marx's economic theory fails to fully capture a process of historical change, and therefore is insufficiently dialectical. I expect this chapter to raise as many questions as it answers (both for those for and against its conclusions). The argument of Chapter 3 could be greatly expanded, and deepened, by a closer and more detailed investigation of the textual evidence in Marx and Engels' works.

Piero Sraffa's unpublished writings are held at Trinity College Library, Cambridge. Unfortunately, this material is not available online, and scholars must gain permission to quote. Chapter 4 supplies an interpretation, and imputes an intellectual motive, to some of the arguments in *Production of Commodities by Means of Commodities*. The argument could either be deepened or critiqued with reference to Sraffa's unpublished works. For instance, Carter (2013) argues that Sraffa's working notes indicate that his construction of the standard commodity was inspired by Marx's theory of the transformation, in particular Marx's discussion of sectors with an average organic composition of capital.

Chapter 5 simplifies Pasinetti's growing subsystems model by assuming that the inter-industry matrix, **B**, equals the identity matrix, **I**, and is therefore inactive. Relaxing this assumption, in order to generalise Theorem 5, which establishes that the production-prices of Pasinetti's non-uniformly growing economy are proportional to super-integrated labour coefficients, would be a useful entry point into Pasinetti's work and its relation to value theory. Pasinetti's structural economic dynamics is a very rich and important body of economic work that not only reveals some of the necessary and invariant features of all economic systems but also specifies normative criteria that can inform the design of post-capitalist institutions that aim to abolish economic exploitation (e.g., Pasinetti and Garbellini (2014)). I think it would be profitable to further explore the relationship between Marx's critique of capitalism and Pasinetti's normative analyses within the context of the more general labour theory of value outlined in this thesis.

Chapter 7 represents, unfortunately, only a small step into the wider and largely unexplored field of formal, dynamic approaches to classical economic theory. The most important unsolved problem is a proof of the stability of the classical macrodynamic system. Numerical simulations clearly indicate that the system is locally asymptotically stable for a wide range of parameter settings. However, a formal proof will provide insight into the domain of attraction of the natural price equilibrium, and the rate of convergence. The specification of the adjustment rules of the classical macrodynamic model should be generalised to include a wider variety of functional forms. This generalisation, it itself, might make a proof easier to formulate. A suitably general theorem will lay the foundations for a nonlinear dynamic production theory, which would significantly generalise linear production theory, and also provide a foundational and fully general explanation of economic coordination in terms of multi-sectoral, cross-dual dynamics.

The classical macrodynamic model should be generalised to include variable returns to scale technology. This requires replacing the coefficients of the technology matrix with production functions. Initial numerical experiments can quickly explore how different choices of production functions (e.g., increasing or decreasing returns to scale) affect the convergence properties of the model and the equilibrium state. The analysis should attempt to quantity over families of production functions in order to generalise, or circumscribe, the propositions of Chapter 7, which will then become special-cases of a more general theory.

A sub-field of the Post Keynesian literature explores stock-flow consistent models after the pioneering work of Wynne Godley (Godley and Cripps, 1983; Godley and Lavoie, 2007). In general, this class of models includes a more sophisticated representation of the institutional setup of capitalist economies, compared to the model in Chapter 7, but less sophisticated multi-sectoral adjustment rules. From a purely technical point-of-view, the combination of classical cross-dual dynamics with stock-flow consistent modelling is likely to be fruitful. For example, the robustness of the conclusions of this thesis could be tested and generalised by replacing the classical loanable funds theory with endogenous credit money. Again, the aim, from the point-of-view of value theory, would be to understand the semantics and functional role of kinds of prices in different institutional contexts, especially with respect to the labour process and the allocation of the total labour of society.

My analysis of the classical macrodynamic model, presented in Chapter 7, is preliminary. Many more questions could be asked of it, especially of elaborations that introduce more of the institutional structure of capitalist societies (see above). For example, one direction that may be fruitful, would be to relate the (somewhat implicit) theory of income distribution of the classical macrodynamic model to existing theories in the Post Keynesian literature. I am unsure to what extent multi-sectoral foundations, and cross-dual dynamics, might affect the conclusions of existing theories of the distribution of income.

Marx introduces the modifier "socially necessary", in Volume 1 of *Capital*, to control for the distribution of labour productivity within sectors of production. In this thesis, as mentioned in the introduction, I assume homogeneous productivity in each sector. In my view, Marx's treatment of this issue is not satisfactory. I think relaxing this assumption and then re-addressing the themes of this thesis would be fruitful, especially in the context of dynamic models of the kind featured in Chapter 7. The assumption of a uniform wage-rate, in contrast to a distribution of wage-rates, should also be relaxed. In this overall context, the theoretical problem of 'defining' reduction coefficients of skilled to unskilled labour can also be tackled (some potentially relevant, but by no means exhaustive, work in this area includes Krause (1982) and Wright (2008)).

This thesis is primarily conceptual or theoretical. Deterministic models define particularly crisp and transparent objects from which to draw theoretical conclusions. However, deterministic models, although important and useful, have difficulty capturing and representing the full variability and richness of economic life.

In contrast, probabilistic models can do better here, and make direct contact with empirical data. Some examples of this kind of approach are Farjoun and Machover (1989); Aoki (1996, 2002); Wright (2005, 2009); Cockshott, Cottrell, , Michaelson, Wright, and Yakovenko (2009). All of the issues of value theory, addressed in this thesis, could in principle be re-expressed, and re-examined, in the context of models that combine the approach of agent-based probabilistic models with some of the deterministic structure of linear production theory.

In addition to deepening the arguments of this thesis there are three important areas that remain unresolved, which are especially important for the development of the labour theory of value.

8.2.1 Shaikh-like results

Ricardo (2005a, p. 404) stated that income distribution is a "less powerful cause", compared to labour values, of the variation of natural prices. Stigler (1958) therefore characterised Ricardo's theory as a "93% labour theory of value". A highly robust finding in Marxian empirical economics is 'Shaikh's result' (Trigg, 2002a) named after Anwar Shaikh's groundbreaking analysis of Italian and American input-output tables, which discovered a high correlation between sectoral market prices and classical labour-values (Shaikh, 1984).

Shaikh's work initiated an important, and ongoing, empirical research program to replicate his results and measure the size of the correlations using diverse national accounts data; e.g., Shaikh and Tonak (1994); Petrovic (1987); Ochoa (1988); Cockshott, Cottrell, and Michaelson (1995); Cockshott and Cottrell (1997b, 2003); Tsoulfidis and Maniatis (2002); Zachariah (2006); Fröhlich (2013).

The empirical results consistently indicate that *both* classical labour-values and production-prices "show nearly identical fits in explaining market prices" (Fröhlich, 2013) with coefficients of correlation of at least 0.9 and often higher. Classical labour-values and production-prices therefore explain a major component of the variation of (highly aggregated) market prices.

Shaikh's result, at first glance, is surprising because we would not expect *either* production-prices or classical labour-values to correlate well with market prices since it seems highly unlikely that a capitalist economy achieves a natural price equilibrium due to interfering factors such as ceaseless technical progress. However, given that they do, then it is not surprising that both measures exhibit similar explanatory power since we know, from theoretical analyses consistent with Ricardo's intuition, that income distribution has a "very small" (Shaikh, 1984) affect on the divergence of production-prices from classical labour-values.

A further empirical surprise is Cockshott and Cottrell's related discovery that alternative real cost bases, such as electricity-value, iron-value, steel-value, oil-value etc., are not as highly correlated with empirical prices (Cockshott and Cottrell, 1997b). Classical labour-values, that is technical real costs of production measured in labour time, appear to be especially explanatory.

These empirical results motivate some authors, inspired by Shaikh's original presentation, to claim that Marx had no need to transform his Volume 1 theory of value. For example, Cockshott and Cottrell (1997a) argue that the classical labour theory of value should be preferred to the theory of production prices since it is the simpler of the two theories; and Flaschel, Fröhlich, and Veneziani (2013) state "there is no 'transformation problem' to be solved in Marx's labour theory of value" and "labour values are not meant to provide an explanation of classical production prices". Their point is that classical labour-values are excellent empirical predictors of aggregate input-output prices, and also appear to explain the main components of industrial profit-rates.

Notwithstanding the importance and interest of these empirical results the existence of high statistical correlation between classical labour-values and aggregate input-output prices does not address the theoretical problems of the classical labour theory of value, as these authors claim. Sraffa, for example, distinguishes between two kinds of measurement error:

> "...one should emphasize the distinction between two types of measurement. First...the one in which the statisticians are mainly interested. Second... measurement in theory. The statisticians' measures were only approximated and provided a suitable field for work in solving index

> number problems. The theoretical measures required absolute precision. Any imperfections in those theoretical measures were not merely upsetting, but knocked down the whole theoretical basis" (quoted in Lutz and Hague (1961, pp. 305–306)).

We know, on theoretical grounds alone, that the existence of capitalist profit introduces an additional degree-of-freedom that deforms natural prices away from classical labour-values. In consequence, classical labour-values cannot be their measure, and the value-theoretic claims of the classical labour theory of value are left hanging, a point the classical economists understood very well. The fact that their theoretical measures lacked absolute precision indicated a logical, not an empirical, problem.

Furthermore, the 'correctness' of the labour theory of value cannot depend on the empirical "strength" of aggregate correlations. For example, at what level of correlation should we decide to reject the classical labour theory? Below 95%, 90% – or is 85% ok? (on this issue see also Laibman (2002, p. 164)). Also, the correlations vary over time. Should we therefore consider that the classical labour theory was more 'correct' in 2011 compared to 2014? Perhaps the residual lack of correlation, however 'small', is precisely a reward to money-capitalists for their patience?

Nonetheless these empirical results, widely reported and discussed in the literature, are very important and have yet to be adequately explained. Correlations are a beginning. But substantive scientific progress requires the identification of causal laws. My guess is that we can explain Shaikh-like results by combining the perspective of the general law of value, outlined in this thesis, with Marx's irreducibly dynamic account of the production of (new) absolute and relative surplus-value (Marx, [1867] 1954, Pts. 3–5). I will briefly, and necessarily tentatively, sketch what such an explanation might look like.

Begin with the classical macrodynamic system presented in Chapter 7. Observe that human agency is the actual cause of technical progress. Conjecture, therefore, that technical progress in each sector of production is positively correlated with the direct labour employed in that sector (and not correlated with the direct corn, oil or electricity inputs used-up etc.) in virtue of the material activity of workers in each sector. Recall Marx's axiom that only "living labour" creates surplus-value, rather than "dead labour" in the form of means of production. Assume then, for the sake of brevity, that the distribution of out-of-equilibrium, profit-of-enterprise is positively correlated with technical progress. Flaschel et al. (2013) present theoretical and empirical work that deepens and provides evidence for this overall vision.

At this point, conjecturally suspend the operation of the general law of value, such that capital is not yet reallocated according to the differential profit-rate signals. The economy, in this state, corresponds to Marx's 'pre-transformation' conditions where "we find considerably different rates of profit" across sectors that are proportional to the direct labour employed (see Chapter 2). In these 'pre-transformation' conditions, prices are proportional to classical labour-values.

Human labour, unlike any other economic 'input', can be intensified, trained, disciplined, but also can modify, alter and improve its own causal powers. Let's therefore postulate the existence of a mechanism that causes market prices to 'gravitate' toward classical labour-values in virtue of the material activity of labour-power, which has "the peculiar property of being a source of value" (Marx, [1867] 1954), in the sense that its productivity is variable, not constant.[2] Call this mechanism the "law of surplus-value".

The economy, then, includes two mechanisms in real contradiction with each other. The attractor of the general law of value is an equilibrium with production-prices proportional to super-integrated labour-values. The attractor of the law of surplus-value is a growth equilibrium with prices proportional to classical labour-values. The law of value reduces the variance of profit-rates and pushes market prices toward super-integrated labour-values; in contrast, the law of surplus-value increases the variance of profit-rates and pushes market prices toward classical labour-values. The empirical facts that (i) both production-prices and classical labour-values are highly correlated with market prices, and (ii) labour-values are superior predictors of market prices compared to alternative real cost bases, might therefore be explained by the interaction of a stabilising law of value and a destabil-

[2] It is for precisely this reason that Marx calls labour-power "variable capital" whereas means of production are "constant capital".

ising law of surplus-value.

An area of further work, then, is to construct a formal model of the law of surplus-value, and then examine its interaction with the general law of value. A necessary theoretical prerequisite is a careful and thoughtful consideration of the relationship between the material activity of human labour, technical progress and the source of profit (e.g., see Flaschel, Franke, and Veneziani (2012) and Flaschel, Veneziani, and Franke (2015)).

8.2.2 Non-reproducibles

The law of value dissipates the out-of-equilibrium scarcity prices of reproducible commodities. But what happens if we introduce non-reproducibles, such as land, to the classical macrodynamic model?

The classical authors, of course, proposed theories of the relationship between the labour theory of value and rent. Notable examples are Ricardo's theory of rent and Marx's theory of differential and absolute rent. A topic for future work, therefore, is to explore how the general law of value relates to classical theories of rent.

The natural prices of a simple commodity economy, as we saw in Chapters 2, 4 and 7, are proportional to, and determined by, classical labour-values. In this simple case there is a single non-reproducible factor of production, which is the stock of labour.

However, the introduction of capitalists, who own money-capital and charge for its use, alters the natural price equilibrium, which now includes an interest-rate. The stock of money-capital available to lend, at least in a loanable funds framework, is a non-reproducible factor of production. The natural prices of a capitalist economy, in this sense, are already deformed by the (institutionally imposed) scarcity price of loanable funds. In this more complex case, natural prices are proportional to super-integrated labour-values, which include the labour cost of supplying the owners of money-capital with their real income. The more general theory, outlined in this thesis, therefore implicitly defines an approach to integrating non-reproducible factors of production with the labour theory of value.

Total labour costs, in an economy with landowners who charge rent for the use of land, include the additional super-indirect labour supplied to produce the real income of landowners. A formal demonstration of this proposition requires extending the classical macrodynamic to include land as an input to production, and out-of-equilibrium adjustment rules for its rental price. I conjecture that market prices, in this more general model, will also gravitate toward natural prices proportional to total labour costs. The introduction of rent, therefore, alters the institutional setup of the economy and its distributive rules, but it does not modify the law of value.

A simple point stands behind this complexity: regardless of the causes of a particular equilibrium price structure, and the kinds of institutional setups, prices normally function to allocate the total labour of society to different branches of production. Is it any wonder, then, that prices and labour-values are indissolubly linked, or that in equilibrium states, with a stable division of labour, they are dual to each other? The more important question, once the scales fall from our eyes, is why economic science ever thought they could not be. The answer, proposed in Chapter 2, is that the classical authors bequeathed a set of theoretical problems stated in a conceptual framework that was itself faulty. The classical category-mistake misdirected theoretical attention towards the problems and away from the fact that a commodity's natural price is the wage bill of the total coexisting labour supplied to produce it (Theorem 1). Breaking free of the classical category-mistake is difficult because it requires adopting a new and more general kind of conceptual framework. However, from the more general vantage point, the kinds of proposition exemplified by Theorem 1 are no longer surprising, but necessary.

8.2.3 The semantic question

The problems of the classical labour theory of value imply that "labour" cannot be an answer to the value question. The argument of this thesis, culminating in the formulation of a general law of value, imply that "labour" can be an answer. The existence of a lawful relation between a representation and its referent is a necessary condition for the ascription of semantic content. The general law of value supplies that necessary condition.

But it is not sufficient. In this thesis, therefore, I do not supply a definitive answer to the value question, although I certainly point in a certain direction. A definitive answer requires addressing a prior, and more fundamental, semantic question: *What are the necessary and sufficient conditions for parts of the world to refer to other parts?* On what grounds can we justify or reject the claim that a monetary phenomenon, such as a price, denotes, refers to, or measures some non-monetary phenomenon, such as real costs measured in labour time? The value question, therefore, requires us to dig deeper and ask the semantic question.

For instance, we can measure the "difficulty of production" of commodities in terms of physical units of any commodity, such as units of corn, iron etc; that is, there are as many kinds of standards, or real cost bases, as there are kinds of commodities. Bródy (1970) defines real-cost analogues of classical labour-values, and I have defined similar analogues for total labour-values (Wright, 2007). It is relatively easy to prove that production-prices are proportional to super-integrated corn-values, iron-values, etc., and therefore the general law of value, described in Chapter 7, is simultaneously a process by which market prices converge to objective costs of production measured in terms of any real cost basis. In consequence, the general labour theory of value, outlined in this thesis, is merely one of many real cost theories that differ in their choice of measuring unit, such as a general corn theory of value, a general energy theory of value etc. Critics of Marx's theory of value have often complained, with some justification, that Marx's argument that labour is special is unsatisfactory (e.g., Keen (1993) and Gintis and Bowles (1981)).

Ricardo to some extent perceived that a single solution to the problem of an invariable measure immediately implies a plurality of solutions. He writes, in response to Malthus' disquisition on the subject,

> "Labour says Mr. Malthus never varies in itself, a day's labour is always worth a day's labour, therefore labour is invariable and a good measure of value. In this way I might prove that no commodity ever varied and therefore that any one was equally applicable as a measure of value, as for example gold never varies in itself and therefore is an invariable measure of value – cloth never varies in itself and therefore is an invari-

able measure of value ... " (Ricardo, 2005a, p. 392)

Of course, Ricardo immediately dismisses this possibility since he believes labour cannot be an invariable standard (Ricardo, 2005a, p. 393). However, the contributions of this thesis now imply, on purely logical grounds, that we have a surfeit of objective costs-of-production theories to choose from.

Ricardo wanted to find an external standard to measure economic value. Any standard would do, although he favoured labour. Marx, in contrast, claimed that labour is the measure of value in virtue of the causal regularities of commodity exchange (see Brown (2008)). In a sense, Ricardo wanted to measure value whereas Marx wanted a theory of how economic value in fact measures. Each author represents a different kind of answer to the semantic question.

Earlier, I suggested that possible answers to the value question include "some specific thing", "many things" or "nothing". However, other kinds of answers are possible. Both human speech acts and mechanical control systems exhibit forms with imperative content: for example, the command to "sit down" or the control signal produced by a thermostat connected to a heating element. Imperatives do not denote an actual state-of-affairs and therefore do not measure anything. Rather, imperatives seem to be related to the absence, rather than the presence, of things. In consequence, the classical focus on measurement may constitute an insufficient basis for a complete theory of economic value.

Answering the semantic question requires stepping outside the field of economics, for a time, in order to investigate the philosophical literature on theories of semantic content.[3] I think this type of interdisciplinary work will be necessary in order to fully, and satisfactorily, answer the value question.

8.3 Final remarks

The generalisation of the classical labour theory of value, proposed in this thesis, preserves its hard core, yields a theory with greater explanatory power, and opens up new research directions that promise to extend its empirical content. The clas-

[3] For those interested in the semantic question see Wright (2014c).

sical theory therefore constitutes a progressive research programme in the sense of Lakatos (1978).

Marx's letter to Kugelmann, quoted at the front of this thesis, sketches a vision of a market economy as a social practice that adaptively allocates the total labour of society according to social need via monetary control signals. The general law of value, proposed in this thesis, supports Marx's claim that money represents labour time in virtue of the causal regularities that emerge from this practice.

Inevitably, the classical approach to economic value, due to its radical implications, remains a path less trodden. Nonetheless, the results of this thesis imply that the science of economics – if it aims to understand the relationship between the material activity of human labour and the monetary forms of an economy – will make progress by taking it.

Chapter 9

Appendices

9.1 Appendix to Chapter 2

The super-integrated labour-values, $\tilde{\mathbf{v}}$, can be directly measured without reference to monetary phenomena and the construction of the capitalist consumption matrix, $\mathbf{C}$. First note the following alternative, but equivalent, definition of $\tilde{\mathbf{v}}$:

Proposition 15. *"Super-integrated labour-values", $\tilde{\mathbf{v}}$, are the solution to the system of equations,*

$$\tilde{\mathbf{v}}\mathbf{A}^{+} = \tilde{\mathbf{v}}\lambda \tag{9.1}$$

$$\tilde{\mathbf{v}}\mathbf{w}^{T} = \mathbf{l}\mathbf{q}^{T}, \tag{9.2}$$

where $\mathbf{A}^{+} = \mathbf{A} + \frac{1}{\mathbf{l}\mathbf{q}^{T}}\mathbf{w}^{T}\mathbf{l}$ is the technique augmented by workers consumption, and λ is the dominant eigenvalue of $\mathbf{A}^{+}$.

Proof. In a steady-state economy (definition 4), $\mathbf{p} = (\mathbf{pA}+\mathbf{l}w)(1+r)$ and $\mathbf{p}\mathbf{w}^{\mathrm{T}} = \mathbf{l}\mathbf{q}^{\mathrm{T}}w$. Hence, $w = \frac{\mathbf{p}\mathbf{w}^{\mathrm{T}}}{\mathbf{l}\mathbf{q}^{\mathrm{T}}}$, and $\mathbf{p} = (\mathbf{pA} + \mathbf{l}\frac{\mathbf{p}\mathbf{w}^{\mathrm{T}}}{\mathbf{l}\mathbf{q}^{\mathrm{T}}})(1+r) = \mathbf{p}(\mathbf{A} + \frac{1}{\mathbf{l}\mathbf{q}^{\mathrm{T}}}\mathbf{w}^{\mathrm{T}}\mathbf{l})(1+r)$. By Theorem 1, $\tilde{\mathbf{v}} = \tilde{\mathbf{v}}(\mathbf{A} + \frac{1}{\mathbf{l}\mathbf{q}^{\mathrm{T}}}\mathbf{w}^{\mathrm{T}}\mathbf{l})(1+r) = \tilde{\mathbf{v}}\mathbf{A}^{+}(1+r)$, which implies $\tilde{\mathbf{v}}\mathbf{A}^{+} = \tilde{\mathbf{v}}\lambda$, where $\lambda = \frac{1}{1+r}$. Also, by Theorem 1, $\mathbf{p}\mathbf{w}^{\mathrm{T}} = \mathbf{l}\mathbf{q}^{\mathrm{T}}w$ and therefore $\tilde{\mathbf{v}}\mathbf{w}^{\mathrm{T}} = \mathbf{l}\mathbf{q}^{\mathrm{T}}$. Equation (9.1) is an eigenvalue equation, which given standard restrictions on the technique augmented by workers consumption, yields a single dominant eigenvalue, $\lambda^{\star}$, and a system of relative super-integrated labour-values. Equation (9.2) fixes the absolute scale. □

In consequence, given a steady-state economy, we can compute its super-integrated labour-values by observing the technique, $\mathbf{A}$, direct labour coefficients,

$\mathbf{l}$, the real wage, $\mathbf{w}$, and the total labour supplied to production, $L = \mathbf{l}\mathbf{q}^{\mathrm{T}}$. Note also this is sufficient information to compute the uniform profit-rate, $r = \frac{1}{\lambda} - 1$.

9.2 Appendix to Chapter 4

For clarity I include a complete numerical example of Theorems 2 and 3.

9.2.1 Numerical example of Theorem 2

We start with a given distribution of real income. The observed parameters are the technique $\mathbf{A} = \begin{bmatrix} 0.1 & 0.2 \\ 0.01 & 0.3 \end{bmatrix}$, direct labour coefficients $\mathbf{l} = \begin{bmatrix} 0.1 & 0.5 \end{bmatrix}$, real wage $\mathbf{w} = \begin{bmatrix} 0.5 & 0.2 \end{bmatrix}$, and capitalist consumption $\mathbf{c} = \begin{bmatrix} 0.05 & 0.01 \end{bmatrix}$.

Quantities, from equation (4.2), are $\mathbf{q} = \begin{bmatrix} 0.68 & 0.31 \end{bmatrix}$. The profit-rate consistent with this distribution of real income, from equation (4.5), is $r = 0.15$. Prices, from equation (4.1), are $\mathbf{p} = \begin{bmatrix} 0.12 & 0.81 \end{bmatrix} w$. The capitalist consumption matrix, from equation (4.6), is $\mathbf{C} = \begin{bmatrix} 0.011 & 0.14 \\ 0.0021 & 0.028 \end{bmatrix}$. The super-integrated labour-values, from equation (4.8), are $\tilde{\mathbf{v}} = \begin{bmatrix} 0.12 & 0.81 \end{bmatrix}$. Hence $\mathbf{p} = \tilde{\mathbf{v}} w$, as per Theorem 2.

Alternatively, start with the observed technique, $\mathbf{A}$, direct labour coefficients, $\mathbf{l}$, and capitalist consumption matrix, $\mathbf{C}$. This is sufficient information to compute the super-integrated labour-values, $\tilde{\mathbf{v}} = \begin{bmatrix} 0.12 & 0.81 \end{bmatrix}$, which then determine the structure of production-prices, $\mathbf{p}$.

9.2.2 Numerical example of Theorem 3

Continuing our example: the 'standard commodity' for this economy, from eigenvector equation (4.10), is $\mathbf{b} = \begin{bmatrix} 1 & 1.048 \end{bmatrix}$ with dominant eigenvalue $\lambda = 0.31$. Sraffa reserves the term 'standard commodity' for the normalised bundle $\alpha\mathbf{b}$, where $\alpha = (1-\lambda)/\mathbf{l}\mathbf{b}^{\mathrm{T}} = 1.107$. Sraffa's "variable quantity of labour", from equation (4.14), is then

$$\omega = \alpha \frac{\mathbf{p}\mathbf{b}^{\mathrm{T}}}{w} = 1.107 \times 0.968 = 1.07.$$

The super-integrated labour-value of the standard commodity, from equation (4.15), is $\tilde{\mathbf{v}}\alpha\mathbf{b}^{\mathrm{T}} = 1.07$, which equals Sraffa's "variable quantity of labour", as per Theorem 3.

9.3 Appendix to Chapter 5

Here, for clarity, I present numerical examples of Theorems 4 and 5 for a $n = 2$ economy, followed by a brief discussion of time-varying final demand.

9.3.1 Numerical example of Theorem 4

The following proposition, which links the technique, capitalist consumption and the profit-rate, will be useful:

Proposition 16. *In a steady-state economy with production prices,* $\mathrm{Tr}(\mathbf{C}\mathbf{A}^{-1}) = \pi$.

Proof. Since capitalists spend what they earn, $\mathbf{pAq}^{\mathrm{T}}\pi = \mathbf{pc}^{\mathrm{T}}$. Hence, $1/\mathbf{pAq}^{\mathrm{T}} = \pi/\mathbf{pc}^{\mathrm{T}}$. From equation (5.11), $\mathbf{CA}^{-1} = (1/\mathbf{pAq}^{\mathrm{T}})\mathbf{c}^{\mathrm{T}}\mathbf{p}$. Substitute for $1/\mathbf{pAq}^{\mathrm{T}}$ to yield, $\mathbf{CA}^{-1} = (\pi/\mathbf{pc}^{\mathrm{T}})\mathbf{c}^{\mathrm{T}}\mathbf{p}$. An inner product is the trace of its outer product, i.e. $\mathrm{Tr}(\mathbf{c}^{\mathrm{T}}\mathbf{p}) = \mathbf{pc}^{\mathrm{T}}$. Hence, $\mathrm{Tr}(\mathbf{CA}^{-1}) = \pi$. □

Consider a stationary economy with (i) technique, $\mathbf{A} = \begin{bmatrix} 0 & 0.5 \\ 0.25 & 0 \end{bmatrix}$ and $\mathbf{l} = \begin{bmatrix} 1 & 2 \end{bmatrix}$ and (ii) capitalist consumption matrix, $\mathbf{C} = \begin{bmatrix} 0 & 0 \\ 0.005 & 0.006 \end{bmatrix}$.

The technique and capitalist consumption matrix determine the super-integrated labour coefficients, i.e. from definition 9, $\hat{\mathbf{v}} = \mathbf{l}(\mathbf{I} - (\mathbf{A} + \mathbf{C}))^{-1} = \begin{bmatrix} 1.74 & 2.89 \end{bmatrix}$.

The technique and capitalist consumption matrix determine the profit-rate, i.e. from Proposition 16, $\pi = \mathrm{Tr}\left(\begin{bmatrix} 0 & 0 \\ 0.012 & 0.02 \end{bmatrix}\right) = 0.02$. The profit-rate determines production-prices, i.e. from equation (5.7), $\mathbf{p} = \mathbf{l}(\mathbf{I} - \mathbf{A}(1 + \pi))^{-1}w = \begin{bmatrix} 1.74 & 2.89 \end{bmatrix} w$.

Hence, $\mathbf{p} = \hat{\mathbf{v}}w$, as per Theorem 4.

9.3.2 Numerical example of Theorem 5

The following proposition, which links the technique, non-uniform growth, capitalist consumption and the profit-rate, will be useful:

Proposition 17. *In a non-uniformly growing economy with production prices,*

$$\mathrm{Tr}\left(\mathbf{C}\left(\mathbf{A}(\pi-g)-\mathbf{\Gamma}\right)^{-1}\right)=1.$$

Proof. Since capitalists spend what they earn, hence $1/Y = 1/\mathbf{p}\mathbf{c}^{\mathrm{T}}$. From equation (5.14), $\mathbf{C}(\mathbf{A}(\pi-g)-\mathbf{\Gamma})^{-1} = (1/Y)\mathbf{c}^{\mathrm{T}}\mathbf{p}$. Substitute for $1/Y$ to yield, $\mathbf{C}(\mathbf{A}(\pi-g)-\mathbf{\Gamma})^{-1} = (1/\mathbf{p}\mathbf{c}^{\mathrm{T}})\mathbf{c}^{\mathrm{T}}\mathbf{p}$. An inner product is the trace of its outer product, i.e. $\mathrm{Tr}(\mathbf{c}^{\mathrm{T}}\mathbf{p}) = \mathbf{p}\mathbf{c}^{\mathrm{T}}$. Hence, $\mathrm{Tr}(\mathbf{C}(\mathbf{A}(\pi-g)-\mathbf{\Gamma})^{-1})=1$. □

Consider a non-uniformly growing economy at time t with (i) technique, $\mathbf{A} = \begin{bmatrix} 0 & 0.5 \\ 0.25 & 0 \end{bmatrix}$ and $\mathbf{l} = \begin{bmatrix} 1 & 2 \end{bmatrix}$, (ii) capitalist consumption matrix, $\mathbf{C} = \begin{bmatrix} 0 & 0 \\ 0.0063 & 0.00088 \end{bmatrix}$, (iii) $g = 0.01$, and (iv) $\mathbf{\Gamma} = \begin{bmatrix} 0.00095 & 0 \\ 0 & 0.0074 \end{bmatrix}$.

This is sufficient information to compute the super-integrated labour-values. From definition 11, $\hat{\mathbf{v}} = \mathbf{l}(\mathbf{I}-\mathbf{A}(1+g)-\mathbf{\Gamma}-\mathbf{C})^{-1} = \begin{bmatrix} 1.75 & 2.91 \end{bmatrix}$.

From Proposition 17, $\mathrm{Tr}\left(\mathbf{C}(\mathbf{A}(\pi-g)-\mathbf{\Gamma})^{-1}\right) = 1$. Solve to yield the profit-rate, $\pi = 0.037$. Production prices, from equation (5.7), are then $\mathbf{p} = \mathbf{l}(\mathbf{I}-\mathbf{A}(1+\pi))^{-1}w = \begin{bmatrix} 1.75 & 2.91 \end{bmatrix}w$.

Hence $\mathbf{p} = \hat{\mathbf{v}}w$, as per Theorem 5.

9.3.3 The trajectory of final demand

Final demand, $\mathbf{n}(t)$, grows exponentially, such that $\mathbf{n}(t) = \mathbf{w}(t) + \mathbf{c}(t) = [w_i(0)\mathrm{e}^{(g+r_i)t}] + [c_i(0)\mathrm{e}^{(g+r_i)t}]$ (see section 5.4.2). In general, given non-uniform growth, that is $r_i \neq r_j$ for some i and j, then matrices $\mathbf{\Gamma}$ and $\mathbf{C}$ and the profit-rate, π, are not constant but vary with time. In consequence, both production-prices, $\mathbf{p}$, and the vertically super-integrated labour coefficients, $\hat{\mathbf{v}}$, vary along the growth trajectory. Nonetheless, Theorem 5 holds at every instant of time. In consequence, the proportionality of production-prices and super-integrated labour-values is a time-invariant property of Pasinetti's model.

9.4 Appendix to Chapter 7

The numerical simulation is available as a Mathematica CDF (computable document format) (Wright, 2014b). Download it from `https://dataverse.harvard.edu/dataset.xhtml?persistentId=doi:10.7910/DVN/27472` (click on `WRIGHT-ClassicalMacrodynamics.cdf` and then click download). To run the code you must also download Wolfram's CDF reader from `http://www.wolfram.com/cdf-player/`.

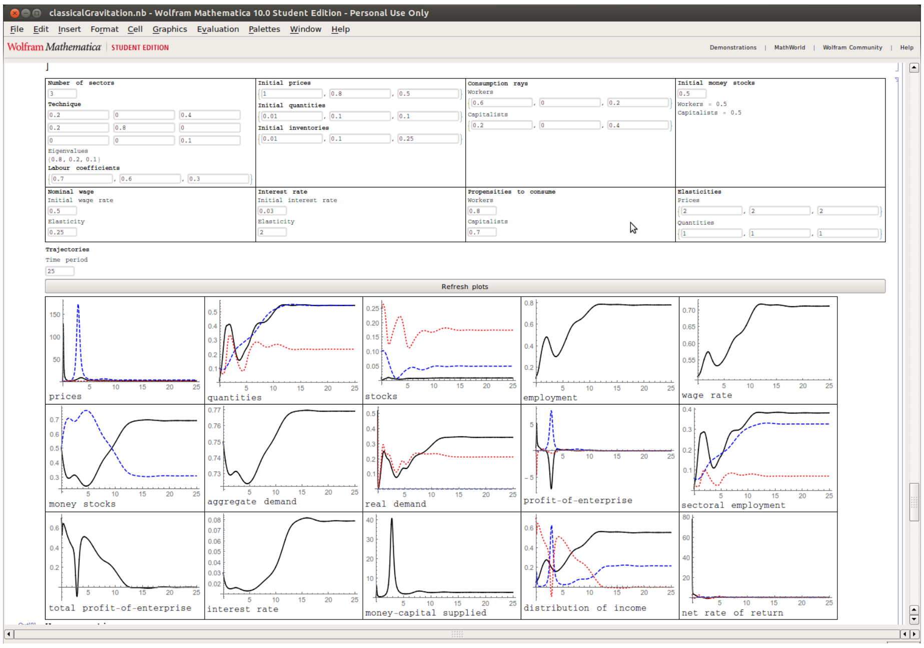
Playing with the simulation is a good starting point for appreciating the rich dynamics the model generates. Start by clicking the bar labelled "Refresh plots" – this will run and plot the results of the simulation. The default parameter settings are those discussed in Chapter 7.

You can interactively change the parameters of the simulation, including the number of sectors, immediately plot the results and also examine the equation system (see Figure 9.1).

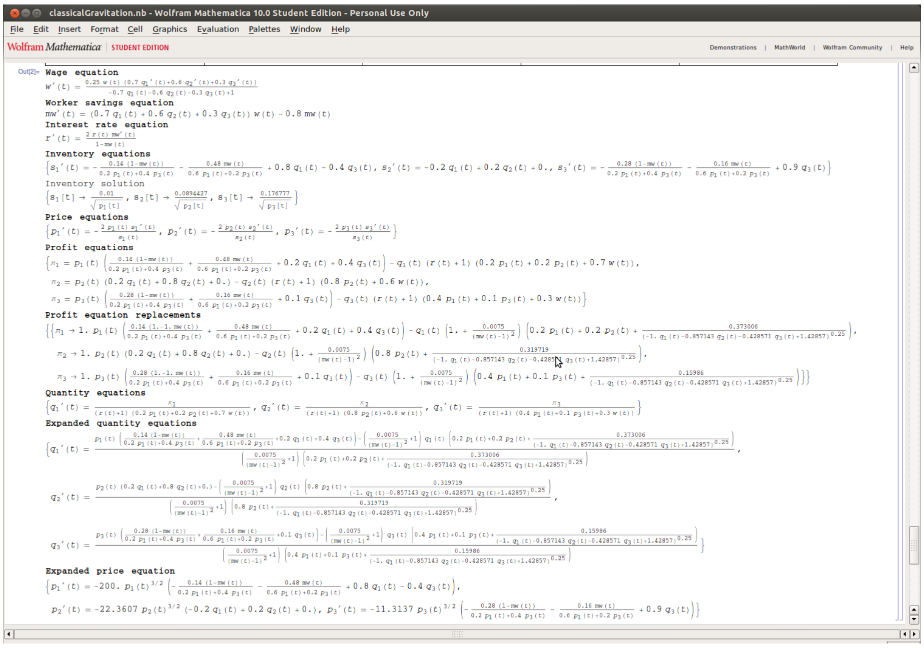
9.4.1 Mathematica code

For experimental reproducibility I include the Mathematica code for the classical macrodynamic system below.

```
DynamicModule[
{
mat={{0.2,0,0.4},{0.2,0.8,0},{0,0,0.1}},
l={0.7,0.6,0.3},
P=Table[Subscript[p, i][t],{i,1,3}],
P0={1,0.8,0.5},
Q=Table[Subscript[q, i][t],{i,1,3}],
Q0={0.01,0.1,0.1},
W={0.6,0,0.2},
K={0.2,0,0.4},
S0={0.01,0.1,0.25},
S=Table[Subscript[s, i][t],{i,1,3}],
NP={2,2,2},
NQ={1,1,1},
w0=0.5,
```



(a)



(b)

Figure 9.1: Screenshots of the interactive numerical simulation of the classical macrodynamic system.

```
nw=0.25,
r0=0.03,
nr=2,
mw0=0.5,
\[Alpha]w=0.8,
\[Alpha]c=0.7,
workersSavingEq=Hold[D[mw[t],t]==l.Q w[t]-\[Alpha]w mw[t]],
wageEq=Hold[D[w[t],t]==nw l.D[Q,t] 1/(1-l.Q) w[t]],
interestRateEq=Hold[D[r[t],t]==nr 1/(1-mw[t]) D[mw[t],t]r[t]],
stockEqSystem=Hold[Table[D[Subscript[s, i][t],t]==Subscript[q, i][t]-(mat[[i]].Q+W[[i]](\[Alpha]w mw[t]/P.W)+K[[i]](\[Alpha]c (1-mw[t])/P.K)),{i,1,3}]],
priceEqSystem=Hold[Table[D[Subscript[p, i][t],t]==-NP[[i]] D[Subscript[s, i][t],t] Subscript[p, i][t]/Subscript[s, i][t],{i,1,3}]],
profitEqSystem=Hold[Table[Subscript[\[Pi], i]==Subscript[p, i][t](mat[[i]].Q+\[Alpha]c (1-mw[t])/P.K K[[i]]+(\[Alpha]w mw[t])/P.W W[[i]])-(P.(Transpose[mat][[i]])+l[[i]]w[t])Subscript[q, i][t](1+r[t]),{i,1,3}]],
quantityEqSystem=Hold[Table[D[Subscript[q, i][t],t]==NQ[[i]] Subscript[\[Pi], i]/((P.(Transpose[mat][[i]])+l[[i]]w[t])(1+r[t])),{i,1,3}]],
wageEqReplacement, interestRateEqReplacement, workersSavingEqReplacement, stockEqSystemReplacement, profitEqSystemReplacement, expandedQuantityEqSystem, expandedPriceEqSystem, computeTrajectories, trajectories, timePeriod=25, computeClassicalLabourValues, plotPrices, pricePlot="", plotQuantities, quantityPlot="", plotStocks, stockPlot="", plotEmployment, employmentPlot="", plotWage, wagePlot="", plotMoneyWealth, moneyWealthPlot="", plotAggregateDemand, aggregateDemandPlot="", plotRealDemand, realDemandPlot="", plotProfitOfEnterprise, profitOfEnterprisePlot="", plotTotalProfitOfEnterprise, totalProfitOfEnterprisePlot="", plotSectoralEmployment, sectoralEmploymentPlot="", plotInterestRate, interestRatePlot="", plotMoneyCapitalSupplied, moneyCapitalSuppliedPlot="", plotDistributionOfIncome, distributionOfIncomePlot="", plotRateOfReturn, rateOfReturnPlot1="", rateOfReturnPlot2="",
styles={Directive[Black],Directive[Dashed,Blue],Directive[Dotted,Red]}
},
(* Remove w[t] by symbolically solving differential equation. *)
wageEqReplacement[wageEq_,Q0_,w0_]:=Module[
{wageEqSolution,wageEqConstants},
wageEqSolution=Flatten[DSolve[Map[#[[1]]==#[[2]]&,Flatten[Solve[wageEq,D[w[t],t]]]],w[t],t]];
wageEqConstants=Flatten[Solve[Map[#[[1]]==#[[2]]&,wageEqSolution],{C[1]}]/.{t->0}];
wageEqSolution/.wageEqConstants/.MapIndexed[Subscript[q, #2[[1]]][0]->#1&,Q0]/.{w[0]->w0}
];
(* Remove r[t] by symbolically solving differential equation *)
```

```
interestRateEqReplacement[interestRateEq_,mw0_,r0_]:=Module[
{interestRateEqSolution,interestRateEqConstants},
interestRateEqSolution=Flatten[DSolve[Map[#[[1]]==#[[2]]&,Flatten[Solve[
    interestRateEq,D[r[t],t]]]],r[t],t]];
interestRateEqConstants=Flatten[Solve[Map[#[[1]]==#[[2]]&,
    interestRateEqSolution],{C[1]}]/.{t->0}];
interestRateEqSolution/.interestRateEqConstants/.{mw[0]->mw0}/.{r[0]->r0}
];
(* Remove mw[t] by symbolically solving differential equation *)
workersSavingEqReplacement[workersSavingEq_,mw0_]:=Module[
{workersSavingEqSolution,workersSavingEqConstants},
workersSavingEqSolution=Flatten[DSolve[Map[#[[1]]==#[[2]]&,Flatten[Solve[
    workersSavingEq,D[mw[t],t]]]],mw[t],t]];
workersSavingEqConstants=Flatten[Solve[Map[#[[1]]==#[[2]]&,
    workersSavingEqSolution],{C[1]}]/.{t->0}];
workersSavingEqSolution/.workersSavingEqConstants
];
(* Remove Subscript[s, i][t] by symbolically solving differential equation.
    *)
stockEqSystemReplacement[stockEqSystem_,priceEqSystem_,numSectors_,P0_,S0_
    ]:=Module[
{stockEqSystemSolution,stockEqSystemConstants},
stockEqSystemSolution=Flatten[DSolve[Map[#[[1]]==#[[2]]&,Flatten[Solve[
    priceEqSystem,D[S,t]]]],S,t]]/.{List[x_,1,t]->List[x,0,t]};
stockEqSystemConstants=Flatten[Solve[MapIndexed[#1[[1]]==#1[[2]]&,
    stockEqSystemSolution],Table[C[i],{i,1,numSectors}]]/.{t->0}];
stockEqSystemSolution/.stockEqSystemConstants/.MapIndexed[Subscript[p,
    #2[[1]]][0]->#1&,P0]/.MapIndexed[Subscript[s, #2[[1]]][0]->#1&,S0]
];
(* Generate expanded quantity equations by substitution *)
expandedQuantityEqSystem[quantityEqSystem_,profitEqSystem_,interestRateEq_,
    mw0_,r0_,wageEq_,Q0_,w0_]:=Module[
{},
quantityEqSystem/.Map[#[[1]]->#[[2]]&,profitEqSystem]/.
    interestRateEqReplacement[interestRateEq,mw0,r0]/.wageEqReplacement[
    wageEq,Q0,w0]
];
(* Generate expanded profit equations by substitution *)
profitEqSystemReplacement[profitEqSystem_,numSectors_,interestRateEq_,mw0_,
    r0_,wageEq_,Q0_,w0_]:=Module[{},
Solve[profitEqSystem,Table[Subscript[\[Pi], i],{i,1,numSectors}]]/.
    interestRateEqReplacement[interestRateEq,mw0,r0]/.wageEqReplacement[
    wageEq,Q0,w0]
];
(* Generate expanded price equations by substitution *)
```

```
expandedPriceEqSystem[priceEqSystem_,stockEqSystem_,numSectors_,P0_,S0_]:=
    Module[{},
priceEqSystem/.Map[#[[1]]->#[[2]]&,stockEqSystem]/.stockEqSystemReplacement[
    stockEqSystem,priceEqSystem,numSectors,P0,S0]
];
(* Solve differential equation system *)
computeTrajectories[expandedQuantityEqSystem_,expandedPriceEqSystem_,
    workersSavingEq_,wageEqReplacement_,timePeriod_,P_,Q_,P0_,Q0_,mw0_]:=
    Module[
{initialPrices,initialQuantities},
initialPrices=MapIndexed[P[[#2]][[1]]==#1&,P0]/.{t->0};
initialQuantities=MapIndexed[Q[[#2]][[1]]== #1&,Q0]/.{t->0};
Flatten[NDSolve[expandedQuantityEqSystem~Join~expandedPriceEqSystem~Join~{
    workersSavingEq/.wageEqReplacement}~Join~initialQuantities~Join~
    initialPrices~Join~{mw[0]==mw0},P~Join~Q~Join~{mw[t]},{t,0,timePeriod}]]
];
(* Store trajectories *)
trajectories=Dynamic[computeTrajectories[expandedQuantityEqSystem[
    ReleaseHold[quantityEqSystem],ReleaseHold[profitEqSystem],ReleaseHold[
    interestRateEq],mw0,r0,ReleaseHold[wageEq],Q0,w0],expandedPriceEqSystem[
    ReleaseHold[priceEqSystem],ReleaseHold[stockEqSystem],Dimensions[mat
    ][[2]],P0,S0],ReleaseHold[workersSavingEq],wageEqReplacement[ReleaseHold
    [wageEq],Q0,w0],timePeriod,P,Q,P0,Q0,mw0]];
(* Compute initial plots *)
plotQuantities[trajectories_,Q_,timePeriod_]:=Module[{},
Plot[Evaluate[Q/.First[trajectories]],{t,0,timePeriod},FrameLabel->{None,
    Style["Subscript[q, i]",FontSize->12]},Frame->False,RotateLabel->True,
    PlotRange->All,Axes->True,ImageSize->250,PlotStyle->styles]
];
plotPrices[trajectories_,P_,timePeriod_]:=Plot[Evaluate[P/.First[
    trajectories]],{t,0,timePeriod},FrameLabel->{None,Style["Subscript[p, i
    ]",FontSize->12]},Frame->False,RotateLabel->True,PlotRange->All,Axes->
    True,ImageSize->250,PlotStyle->styles];
plotStocks[trajectories_,S_,timePeriod_,stockEqSystem_,priceEqSystem_,
    numSectors_,P0_,S0_]:=Module[{},
Plot[Evaluate[S/.stockEqSystemReplacement[ReleaseHold[stockEqSystem],
    ReleaseHold[priceEqSystem],numSectors,P0,S0]/.First[trajectories]],{t,0,
    timePeriod},FrameLabel->{None,Style["Subscript[s, i]",FontSize->12]},
    Frame->False,RotateLabel->True,PlotRange->All,Axes->True,ImageSize->250,
    PlotStyle->styles]
];
plotEmployment[trajectories_,Q_,l_,timePeriod_]:=Module[{},
Plot[Evaluate[l.Q/.First[trajectories]],{t,0,timePeriod},FrameLabel->{None,
    Style["lq^T",FontSize->12]},Frame->False,RotateLabel->True,PlotRange->
    All,Axes->True,ImageSize->250,PlotStyle->First[styles]]
];
```

```
plotWage[trajectories_,Q0_,w0_,timePeriod_]:=Module[{},
Plot[Evaluate[w[t]/.wageEqReplacement[ReleaseHold[wageEq],Q0,w0]/.First[
    trajectories]],{t,0,timePeriod},FrameLabel->{None,Style["w",FontSize
    ->12]},Frame->False,RotateLabel->True,PlotRange->All,Axes->True,
    ImageSize->250,PlotStyle->First[styles]]
];
plotMoneyWealth[trajectories_,timePeriod_]:=Module[{},
Plot[Evaluate[{mw[t],1-mw[t]}/.First[trajectories]],{t,0,timePeriod},
    FrameLabel->{None,Style["{Subscript[m, w],Subscript[m, c]}",FontSize
    ->12]},Frame->False,RotateLabel->True,PlotRange->All,Axes->True,
    ImageSize->250,PlotStyle->Take[styles,2]]
];
plotAggregateDemand[trajectories_,\[Alpha]w_,\[Alpha]c_,timePeriod_]:=Module
    [{},
Plot[Evaluate[\[Alpha]w mw[t]+\[Alpha]c (1-mw[t])/.First[trajectories]],{t
    ,0,timePeriod},FrameLabel->{None,Style["Subscript[\[Alpha], w]Subscript[
    m, w]+Subscript[\[Alpha], c]Subscript[m, c]",FontSize->12]},Frame->False
    ,RotateLabel->True,PlotRange->All,Axes->True,ImageSize->250,PlotStyle->
    First[styles]]
];
plotRealDemand[trajectories_,\[Alpha]w_,\[Alpha]c_,P_,W_,K_,timePeriod_]:=
    Module[{},
Plot[Evaluate[(\[Alpha]w mw[t])/P.W W+(\[Alpha]c (1-mw[t]))/P.K K/.First[
    trajectories]],{t,0,timePeriod},FrameLabel->{None,Style["(Subscript[\[
    Alpha], w]Subscript[m, w]/pw^T)Subscript[w, i]+(Subscript[\[Alpha], c]
    Subscript[m, c]/pc^T)Subscript[c, i]",FontSize->12]},Frame->False,
    RotateLabel->True,PlotRange->All,Axes->True,ImageSize->250,PlotStyle->
    styles]
];
plotTotalProfitOfEnterprise[trajectories_,mat_,mw0_,r0_,Q0_,w0_,timePeriod_
    ]:=Module[{},
Plot[Evaluate[Sum[Subscript[\[Pi], i],{i,1,Dimensions[mat][[2]]}]/.
    profitEqSystemReplacement[ReleaseHold[profitEqSystem],Dimensions[mat
    ][[2]],ReleaseHold[interestRateEq],mw0,r0,ReleaseHold[wageEq],Q0,w0]/.
    First[trajectories]],{t,0,timePeriod},FrameLabel->{None,Style["\[Sum]
    Subscript[\[Pi], i]",FontSize->12]},Frame->False,RotateLabel->True,
    PlotRange->All,Axes->True,ImageSize->250,PlotStyle->First[styles]]
];
plotProfitOfEnterprise[trajectories_,mat_,mw0_,r0_,Q0_,w0_,timePeriod_]:=
    Module[{},
Plot[Evaluate[Table[Subscript[\[Pi], i],{i,1,Dimensions[mat][[2]]}]/.
    profitEqSystemReplacement[ReleaseHold[profitEqSystem],Dimensions[mat
    ][[2]],ReleaseHold[interestRateEq],mw0,r0,ReleaseHold[wageEq],Q0,w0]/.
    First[trajectories]],{t,0,timePeriod},FrameLabel->{None,Style["Subscript
    [\[Pi], i]",FontSize->12]},Frame->False,RotateLabel->True,PlotRange->All
    ,Axes->True,ImageSize->250,PlotStyle->styles]
```

```
];
plotSectoralEmployment[trajectories_,Q_,l_,timePeriod_]:=Module[{},
Plot[Evaluate[l Q/.First[trajectories]],{t,0,timePeriod},FrameLabel->{None,
    Style["Subscript[l, i]Subscript[q, i]",FontSize->12]},Frame->False,
    RotateLabel->True,PlotRange->All,Axes->True,ImageSize->250,PlotStyle->
    styles]
];
plotInterestRate[trajectories_,interestRateEq_,mw0_,r0_,timePeriod_]:=Module
    [{},
Plot[Evaluate[r[t]/.interestRateEqReplacement[ReleaseHold[interestRateEq],
    mw0,r0]/.First[trajectories]],{t,0,timePeriod},PlotRange->All,FrameLabel
    ->{None,Style["r",FontSize->12]},Frame->False,RotateLabel->True,Axes->
    True,ImageSize->250,PlotStyle->First[styles]]
];
plotMoneyCapitalSupplied[trajectories_,mat_,P_,Q_,l_,wageEq_,Q0_,w0_,
    timePeriod_]:=Module[{},
Plot[Evaluate[(P.mat+l w[t]).Q/.wageEqReplacement[ReleaseHold[wageEq],Q0,w0
    ]/.First[trajectories]],{t,0,timePeriod},PlotRange->All,FrameLabel->{
    None,Style["mq^T",FontSize->12]},Frame->False,RotateLabel->True,Axes->
    True,ImageSize->250,PlotStyle->First[styles]]
];
plotDistributionOfIncome[trajectories_,l_,Q_,wageEq_,Q0_,w0_,P_,mat_,
    interestRateEq_,mw0_,r0_,profitEqSystem_,timePeriod_]:=Module[
{wageIncome,interestIncome,profitIncome},
wageIncome=l.Q w[t]/.wageEqReplacement[ReleaseHold[wageEq],Q0,w0];
interestIncome=(P.mat+l w[t]).Q r[t]/.wageEqReplacement[ReleaseHold[wageEq],
    Q0,w0]/.interestRateEqReplacement[ReleaseHold[interestRateEq],mw0,r0];
profitIncome=Sum[Subscript[\[Pi], i],{i,1,Dimensions[mat][[2]]}]/.
    profitEqSystemReplacement[ReleaseHold[profitEqSystem],Dimensions[mat
    ][[2]],ReleaseHold[interestRateEq],mw0,r0,ReleaseHold[wageEq],Q0,w0];
Plot[Evaluate[{wageIncome,interestIncome,profitIncome}/.First[trajectories
    ]],{t,0,timePeriod},FrameLabel->{None,Style["{lq^Tw,(pA+lw)q^Tr,\[Sum]
    Subscript[\[Pi], i]}",FontSize->12]},Frame->False,RotateLabel->True,Axes
    ->True,PlotRange->All,ImageSize->250,PlotStyle->styles]
];
plotRateOfReturn[trajectories_,mat_,Q_,W_,\[Alpha]w_,P_,K_,\[Alpha]c_,l_,
    wageEq_,interestRateEq_,mw0_,r0_,Q0_,w0_,timeStart_,timeStop_]:=Module[
{mee,insetPlot},
(* Marginal efficiency of investment *)
mee=Table[(Subscript[p, i][t] (mat[[i]].Q+W[[i]](\[Alpha]w mw[t]/P.W)+K[[i
    ]](\[Alpha]c (1-mw[t])/P.K))-Subscript[q, i][t] (P.Take[mat,All,{i,i}]+l
    [[i]]w[t])[[1]])/(Subscript[q, i][t] (P.Take[mat,All,{i,i}]+l[[i]]w[t])
    [[1]])-r[t],{i,1,Dimensions[mat][[2]]}]/.wageEqReplacement[ReleaseHold[
    wageEq],Q0,w0]/.interestRateEqReplacement[ReleaseHold[interestRateEq],
    mw0,r0];
```

```
Plot[Evaluate[mee/.First[trajectories]],{t,timeStart,timeStop},FrameLabel->{None,Style["Subscript[r, i]-r",FontSize->12]},Frame->False,RotateLabel->True,Axes->True,PlotRange->All,ImageSize->250,PlotStyle->styles]
];
(* Calculate classical labour values *)
computeClassicalLabourValues[l_,mat_]:=Module[{},
l.Inverse[IdentityMatrix[Dimensions[mat][[2]]]-mat]
];
(* Display interactive dashboard. *)
Column[{
Style[Grid[{
{Column[{
Style["Number of sectors",Bold],
InputField[Dynamic[Dimensions[mat][[1]],(
mat=ConstantArray[0.1,{#,#}];
l=Table[0.1,{i,1,#}];
P=Table[Subscript[p, i][t],{i,1,#}];
P0=Table[1,{i,1,#}];
Q=Table[Subscript[q, i][t],{i,1,#}];
Q0=Table[0.1,{i,1,#}];
W=Table[1,{i,1,#}];
K=Table[1,{i,1,#}];
S0=Table[1,{i,1,#}];
S=Table[Subscript[s, i][t],{i,1,#}];
NP=Table[1,{i,1,#}];
NQ=Table[1,{i,1,#}];
stockEqSystem=Hold[Table[D[Subscript[s, i][t],t]==Subscript[q, i][t]-(mat[[i]].Q+W[[i]](\[Alpha]w mw[t]/P.W)+K[[i]](\[Alpha]c (1-mw[t])/P.K)),{i,1,#}]];
priceEqSystem=Hold[Table[D[Subscript[p, i][t],t]==-NP[[i]] D[Subscript[s, i][t],t] Subscript[p, i][t]/Subscript[s, i][t],{i,1,#}]];
profitEqSystem=Hold[Table[Subscript[\[Pi], i]==Subscript[p, i][t](mat[[i]].Q+\[Alpha]c (1-mw[t])/P.K K[[i]]+(\[Alpha]w mw[t])/P.W W[[i]])-(P.(Transpose[mat][[i]])+l[[i]]w[t])Subscript[q, i][t](1+r[t]),{i,1,#}]];
quantityEqSystem=Hold[Table[D[Subscript[q, i][t],t]==NQ[[i]] Subscript[\[Pi], i]/((P.(Transpose[mat][[i]])+l[[i]]w[t])(1+r[t])),{i,1,#}]];
)&],
FieldSize->4
],
Style["Technique",Bold],
Dynamic[
Table[
With[{x=x,y=y},InputField[Dynamic[mat[[x,y]]],ImageSize->100]],
{x,1,Dimensions[mat][[1]]},{y,1,Dimensions[mat][[2]]}
]//Grid
],
```

```
"Eigenvalues",
Dynamic[Eigenvalues[mat]],
Style["Labour coefficients",Bold],
Dynamic[Table[With[{y=y},InputField[Dynamic[l[[y]]],ImageSize->100]],{y,1,Dimensions[mat][[2]]}]]
}],
Column[{
Style["Initial prices",Bold],
Dynamic[Table[With[{y=y},InputField[Dynamic[P0[[y]]],ImageSize->100]],{y,1,Dimensions[mat][[2]]}]],
Style["Initial quantities",Bold],
Dynamic[Table[With[{y=y},InputField[Dynamic[Q0[[y]]],ImageSize->100]],{y,1,Dimensions[mat][[2]]}]],
Style["Initial inventories",Bold],
Dynamic[Table[With[{y=y},InputField[Dynamic[S0[[y]]],ImageSize->100]],{y,1,Dimensions[mat][[2]]}]]
}],
Column[{
Style["Consumption rays",Bold],
"Workers",
Dynamic[Table[With[{y=y},InputField[Dynamic[W[[y]]],ImageSize->100]],{y,1,Dimensions[mat][[2]]}]],
"Capitalists",
Dynamic[Table[With[{y=y},InputField[Dynamic[K[[y]]],ImageSize->100]],{y,1,Dimensions[mat][[2]]}]]
}],
Column[{
Style["Initial money stocks",Bold],
InputField[Dynamic[mw0],FieldSize->4],
Row[{"Workers = ",Dynamic[mw0]}],
Row[{"Capitalists = ",Dynamic[1-mw0]}]
}]
},
{Column[{
Style["Nominal wage", Bold],
"Initial wage rate",
InputField[Dynamic[w0],FieldSize->4],
"Elasticity",
InputField[Dynamic[nw],FieldSize->4]
}],
Column[{
Style["Interest rate", Bold],
"Initial interest rate",
InputField[Dynamic[r0],FieldSize->4],
"Elasticity",
InputField[Dynamic[nr],FieldSize->4]
```

```
}],
Column[{
Style["Propensities to consume",Bold],
"Workers",
InputField[Dynamic[\[Alpha]w],FieldSize->4],
"Capitalists",
InputField[Dynamic[\[Alpha]c],FieldSize->4]
}],
Column[{
Style["Elasticities",Bold],
"Prices",
Dynamic[Table[With[{y=y},InputField[Dynamic[NP[[y]]],ImageSize->100]],{y,1,Dimensions[mat][[2]]}]],
"Quantities",
Dynamic[Table[With[{y=y},InputField[Dynamic[NQ[[y]]],ImageSize->100]],{y,1,Dimensions[mat][[2]]}]]
}]
}
},Frame->All,Alignment->{Left,Top}],
Small],
Style["Trajectories",Bold,Small],
Style["Time period",Small],
Dynamic[Style[InputField[Dynamic[timePeriod],FieldSize->4],Small]],
(* Graph trajectories *)
Button["Refresh plots",
wagePlot=plotWage[trajectories,Q0,w0,timePeriod];
pricePlot=plotPrices[trajectories,P,timePeriod];
quantityPlot=plotQuantities[trajectories,Q,timePeriod];
stockPlot=plotStocks[trajectories,S,timePeriod,ReleaseHold[stockEqSystem],ReleaseHold[priceEqSystem],Dimensions[mat][[2]],P0,S0];
employmentPlot=plotEmployment[trajectories,Q,l,timePeriod];
moneyWealthPlot=plotMoneyWealth[trajectories,timePeriod];
aggregateDemandPlot=plotAggregateDemand[trajectories,\[Alpha]w,\[Alpha]c,timePeriod];
realDemandPlot=plotRealDemand[trajectories,\[Alpha]w,\[Alpha]c,P,W,K,timePeriod];
totalProfitOfEnterprisePlot=plotTotalProfitOfEnterprise[trajectories,mat,mw0,r0,Q0,w0,timePeriod];
profitOfEnterprisePlot=plotProfitOfEnterprise[trajectories,mat,mw0,r0,Q0,w0,timePeriod];
sectoralEmploymentPlot=plotSectoralEmployment[trajectories,Q,l,timePeriod];
interestRatePlot=plotInterestRate[trajectories,interestRateEq,mw0,r0,timePeriod];
moneyCapitalSuppliedPlot=plotMoneyCapitalSupplied[trajectories,mat,P,Q,l,wageEq,Q0,w0,timePeriod];
```

```
distributionOfIncomePlot=plotDistributionOfIncome[trajectories,l,Q,wageEq,Q0,w0,P,mat,interestRateEq,mw0,r0,profitEqSystem,timePeriod];
rateOfReturnPlot1=plotRateOfReturn[trajectories,mat,Q,W,\[Alpha]w,P,K,\[Alpha]c,l,wageEq,interestRateEq,mw0,r0,Q0,w0,0,timePeriod];
],
Grid[{
{Column[{Dynamic[pricePlot],"prices"}],Column[{Dynamic[quantityPlot],"quantities"}],Column[{Dynamic[stockPlot],"stocks"}],Column[{Dynamic[employmentPlot],"employment"}],Column[{Dynamic[wagePlot],"wage rate"}]},
{Column[{Dynamic[moneyWealthPlot],"money stocks"}],Column[{Dynamic[aggregateDemandPlot],"aggregate demand"}],Column[{Dynamic[realDemandPlot],"real demand"}],Column[{Dynamic[profitOfEnterprisePlot],"profit-of-enterprise"}],Column[{Dynamic[sectoralEmploymentPlot],"sectoral employment"}]},
{Column[{Dynamic[totalProfitOfEnterprisePlot],"total profit-of-enterprise"}],Column[{Dynamic[interestRatePlot],"interest rate"}],Column[{Dynamic[moneyCapitalSuppliedPlot],"money-capital supplied"}],Column[{Dynamic[distributionOfIncomePlot],"distribution of income"}],Column[{Dynamic[rateOfReturnPlot1],"net rate of return"}]}
},Frame->All,Alignment->{Left,Top}],
(* Display equations *)
Style["Wage equation",Bold],
Dynamic[ReleaseHold[wageEq]//TraditionalForm],
Style["Worker savings equation",Bold],
Dynamic[ReleaseHold[workersSavingEq]//TraditionalForm],
Style["Interest rate equation",Bold],
Dynamic[ReleaseHold[interestRateEq]//TraditionalForm],
Style["Inventory equations",Bold],
Dynamic[ReleaseHold[stockEqSystem]//TraditionalForm],
"Inventory solution",
Dynamic[stockEqSystemReplacement[ReleaseHold[stockEqSystem],ReleaseHold[priceEqSystem],Dimensions[mat][[2]],P0,S0]],
Style["Price equations",Bold],
Dynamic[ReleaseHold[priceEqSystem]//TraditionalForm],
Style["Profit equations",Bold],
Dynamic[ReleaseHold[profitEqSystem]//TraditionalForm],
Style["Profit equation replacements",Bold],
Dynamic[profitEqSystemReplacement[ReleaseHold[profitEqSystem],Dimensions[mat][[2]],ReleaseHold[interestRateEq],mw0,r0,ReleaseHold[wageEq],Q0,w0]//TraditionalForm],
Style["Quantity equations",Bold],
Dynamic[ReleaseHold[quantityEqSystem]//TraditionalForm],
Style["Expanded quantity equations", Bold],
Dynamic[expandedQuantityEqSystem[ReleaseHold[quantityEqSystem],ReleaseHold[profitEqSystem],ReleaseHold[interestRateEq],mw0,r0,ReleaseHold[wageEq],Q0,w0]//TraditionalForm],
```

```
Style["Expanded price equation",Bold],
Dynamic[expandedPriceEqSystem[ReleaseHold[priceEqSystem],ReleaseHold[
    stockEqSystem],Dimensions[mat][[2]],P0,S0]//TraditionalForm]
}]
]
```

Chapter 10

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