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ON THE APPLICATION OF MATHEMATICS TO POLITICAL ECONOMY. The Edgeworth-Walras-Bortkievicz Controversy, 1889-1891

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On the Application of Mathematics to Political Economy.

The Edgeworth-Walras-Bortkievicz Controversy, 1889-1891

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I. Introduction

In a letter to the French economist Charles Gide on 3 November 1889, Léon Walras wrote that he had always thought that the application of mathematical reasoning to political economy, 'would have resulted in throwing out phraseology and charlatanism, and in making precision and consciousness reign ('*aurait pour 1er résultat d'en chasser la phraséologie et le charlatanisme et d'y faire régner la précision et la conscience*'). Hence it would have solved any controversies that could remain among the economists. However, the situation was very different in the period of first spread and consolidation of mathematical economics in the 1880s and 1890s after the pioneering works by Jevons, Walras, Marshall and Edgeworth. The new mathematical economics was characterized by some lacerating controversies which involved the most important economists of the time - who favoured the application of mathematics to political economy (or economics, as the economists began to call their discipline) - on different sides of the fence. A dramatically important debate on Walras's theory of exchange involved Edgeworth, Walras and Bortkievicz directly and Marshall indirectly in the years 1889-1891. It was a bitter controversy, in which accusations of abuse of mathematics were leveled by Edgeworth against Walras and of ignorance, lack of understanding, even charlatanism, by Walras against Edgeworth (and Marshall). The theoretical relevance of the topics under discussion and the importance of the economists involved make this controversy not only the most important of that time, but also the most representative of a typical dispute between economists in the history of economics.

The literature on this controversy was dominated for a long time by Schumpeter's pro-Walrasian judgement (1954) that the debate showed Edgeworth's failure to understand Walras, who, according to Schumpeter, achieved a clearness, rigor and theoretical unity that was much greater than Marshall's and Edgeworth's. Jaffé's writings, which eshablished him as *the* authority on Walras, dealt only partially with the debate, without affecting the Schumpeterian judgement. Creedy (1990), Walker (1987a, 1987b, 1996) and some recent papers raised by Walker's challenge to Jaffé's received interpretation of Walras (De Vroey 1999, Bridel 2002), tend to play down Schumpeter's judgement and giving more importance to Edgeworth. However, this recent literature fails to recognize that the central unifying issue in the debate revolved around the role and extent of the use of mathematics in economics. The economists involved in the controversy agreed that mathematics was necessary for deductive reasoning in economics, but disagreed on the extent of its use, an issue that is still not a trivial one in the contemporary economics.

This paper is devoted to the reconstruction and analysis of this debate. Section II traces the history of the controversy, analyzes the two main issues at stake - the equilibrium characteristics and the

process towards equilibrium - and discusses the meaning of the controversy over the role of mathematical reasoning in economics. Section III puts the debate in a historical perspective.

II. The Edgeworth-Walras-Bortkievicz controversy, 1888-1891

A. The history of the controversy

In the summer 1888 Walras, who was preparing the second edition of the *Eléments d'économie politique pure*, sent the proofs of his work to a list of correspondents which included Edgeworth, at that time Professor of Political Economy at Kings College, London. In the letter of 10 August, Walras invited Edgeworth, whom he considered more mathematician than himself ('*plus mathématicien que je ne le suis*'), to make comments on his theories. Edgeworth answered him on 8 November 1888 without making comments on Walras's theories but showing strong appreciation for Walras's work: 'You share with Jevons – Edgeworth wrote - the honour of having dug down to the roots of economic science, of having laid the corner stone of the mighty edifice' (Jaffé 1965, p. 274), i.e. the concept of marginal utility. Then, on February 1889, Edgeworth received the galley proofs of the 'Theorem of the maximum utility of new capital goods' ('Théorème de l'utilité maxima des capitaux neufs') through the English economist Foxwell. On this subject there was an exchange of letters in March and May 1889. Edgeworth explained his doubts and criticisms and Walras attempted to overcome these by trying to explain that Edgeworth simply had not fully understood his thought. At last, in July, 1889, the second edition of Walras's *Eléments* was published. Edgeworth reviewed it in *Nature*, on 5 September 1889. A few days after the publication of the review, on 12 September 1889, Edgeworth delivered his *Presidential Address* to section F of the British Association for the Advancement of Science, entitled 'On the Application of Mathematics to Political Economy', which was then published in *Nature* on 19 September. Here he referred to the theories of the 'Helvetian Jevons', as he called Walras rather pointedly. In these writings Edgeworth criticized three points that Walras considered fundamental to his theoretical work. Firstly, he criticized the theorem on the maximum utility of new capital goods already under fire in his correspondence; secondly the theory of the entrepreneur; and thirdly, the theory of *tâtonnement*. Moreover, Edgeworth declared that he agreed with Walras 'in his plea for the use of mathematical reasoning in economics', but added that the French economist prejudiced 'the case by his advocacy', because of his excessive use of symbols. Actually, the idea that there is an 'excessive elaboration' of mathematical reasoning in the *Eléments*, 'in such a manner as to justify the

particular prejudice against it' (Edgeworth 1889, p. 435) is the factor that unifies Edgeworth's criticism.

The correspondence had made Walras afraid that Edgeworth, whom he considered to be 'a bit enfeoffed to Marshall' ('*un peu inféodé à Marshall'*) (letter to Luigi Perozzo, 13 October 1889), had some reservations about his work[1]. In any case, Walras showed anger and disappointment when he reacted to Edgeworth's review and *Opening Address*, which he read only at the end of October. He called Edgeworth 'a competent mathematician but a mediocre economist' ('*un homme qui paraît assez habil comme mathématicien, mais médiocrement économiste*') in the letter to Maffeo Pantaleoni of 5 January 1890. He accused Edgeworth of 'mathematical phraseology and charlatanism' ('*phraséologie et charlatanisme mathématiques*'). He took Edgeworth to task for unjustifiably criticizing 'the points that I was the surest of' ('*les points dont je suis le plus assuré*') (letter to Charles Gide, 3 November 1889). On October 17 Walras wrote a letter to the young Russian-Polish student Ladislaus Bortkievicz, a person whom he deemed capable of arguing his theory. He wrote: 'It seems to me that the moment has come to clarify this point [the object of pure economics] so that mathematical economics will not wander off on all kinds of sterile fantasies that will discredit it' ('*Il me semble que le moment serait venu de bien fixer ce point si l'on ne veut voir l'économie politique mathématique s'égarer en toutes sortes de fantaisies stériles qui la déconsidèreront*'). He asked Bortkievicz to reply to Edgeworth. In early December Walras received Bortkievicz's paper and was very satisfied of it: 'I found a man capable of reading me attentively and understanding me perfectly, and capable of defending my point of view as well as I can, if not better' ('*J'ai trouvé un homme capable de me lire attentivement, de me comprendre parfaitement et de défendre mon point de vue aussi bien, sinon mieux, que je pourrais le faire moi-même*') (letter of 8 December 1889). He sent it (with a few changes) to Gide, the editor of the *Revue d'économie politique*. He wrote: 'I am sending you an excellent paper that offers an exact idea of my work in the form of a rejoinder (incontestable, according to me) to Edgeworth's criticism' ('*Je vous envoie sous ce pli séparé un article excellent qui, sous forme d'une réponse (tout à fait irréfutable, selon moi) aux critiques d'Edgeworth, donne une idée parfaitement exacte de mon ouvrage*') (26 December). Bortkievicz's paper was published at the beginning of 1890. On 20 February 1890, Walras wrote proudly to Edgeworth : 'This is the answer to your critiques' ('*Voici la réponse à vos critiques*'). Edgeworth answered on 5 March. He still disagreed 'with the writer, but admitted that 'he hits one weak point, namely that I ought not in criticising an author of such eminence to have expressed myself so succintly and without a full array of proofs'. Edgeworth replied the following

[1] On 19 September 1889, Walras also received a short letter from Marshall who gave the *Eléments* a cold reception: 'The right place of mathematics in a treatise on economics is in the back-ground. But I think it is most desirable that different seekers after truth should take different routes'.

year in a long article entitled 'The Mathematical theory of supply and demand and the cost of production' ('La théorie mathématique de l'offre et de la demande et le coût de production') published in the same *Revue d'économie politique*. This article contained a 'fuller statement of [his] matured views', as he put it when he informed Walras about it in advance (letter of 5 March 1890). Walras sent the proofs of Edgeworth's article (in Gide's French translation) to Bortkievicz, hoping that he was ready to go on with the controversy. However, Bortkievicz recognized that Edgeworth raised some actual difficulties and expressed some doubts on his capability of continuing the controversy (see letter of 4/16 February 1891). He then informed Walras of his decision to break off the debate (see letter of 13 September 1891). Next, Walras drew the conclusion that the direct controversy should be stopped:

'Il n'est pas probable que je me décide à intervenir moi-même. Je suis à bout de forces et le moment est venu pour moi de céder la place a d'autres. J'attendrai, s'il le faut, de trouver des hommes sachant que le secret de la science est de mettre au premier plan le cas général et de reléguer au second plan les cas particuliers et les exceptions; car là, en définitive, est le fond de ma querelle avec Edgeworth' (letter to Bortkievicz, 27 February 1891).

From the correspondence with Vilfredo Pareto in the years immediately after the controversy, it seems that Walras hoped that the Italian economist could support his position in the controversy with the 'English school'. Pareto did support Walras's position in his pre-*Cours* writings and in the *Cours* on some specific points of the controversy, but he also expressed an attitude towards the method of economics that was different from Walras's (see Marchionatti 1999). Edgeworth added a short note to his 1889 *Opening Address* when it was re-published in his *Papers Relating to Political Economy* in 1925. Here he took up the controversy again in order to reaffirm his criticism of more than thirty years before and to restate his position on the application of mathematics to economics, which was profoundly different from Walras's.

B. The issues under discussion

The first point under discussion is the concept of the entrepreneur who makes neither a profit nor a loss. This is a characteristic of the equilibrium in production. In the state of perfect equilibrium, when there is equality in the quantities supplied and demanded and equality of price and average cost, profit does not exist, since total profit is the difference between price and average cost multiplied by the number of units of output sold. Hence, in equilibrium the Walrasian entrepreneur makes neither a profit nor a loss (1889, pp. 212-215). The second issue is the process towards equilibrium - the so-called *tâtonnement*. This is the process through which Walras represents the

determination of the equilibrium prices in a competitive market system. Walras calls this 'the very essence of the theory of price determination' ('*l'essence même de la théorie de la détermination des prix*') (letter to Gide, p. 370). This is one of the most controversial issues in Walrasian literature. Walras's aim is to give a presentation of the process of equilibration in the model of exchange and to prove that the relative prices that emerge from the process of free competition are identically the same as the roots of his system of equation. The third issue is a completion of the mathematical theory of equilibrium. This centers around a novelty Walras introduced - the 'Theorem of the maximum utility of new capital goods' (*Théorème de l'utilité maxima des capitaux neufs*) - in the second edition of the *Eléments*. Walras considered his original model of saving incomplete because it did not explain the motives for saving and investment and the way the utility from saving is maximized. Walras also considered his general theorem of maximization of utility inapplicable to new capital goods because of the difference between income and capital. Thus there was a need for a special analysis that took on these issues. Walras considered these three issues fundamental in his equilibrium theory. However, the third issue is not crucial in the Edgeworth's criticism. At first Edgeworth was baffled by this theorem. It was initially rather obscure in Walras's preliminary version. Later Edgeworth came to consider it redundant. He wrote: 'If the price of capital is determined by competition, he wrote, it follows from the general theory of supply and demand that the maximum utility of all the parties concerned is realized in the same sense as in other markets'(Edgeworth 1889a, p. 435). At last he seemed to understand Walras's reasons. In any case he did not consider this theorem in his 1891 article. This reveals Edgeworth's limited interest in deeply penetrating Walrasian logic and following him in his construction of a system. Walras's logic seemed to him too abstract and scarcely useful for the understanding of how the actual equilibrium raises in the markets. The present paper focus on the two other issues, the theory of entrepreneur and the theory of tâtonnement, which represent the core of the controversy.

a. The notion of ideal entrepreneur versus the principle of industrial competition

In his *review* Edgeworth praises Walras for conceiving the entrepreneur 'as buying agencies of production (use of land, labour and capital) and selling finished products in markets, which thus become interdependent' (Edgeworth 1889a, p. 434). However, he strongly criticizes the concept of the 'ideal entrepreneur' who makes neither a profit nor a loss: it is, Edgeworth maintains, an 'extreme abstraction'. He writes: '[Walras] goes too far in the way of abstraction when he insists that the ideal entrepreneur should be regarded as making neither gain nor loss' (ibid.). Walras, Edgeworth continues, confines his attention to final utility. 'His [Walras] view *on this and other*

points would have been more exact', 'if [Walras] had considered the part which the disutility of labour ... plays as a factor of economic equilibrium' (ibid., italics added). In his *Opening Address* Edgeworth repeats this critique in terms of a reproach to Walras for not using, among the factors which determine the equilibrium, the concept of the cost of production considered as importing sacrifice and effort (i.e. in terms of disutility). Edgeworth holds that Walras considers the maximization of individual advantage 'according to the law of final utility' alone. It may be accepted, Edgeworth explains, only to illustrate 'the operation of a simple market' of free competition, where we suppose 'each dealer, before going to market, to write down his scale of requirements – how much he would be willing to buy or sell at each price' (Edgeworth 1889a, p. 545). 'From these data, Edgeworth writes, 'it would be easy to calculate beforehand the rate of exchange which would prevail in the market formed by those individuals' (ibid.). However, according to Edgeworth, Walras's representation cannot be accepted '*when we advance from the simplest type of market to the complexities introduced by division of labour'* (Edgeworth 1889b, p. 281, italics added): in this case 'we could hardly conceive it possible to deduce a priori the position of equilibrium towards which a system so complicated tends' (ibid.), i.e. simple deductive reasoning fails.

Walras and Bortkievicz considered this criticism incorrect. Walras notes that in his system it is assumed that labor does entail disutility: the term personal services is an 'exact synonym for the disutility of labour', he writes to Bortkievicz (27 February 1891). Bortkievicz (1890) writes that Walras left cost of production out of his theory of exchange in which the quantities of the several products were designated as parameters, and introduced the cost of production into his theory of production where these quantities became variables to be determined by a two-fold condition. The condition is that cost of production must equal price and that the quantities demanded of productive services must equal the quantities offered. Hence Walras, Bortkievicz maintains, did not make abstraction of the cost of production considered as importing sacrifice and effort. The reason is that these were included in his theory under another name, 'personal capital services' ('*services des capitaux personelles*'). Pareto (1894) also admits that he did not understand ['*intendere poco*'] much of Edgeworth's criticism. On this point, from the formal point of view, Walras-Bortkievicz reasoning appears to be consistent. As far as the general validity of the Walrasian model is concerned, Bortkievicz thinks that Edgeworth is not clear when he says that the model is valid only in the case of the simplest type of market. Actually, Bortkievicz and Walras (see letters of 25 and 29 December 1889) did not understand what Edgeworth meant by the expression 'complexities

introduced by the division of labour'.[2] Hence Bortkievicz's reply did not touch the real significance of Edgeworth's criticism. In fact, Edgeworth's criticism of the ideal entrepreneur turns out to be a criticism of Walras's mode of conceiving competition, which is considered to be too limited. In particular, with 'complexities introduced by the division of labour' Edgeworth refers to the existence of 'industrial competition', a seemingly classical concept introduced by J. E. Cairnes (1874) and used by H. Sidgwick (1883)[3]. According to Cairnes, industrial competition takes place between the producers of different commodities (industries) and tends to bring wages and profits into correspondence with the sacrifices undergone. On the other hand, commercial competition is what takes place between dealers in the same commodity (industry) and operates towards equality of price (see Cairnes 1874, p. 363). In other words, industrial competition is a force equalizing the remunerations of producers in different industries. Therefore under the assumption of industrial competition, 'normal' values are considered determined by cost of production, as Sidgwick notes (see Sidgwick 1883, p.182). Of course, this expresses a weak version of the theory of cost of production – 'understood in a broad and vague sense', Sidgwick writes. Edgeworth (1889b) used the concept of commercial and industrial competition before his critique of Walras, quoted above. He first considered economic equilibrium without including cost of production explicitly – 'the system of markets .. is that which would arise if all the articles of exchange were periodically rained down like *manna* upon several proprietors'(p. 277) – i.e. commercial competition. Then, he takes account of efforts and sacrifices, so to conceive the equilibrium as the result of the combined effect of utility and cost of production in order to deal with the industrial competition. The final utility of the exchanged articles is equal in equilibrium, Edgeworth writes. Similarly, the final disutilities are equal. Thus the advantages for an individual who balances advantages and costs of an occupation, must be at least as great as in any other position open to him. This condition, Edgeworth writes, can be expressed with the equation of 'the net advantages (or total utilities) in different occupations' – a concept introduced by Marshall in his *Economics of Industry*.[4] The two equations – that of the final

[2] Bortkievicz thought that the expression 'complexities introduced by the division of labour' was an expression lacking in significance ('*dépourvue de toute signification précise*'). Later he acknowledged: "*J'ai cru qu'il avait voulu désigner par là un régime dans lequel chaque individu n'est apte qu'à telle ou telle autre sorte d'occupation ... tandis qu'il avait en vue un état précisément opposé, où chaque individu peut choisir entre différents genres de travail*' (letter to Walras of 4/16 February 1891).

[3] Cairnes and Sidgwick were, together Marshall, the only positive references in these writings of Edgeworth. Cairnes was 'England's first scientific economist' after J. S. Mill's death in 1873 (Schumpeter 1951, 533). Sidgwick was 'one of the greatest English university men ..: milieu-creating, milieu-leading, soul shaping to an extraordinary degree' (Schumpeter 1951, 408). Cairnes's *Some Leading Principles of Political Economy* and Sidgwick's *Principles of Political Economy* expounded the old Ricardian-Millian economics. Their books were usual references in English literature before Marshall's *Principles*. Edgeworth's articles were written - or at least drafted in the case of 1891 article - before the publication of Marshall's *Principles*, as Edgeworth declares in the 1891 article.

[4] Marshall's definition of net advantages is in chapter 7, Book II, of the *Economics of Industry*: 'If the wages and the money equivalents of the other special advantages of a trade be added together into one sum, and the money values of

utility for different kinds of expenditure and that of the net advantages in different occupations – may be considered the conditions of normal economic equilibrium of industrial competition. Consequently, according to Edgeworth, industrial competition, which characterizes the modern economic world, may be represented only by considering the disutility of labour in an 'more explicit' way than Walras's. In 1925 Edgeworth ri-examined in a note the controversy with Walras on this point and reasserts his position:

'Economic theory ... *does require the recognition of the ... industrial competition. ... Walras's peculiar doctrine ... cut him [the entrepreneur] from this [industrial competition] essential principle* ... It is difficult to see how the equality .. of profits in different occupations can be reconciled with this favourite tenet of the Lausanne School. Of course it may be *tolered as an extreme abstraction*, a simplification permissible to a path-breaker. But it seems to deserve pardon rather than praise' (italics added, p. 311).

The fact that Walras did not deal with industrial competition and had stopped short of these complexities is due, according to Edgeworth, to mathematical difficulties. As for the mathematical problem of dealing with industrial competition, Edgeworth writes, and 'it is seen be no longer a straightforward problem in algebra or geometry' (Edgeworth 1889a, p. 545). 'It does not seems easy or helpful to represent by physical analogies' this mode of competition. Unlike commercial competition, which may be likened 'to a system of lakes flowing into each other', industrial competition, Edgeworth writes, may be compared 'to a system of vessels so communicating by means of valves, that when the level in one exceeded that of another to a certain extent, then *per saltum* a considerable portion of the contents of that one (a finite difference as compared with the differentials of the open system) is discharged into the other' (Edgeworth 1889b, p.280). From the mathematical point of view, the introduction of the equation of the net advantages, Edgeworth writes, implies introducing a condition that complicates the problem and goes beyond the resources of the ordinary algebra (in the note h of the *Opening Address* is expressed as equality of the functions of individual utility in different occupations, where the advantage of an individual is a function of his net income, the price of the articles on which his expenditure is made and the disutility of effort)[5]. In his 1891 rejoinder Edgeworth writes that the mathematical tool right for this purpose is the calculus of variations, but he limited himself to a few considerations.[6]

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its special disadvantages be subtracted from the sum, the balance that remain may be called the Net Advantages of the trade' (p. 103). It is a concept which Marshall rarely used after the publication of the *Principles*.

[5] Edgeworth writes: 'The condition that net advantages should be equal in industries between which there is mobility may thus be contemplated. Let us put the advantage of an individual, say r, engaged in the occupation s as a function of his net income, the price of the articles on which his expenditure is made, and the disutility of effort. Say $\phi_{rs}(f_{rs}(\pi_1, \pi_2, \ldots e_{rs}), p_1, p_2 \ldots - e_r)$; where ϕ_{rs} is a utility function, not necessarily the same for the same individual in different occupations ...; f_{rs} ... is the individual's net earnings in the business s, involving prices

From the complexity of the mathematical problem of dealing with industrial competition Edgeworth (1891) deduces the necessity of limiting the use of mathematics:

'Je ne ferai jamais un reproche à un économiste mathématicien de n'avoir pas formulé le problème de la concurrence industrielle. Les représentations abstraites se trouvent toujours en défaut pour représenter la réalité ... J'ai reconnu moi-même qu'au degré de complexité qu'introduit la concurrence industrielle, il est juste de fixer les limites que peut se proposer d'atteindre l'économie politique mathématique' (Edgeworth 1891, p. 26).

Edgeworth thought that Marshall's approach to the issue seemed the correct one. Marshall had just published *Principles*, which Edgeworth enthusiastically reviewed. In the *Principles* Marshall presentsed a concept of competition as a struggle for survival among entrepreneurs which tends to level profits. Marshall gave up the idea of representing this in a general mathematical way. He renounced to use the mathematical tool to deal with a complex phenomenon like industrial competition. Marshall's book, Edgeworth (1891) concludes, makes it unnecessary to discuss the equilibrium of industrial competition mathematically. Edgeworth continues his critique of the Walrasian ideal entrepreneur: 'This entrepreneur who makes neither a profit nor a loss is by now a figure out of place' ('*Cet entrepreneur qui ne fait ni pertes ni gains est désormais un personnage hors de cause*') (p. 28).

Schumpeter (1954) maintains that 'the almost violent aversion displayed towards Walras' concept ... is .. wholly unjustified' [1954, p. 1049, note 59]. According to Schumpeter:

π_1, π_2, etc. of all manner of agents of production, involving also .. the effort e_{rs}; p_1, p_2, etc., are prices of articles of consumption as a function of which the individual's advantage may be obtained ... The last variable in the function, the explicit e_{rs}, has a negative sign prefixed, to indicate that the direct effect of increased fatigue is diminished advantage. The equation of net Advantages imports that the advantage, ϕ_{rr}, of the occupation of which the individual chooses is not less than ϕ_{rs}, the advantage of any other occupation open to him. It is important to observe that for all occupations the complete differential with regard to e is zero; in symbols $\left(\frac{d\phi}{df}\right)\frac{df}{de} + \left(\frac{d\phi}{de}\right) = 0.$" (p. 299-300)

[6] He illustrates the problem as follows: 'Si nous essayons ... de mettre en formules la concurrence industrielle, il convient de considérer les utilités dont on s'occupe non plus simplement comme variant continuellement avec l'accroissement ou le décroissement de variables dont elles représentent une fonction constante, mais aussi *comme variant d'une façon discontinue par suite de changement dans la fonction*. Le problème n'est plus simplement de découvrir ce système de variables, par lequel l'utilité de toutes les personnes que l'on considère se trouve au maximum .., mais de trouver telles fonctions et telles valeurs des variables pour lesquels la formule ne donne pas seulement un maximum, mais la *plus grande valeur possible*.' (Edgeworth 1891, p. 24). In other terms, it is a problem of calculus of variations in which we have to determine that function which maximizes the utility. Edgeworth illustrates the difficulty by reference to a problem discussed in the *Calculus of Variations* by I. Todhunter, a Cambridge mathematician, which has for quaesitum not simply a maximum, but the greater possible value. To determine the course of a ship between two given points, so that the voyage may be accomplished in the shortest possible time. 'A stroke of the pen gives us the marginal conditions, from which it follows that the course must be rectilinear', Edgeworth writes. But a series of tentatives may be required to determine what combination of right lines affords the quickest passage.

'the Marshallian theory, according to which profits have no tendency to vanish, and the Walrasian theory, according to which they do, not only do not contradict one another but, *referred to the same level of abstraction*, turn out to be identical. ... If we are resolved to display the logical properties of perfect equilibrium in pure competition, Marshall's profits will in fact vanish as completely as will Walras'. (p. 1049-50).

Analogously, Walker 1996 maintains that the two constructions are compatible and that the difference between them depends on the differences of definition of profit.[7] These considerations do not seem to catch the essence of Edgeworth's criticism, which lies in the statement that a different conception of economic equilibrium is necessary, able to grasp essentials characteristics of the actual economic world. Edgeworth is interested in understanding the role that mathematics can play in economics in order to improve theoretical reasoning, and in avoiding that, reducing the theoretical structure to mathematical treatment, that structure becomes too simple and poor and therefore too insignificant to interpret the real world. Of course, for Edgeworth, Walras's theory of exchange do not satisfy these conditions because of its use of extreme abstraction.

Pareto showed an ambivalent position. Pareto (1894) is not so different from Edgeworth when he defines the Walrasian case 'an extreme hypothesis' ('*un caso limite*'). On the contrary, in the *Cours* defends Walras against Edgeworth's criticism on the basis of considerations then used by Schumpeter and Walker which were quoted above.[8]

b. Walras's tâtonnement versus Edgeworth's recontracting

In the *Eléments* Walras poses the problem of the relation between the scientific (or theoretical) solution of the exchange and the market solution ('which is solved in practice in the market by the mechanism of free competition', '*celui qui se resout empiriquement sur le marché par le mecanisme de la concurrence*'). He establishes the identity of the two solutions showing that '*le*

[7] To Marshall, profit include interest, and managerial earnings ('what remains of [a businessman's] profits after deducting interest on his capital at the current rate may be called his earnings of undertaking or management' (1890, p. 142, 1920 p. 74)), which are not entrepreneurial earnings in Walras's sense of the term. Marshall's use of the word 'profit' is not the same as Walras's. Edgeworth followed Marshall's usage. 'Far from denying that Marshallian profits exist in equilibrium, Walras emphasized that in that situation, as in disequilibrium, capitalists receive interest and entrepreneurs receive an income as managers of their firms. To Walras, the interest that Marshall called a part of profit and that is earned by the businessman who invests in his own business is earned by that person in his role as a capitalist, not as an entrepreneur' (Walker 1996, p. 297).

[8] '*Spesso la scuola inglese non separa il capitalista dall'imprenditore. 'Col pretesto che in realtà è difficile essere imprenditore senza essere al contempo capitalista non distingue queste due funzioni l'una dall'altra. Il termine profitto, ch'essa impiega, significa perciò, a un tempo, l'interesse del capitale e l'utile dell'impresa' Note 1. 'Gli è ponendosi da questo punto di vista, chiudendo sistematicamente gli occhi dinanzi al punto di vista del tutto diverso da cui si pone Walras, che parecchi autori han criticato la teoria dell'imprenditore che non fa guadagni, né perdite. Han supposto si volesse così annullare il guadagno o la perdita dei possessori di certi capitali, mentre quel che si annulla non è che il guadagno o la perdita che potrebbero determinarsi se l'imprenditore avesse il monopolio o non potesse ritirarsi dall'impresa*' (*Cours*, 2, par. 704, n.1)

mécanisme de la hausse et a baisse des prix sur le marché, combiné avec le faite du détournement des entrepreneurs des entreprises en perte vers les entreprises en bénefice n'est rien autre chose qu'un mode de resolution par tatonnement du systeme des equations de ces problèmes' (*Eléments*, 1889, preface). Walras conceives the general market as an auction market and introduces an auctioneer who continues to change prices until supply and demand imbalances with respect to all commodities disappear. As originally formulated (in the three first editions of the *Eléments*), the *tâtonnement*, is conceived to be the abstract model of functioning of an ideal auction market. It is an ideal abstraction of the spontaneous mechanism of competitive markets from which all the minor disturbances obscuring the generality of the analysis are deleted. Walras considers the *tâtonnement* as the way the mechanism of free competition solves his system of equations. In other words, the *tâtonnement* appears to be the image of the equilibrating process in real markets. As Walras writes to Filippo Virgilii in the period of the controversy (see letter of 17 ottobre 1889), pure economics must:

'faire voir que le mécanisme de la libre concurrence amène précisément la résolution par tâtonnement de ce système d'équation fondé sur la double condition de la satisfaction maxima des besoin et de l'égalité de l'offre et de la demande des services et produits: 'd'où il suit que ce mécanisme produit bien la satisfaction maxima. Ainsi se trouve démontré, avec certaines réserves et dans certaines limites, le principe de la libre concurrence ; et c'est là, à mes yeux, un résultat assez importante pour voloir la peine d'élaborer une science'.

Edgeworth interpreted Walras's theory correctly as an attempt to develop a theory of the equilibrating behavior of real competitive markets. Edgeworth writes:

'what the author professes to demonstrate is the course which the higgling of the market takes – the path, as it were, by which the economic system works down to equilibrium'[9].

[9] The Edgeworthian interpretation of *tâtonnement* as a description of the dynamic path of real markets has been shared by many economists: Pareto (1896-7), Lange (1944), Patinkin (1956), Morishima (1977) and, for a long period, Jaffé (for example 1967). Successively, in the 1980s, Jaffé modified his interpretation radically, maintaining that Walras's model of *tâtonnement* is purely static and it is not an attempt to understand the behavior of real market. 'In the light of Léon Walras's normative bias implicit in his model I am now more inclined to consider that the underlying purpose of Léon Walras's *tâtonnement* theory was to portray an empirical possibility or feasible desideratum rather than an empirical fact' (p. 315). 'Seen .. as a whole, Léon Walras's definite theory of *tâtonnement* in exchange proves to be a theory of virtually timeless' (p. 321). In this interpretation, the adjustment towards the Walrasian general equilibrium ought to be considered as taking place instantaneously, i.e. in logical time. In all the editions of *Eléments* Walras writes that, after defining the conditions of general equilibrium of exchange mathematically, he moves to show how the equilibrium solution emerges in practice [*pratiquement or empiriquement*] by virtue of the forces at work within the competitive market mechanism. However, in the last editions – when Walras became aware of the problem of the

Walras's theory of tatonnement, Edgeworth writes in his *Review*, was 'not a very good idea', because Walras had analysed a process of dynamic adjustment towards equilibrium with a model of static equations. According to Edgeworth the equation of exchange are of a static, not dynamic character. Hence they could provide no information as to the path by which the equilibrium is reached: 'Prof. Walras's laboured lessons indicate a way, but not the way of descent to equilibrium' (p. 435), the economic behavior underlying the dynamic path. The determination of prices cannot be brought under one rule.

Bortkievicz (1890) denies that Walras's *tâtonnement* implied a dynamic approach to equilibrium, in the sense that the market adjustments take place among holders of products and capital goods possessing fixed amounts. However, Bortkievicz does not regard Walras's *tâtonnement* as an essentially static and timeless adjustment process. In this, Bortkievicz's evaluation differs from Jaffé (1981).[10] He maintains that Walras did not treat 'dynamics' if that word 'dynamics' is to be understood in the way that Jevons used it – i.e. in order to mean the analysis of a system undergoing changes in asset holding and preferences. Bortkievicz maintains that Walras's model is not a purely static or a mathematical device. Bortkievicz notes that Walras analyzed the 'dynamic' question of the solution of equations of exchange by the raising and lowering of the price. He thinks that

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exchange at dis-equilibrium prices – this statement has to be considered a survival of the position expressed in the first editions. In fact, Walras's viewpoint about the meaning and role of the *tâtonnement* hypothesis substantially changed over time. What is important to say is that for Walras the *tâtonnement* is not a device of analytical simplification (see Ingrao-Israel (1990). It is an ideal simulation of the mechanism working in the actual markets if free competition were to prevail, even if his position slides progressively (without identifying itself) into one of the neo-Walrasian type where the term *tâtonnement* designates the mathematical technique of iteration used by theorists to find a solution to the general equilibrium system of simultaneous equations. Therefore Jaffé's interpretation cannot be shared: Jaffé neglects the fact that Walras follows a model of scientific inquiry (the classical physics) in which there is a strict relationship between scientific abstraction and empirical evidence. Recently a prominent student of Walras, Donald Walker, has launched a comprehensive offensive against it, aiming at rehabilitating the first interpretation of *tâtonnement*. Walker brings a large body of evidence that Walras regarded *tâtonnement* as the image of the equilibrating process in real markets.

[10] Jaffé did not interpret accurately the position taken by Bortkievicz and Walras. They were writing in 1889 about Walras's theories as they stood in that year, whereas Jaffé represented Bortkievicz as supporting an interpretation of ideas that Walras did not have until ten years after Bortkievicz's remarks were made. It is true that Bortkievicz, and sometimes Walras, used the word 'static' to describe an exchange model in which preferences are assumed to be parameters and asset holdings are assumed to be constant prior to trade at the equilibrium prices. Jaffé followed the same terminology, but he used it with an additional and unwarranted meaning that Walras's model does not contain a dynamic process. Bortkievicz did not do this.

Walrasian *tâtonnement* actually corresponds to 'the real process, effectively employed on the market' (1890, p. 85). He observes that Edgeworth was right when he said that there could be more than one method for solving a system of equations. However, what was under considerations, Bortkievicz writes, was not 'a problem of algebra' but 'a question of showing what is the real procedure, actually used in the market, that constitutes the manner of solution of the given equations'. Pareto reasserted Bortkievicz's position in the *Cours*:

'Mr. Walras has shown that the bargaining established with with free competition is the means of solving the equations of exchange by successive attempts. Mr. Edgeworth has objected that the *tâtonnement* process is only one way. He is right; but the way indicated by Mr. Walras is truly that which describes most cases'.
('*Walras ha fatto vedere che il contrattare tra le parti che si viene manifestando con la libera concorrenza è il mezzo per risolvere per tentativi le equazioni dello scambio. Edgeworth ha obiettato che non è che uno dei mezzi. Egli ha ragione; non di meno il mezzo indicato da Walras rappresenta la parte principale del fenomeno economico*') (par. 59)).

All three authors agree that the *tâtonnement* is an ideal representation of a real process but disagree on to the empirical relevance of the Walrasian description. Bortkievicz and Pareto disagree with Edgeworth's view that Walras's account of *tâtonnement* was unrealistic and lacked sufficient generality. Edgeworth (1891) reasserts that the exchange equations were static and not dynamic and so 'the game of all this higgling by which market prices are determined, the direction which the system follows in order to arrive at the position of equilibrium, does not belong to the sphere of science' ('*le jeu de tout ce marchandage par lequel le prix du marché se trouve déterminé, la direction que suit le système pour arriver à la position d'équilibre, ne rentre pas dans la sphère de la science*' (12)).[11] This issue, Edgeworth writes, can be discussed in an abstract form, offering a stylized description of real markets process. However, the problem is to present 'a conception appropriate for a certain kind of facts' ('*une conception appropriée à un certain ordre de faits*') (p. 13). There are others more appropriate ways (ideal types) to determine the market prices besides Walras's. For example, there are Cournot's way and Edgeworth's own way, which he presented in his *Mathematical Psychics*. In Part II of *Mathematical Psychics* Edgeworth investigates the equilibrium of a system of hedonic forces, each tending to maximum individual utility. He assumes that economic agents are free to communicate within 'fields of competition', and enter into contracts and re-contracts. Edgeworth begins with a case of barter and inquires into when the two individuals will reach equilibrium. He concludes that the contract generally does not supply

[11] According to P. Newman (1990), here Edgeworth was echoing Marshall in the *Pure Theory of Foreign Trade* (1879), where he asserted that 'even if we knew exactly the shapes which the curves assumed in any particular problem, we should not have the data on which to base a calculation of the precise path which the exchange point would describe'.

conditions sufficient enough to determine the solution. What the contract supplies is one condition alone. Edgeworth expresses this condition by writing an equation that corresponds to Jevons's equation of exchange. The locus of the points (*x,y*), which satisfy the equation, is called the contract-curve. The equilibrium point is the point where the individuals's lines of indifference coincide. This case, Edgeworth concludes, clearly illustrates the characteristic evil of indeterminate contract. It is an 'undecidable opposition of interests'. The opposite of this, Edgeworth continues, is 'the smooth machinery of the open market'. Referring to Walras, Edgeworth describes the working of this mechanism in the following way:

'you might suppose each dealer to write down his demand, how much of an article he would take at each price, without attempting to conceal his requirements; and these data having been furnished to a sort of market-machine, the price to be passionlessly evaluated' (p. 30).

The 'equilibrium is attained when the existing contracts can neither be varied without recontract with the consent of the existing parties, nor by recontract within the field of competition' (p. 31). Then Edgeworth investigates the degree to which a contract is determinate in cases of imperfect competition. As he proceeeds, he introduces additional competitors into the field until the limit case of perfect market is reached. Here the contract is determined and Jevons's law of indifference is in force. However, Edgeworth emphasizes, in a way different from Walras's, 'here it is attempted to proceed without postulating the phenomenon of uniformity of price by the longer route of contract-curve' (p. 40):

'Proceeding by degrees from the case of two isolated bargainers to the limit case of a perfect market, we see how contract is more or less indeterminate according as the field is less or more affected with the first imperfection, limitation of numbers' (p. 42).

Edgeworth refers to his own re-contracting process as a general case of Walras's special competitive market approach. In other terms, for Edgeworth his re-contracting hypothesis is not only an alternative mechanism but one more general than Walras's *tâtonnement*, whose validity is narrowly restricted to competitive markets. Edgeworth 1925 reformulates his 1891 critique:

'[Walras] describes a way rather than the way by which economic equilibrium is reached. For we have no dynamical theory determining the path of the economic system from any point assigned at random to a position of equilibrium. We only know the statical properties of the position ... Walras's laboured description of prices set up or cried in the market is calculated to divert attention from a sort of higgling which may be regarded as more fundamental than his conception, the process of recontract ... It is believed to be a more elementary manifestation of the propensity to truck than even the

effort to buy in the cheapest and sell in the dearest. The proposition that there is only one price in a perfect market may be regarded as deducible from the more axiomatic principle of recontract' (II, 311-2).

The Edgeworth-Walras-Bortkievicz debate implies the opposition of two different technologies of exchange that reflect two very different conceptions of the core of the theory of exchange - Walras's competitive markets and Edgeworth's fields of competition. As mentioned above, Walras substantially abandoned the realistic interpretation of the mechanism of *tâtonnement* of the first editions of the *Eléments*. This change stems from theoretical problems easily grasped by modern economists: allowing disequilibrium trade prompts endowment and path-dependency effects. These problems do not make Edgeworth's critique vain, as Jaffé maintained, but strengthens it.

C. Some comments

The controversy between Edgeworth and Walras reveals the clash of two different methodological requirements. On the one hand, Edgeworth's requirement of realism. This implies that he rejected the Walrasian level of abstraction as a representation of the general case. The Walrasian case is acceptable only as a case of extreme simplification. Edgeworth proposed a non-mathematical treatment of the issues considered 'complex'.[12] On the other hand, Walras's requirement of formal rigour, simplicity and reduction of economics to mathematical treatment. Walras considered his simple model of free competition as the general case, and the Edgeworthian approach wrong because it subordinated the general case to particular cases. In essence, the controversy may be traced back to the issue of the role of abstract reasoning and the use of mathematics in economics, and ultimately to the two authors's difference about what economics is.

These economists considered mathematics, the 'sovereign science' (Edgeworth), as the guarantee of scientific quality because it allowed for the adoption of rigorously deductive reasoning. The mechanical analogy from classical physics hade been adopted widely. This helped make mathematical language the natural expression of an economic reasoning that seemed clearer and more precise than the language Ricardo or Stuart Mill used. Mathematical calculus seemed the most effective tool for describing and understanding the general quantitative relations of the hypotheses upon which the theory was based. On the analytical level, this new approach permitted noteworthy achievements in consumer theory and in the theory of exchange. These theories started out from a limited number of abstract premises and went on to show a high generality and simplicity.

[12] On the meaning of complexity in Marshall see Marchionatti (2003).

Nevertheless, what is the role and the extension of the use of mathematics in economics ? On this question the opinions slipt. Walras saw mathematical method and language as the natural expression of reasoning in political economy, which he considered a physical-matematical science like mechanics. The whole theory was mathematical and the mathematical expression of the theory was considered a condition of intelligibility. On the contrary, Edgeworth and Marshall emphasize that mathematics has an instrumental and only limited use in economics. They agreed with Walras that mathematics is necessary for deductive reasoning but they limited its use to simple cases. This limitation was due to a different idea of economics. In the *Principles* (1890) Marshall maintains that economics is a science. In fact, it 'deals mainly with just the class of motives which are measurable, and therefore are specially amenable to treatment by scientific machinery' (1890, p.78). However, economics 'must never lose sight of the real issue of life; and these are all … affected more or less by motives that are not measurable' (ibid.). He emphasizes the complexity of human and social subject, which implies that 'economic laws' have some limitations as to exactness, certitude and precision. Marshall writes:

'There is so much variety of economic problems, economic causes are intermingled with others in so many different ways, that exact scientific reasoning will seldom bring us all the way to the conclusion for which we are seeking. It would be foolish on this account to reject its aid so far as it will reach, but something must be left at the end to be done by practical instinct and trained common sense' (1890, p. 88).[13]

In his *Opening Address*, Edgeworth assumes a substantially Marshallian position as to the role of mathematics in economics. According to him, Marshall, 'of all mathematical economists', 'has best complied with his own maxim that the economist, while he employs systematic reasoning must never lose sight of the real issues of life'. This is what he wrote in his review of the first edition of Marshall's *Principles of Economics* (published in *Nature*, 1890), where he emphasized that Marshall establishes the mathematical method in its proper position. In the *Address* Edgeworth lists the limits of a mathematical theory of economics along Marshallian lines. He emphasizes two points - the lack of quantitative data and the necessary shortness of the abstract reasoning:

'In our subject, unlike physics, it is not often clear what is the prime factor, what elements may be omitted in a first approximation ... Imagine an astronomer hesitating whether in the determination of Jupiter's movements the sun or the planet Saturn played the most important part. That is the condition of many of our speculations ..... Another point of contrast with mathematical physics is the brevity of our calculations. The whole difficulty is in the statement of our

[13] All the references to Marshall are from the first (1890) edition of the *Principles*, which was the edition Edgeworth knew at that time. A discussion of Marshall on nature and method of economics based on the successive editions of the *Principles* is in Marchionatti (2002) and (2003).

problems ... Scarcely has the powerful engine of symbolic language been applied, when the train of reasoning comes to a stop' (p. 551)

Like Edgeworth, Marshall, in the various editions of the *Principles*, emphasizes that deductive reasoning should function in economics not to forge a few long chains of reasonings but rather many short chains and single connecting links. It is illusory to think that there is room for long chains of deductive reasoning in economics since economic material is often inadequate to resist the strains of the mathematician machinery (see *Principles*, Appendix C and D). The nature of economic material is what limits the use of mathematics. The whole of a complex real-life problem cannot be grasped in a series of equations. Edgeworth concludes that 'our little branch of learning is of quite rudimentary form' and that 'the solid structure and regular ramifications of the more developed mathematical sciences are wanting' (ibid).

Hence, these different concepts of the nature of economics that separate Walras, Marshall and Edgeworth explain the difference in their extension of mathematics in economics and the difference in their attitude towards the use of abstraction. The greater realism of the hypotheses and models that is implicit in Marshall and Edgeworth makes them consider Walrasian theories spoiled by an excess of abstraction. On the contrary, Walras considered Marshallian and Edgeworthian claims for realism a proof that their approach was scientifically inadequate.

III. The controversy in a historical perspective. Concluding remarks

The central issue in the Edgeworth-Walras-Bortkievicz controversy was the role and extension of the use of mathematics in economics. Whereas Walras saw in mathematics the natural expression of economic reasoning, Marshall and Edgeworth emphasised that deductive reasoning is essential in dealing with ecnomic questions, but is by no means the economist's type of reasoning.

This crucial issue appears again in the work of Vilfredo Pareto, the leading figure in mathematical economics between the 1890s and the First World War. His position has many points of contact with that of the English economists. In his first theoretical article 'Di un errore del Cournot nel trattare l'economia politica colla matematica' (1891), Pareto warns that: 'the use of algebrical symbols sometimes mislead ... because it gives an apparent rigour to the reasoning' ('*l'uso dei simboli algebrici trae in inganno alcune volte ... perchè da un'apparenza di rigore al ragionamento*') (p. 12). He adds that 'the error to be avoided is to believe that a reasoning which starts from dubious premises acquires rigor only because algebrical symbols are used' ('*L'errore che devesi scansare è di credere che un ragionamento, il quale muove da incerte premesse, acquisti*

maggiore rigore solo perchè vi si usano simboli algebrici') (ibid.). In fact, Pareto continues, Cournot's use of symbols and rigorous reasoning in his treatment of tariff protection did not achieved his goal of clearing up the problems raised in endless controversies. Pareto concludes that 'we must proceed with caution in treating political economy using mathematical analysis' ('*occorre procedere guardinghi nel trattare con l'analisi una scienza come l'economia politica*') (p. 14), because the principles of political economy are not deduced by the rigorous axioms of mechanics and astronomy. In his 'Considerazioni sui principi fondamentali dell'economia politica pura' Pareto's remarks follows Edgeworth's and Marshall's argument:

"quanto più s'allunga la catena delle deduzioni, quanto più il ragionamento accenna a divenire quasi un'operazione meccanica, come avviene coll'uso dei simboli algebrici, tanto maggiori divengono le probabilità di errori, che provengono dall'incertezza delle premesse" (p. 399)

In the Paretian era mathematical economics is dominated by the problem of realism (see Marchionatti 2004). The abstractness of mathematical economics made it extremely difficult to apply its conclusions to the explanation of actual facts. Actually, mathematical economics was in a more unfavorable situation than rational mechanics. The main issues under discussion in the early years of the new century were the theory's excessive abstraction and the unreality of its assumptions and models rather than its formal aspects. These economists generally seemed not to be worried about the formal establishment of equilibrium, an issue that dominated mathematical economics later. They are chiefly interested in the problems connected with the relationship between mathematical expression and experimental reality.

In the 1930s the theoretical and methodological framework was modified. The axiomatization of the economic theory permitted mathematical developments that were free from problems of the realism of the model used. In a certain sense, we could say that this was Walras's revenge. The week link with actual economic reality established by Walras could be abandoned, making the theory free from requirements of realism. This Walrasian project has dominated economic theory for many decades of the tweentieth century. However, the end of the past century saw a renaissance of the Marshallian approach. Is the controversy a never ending one ?

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