

A Profit-Rate Invariant Solution to the Marxian Transformation Problem

By
Jean-Guy Loranger
(Latest revision, July 2002)

Abstract

The aim of this paper is to re-examine Samuelson's answer to Marx solution of the transformation problem between value and price. Although this article around the labour value theory appears to be an old fashion debate, at least for mainstream economists, the role of human capital (or labour force) remains at the heart of the growth of productivity and value added. It will be demonstrated that, although Marx's solution to the transformation problem can be modified, his basic conclusion remains valid: the interdependence between labour values and prices is maintained. The proposed alternative solution presented here is based on the constraints of a general profit rate common to labour values and prices and a money wage level determined simultaneously with prices. The labour cost is determined exogenously in the value space by a given exploitation rate (based on a historical cost as the other input coefficients) while it is determined endogenously in the monetary space. This solution rejects the hypothesis of a given subsistence basket as the basis for the justification of the exploitation rate in the labour value space. Therefore, it diverges from the traditional Marxian or neo-Ricardian solution. This solution also diverges from the Dumenil-Foley-Lipietz "new solution" or interpretation which assumes that the money wage rate is exogenously determined while the general profit rate is endogenously determined with prices. Beyond the historical debate, this new approach shows that the labour value theory can still be a modern alternative approach to the price theory and that it is logically consistent with a static general equilibrium framework even though this view is rejected by the Temporal Single System approach.

Key words: Transformation, value, price, surplus-value, profit, wage, capital, labour, equilibrium
JEL classification: D-4, D-5, J-3, L-1, P-1.

Department of Economics
University of Montreal
P.O. Box 6128, Montreal, QC
H3C-3J7, Canada

Phone (514) 343-5961
Fax (514) 343-7221
E-mail jean.guy.loranger@umontreal.ca

Introduction

«Every man is rich or poor according to the degree in which he can afford to enjoy the necessities, conveniences, and amusements of human life... The far greater part of them he must derive from the labour of other people, and he must be rich or poor according to the quantity of that labour which he can command, or which he can afford to purchase»
A. Smith (1776, ch. 5, p. 31).

The great classical economists, Smith, Ricardo and Marx, all chose labour as the corner-stone of their theory of value, even if there were substantial differences of interpretation between them. *The problem with which they struggled is how labour values are related to prices.* They all rejected scarcity as the main source of value, although they recognized that there are exceptions for some specific goods such as rare old paintings, whose value has nothing to do with labour but depends alone on the taste of wealthy individuals. Another example is the appropriation of land by landowners: it creates scarcity of land but does nothing to create value and wealth. In this case, scarcity of land influences the distribution of value but it does not create value. Apart from these exceptions, which correspond to natural or social phenomena, value depends on labour and not on scarcity or abundance.¹

The transformation problem is nothing else but a repeated attempt to give a satisfactory answer to the question of how prices are related to labour values. Of course, neo-classical economists and many neo-Marxian economists have given up the idea of finding a satisfactory answer, especially within the framework of a general equilibrium approach, since Samuelson, in his famous article in the *Journal of Economic Literature* (1971), ridiculed those who pretended that there is an interdependence between labour values and prices.² Many economists think erroneously that Marx's labour value equation is flawed when compared to the price equation. Nothing can be farther from the truth. What Samuelson's paper stresses is that there is no link between the two equations on the basis of a chosen set of assumptions for transforming the production costs. The first aim of this paper is precisely to challenge Samuelson's claim and show

¹Marx is more explicit on this issue by identifying an object such as a painting as a use-value which has no (labour) value but has a price. If money is the concrete expression of human labour value, a part of the labour value exists under the form of a use-value which has not contributed to wealth creation.

²« Stripped of logical complication and confusion, anybody's method of solving the famous transformation problem is seen to involve returning from the unnecessary detour taken in Volume I's analysis of values. As I have cited in my mathematical paper, such a 'transformation' is precisely like that in which an eraser is used to rub out an earlier entry, after which we make a new start to end up with the properly calculated entry », Samuelson (1971, p. 421).

that there exists a perfectly logical interdependence between labour values and prices provided that a proper set of assumptions is made in full respect of Marx's approach rather than of some other possible (neo-Ricardian) approach. It will be demonstrated that, although Marx's solution can be modified, namely, the rejection of his hypothesis of the non transformation of costs, his basic conclusions remain valid. This demonstration is a counter-tendency to the rejection of the static (dual) general equilibrium approach strongly advocated by the Temporal Single System (TSS) school.

A second aim is to show that, beyond the historical interest of this debate which is now more than a century old, the labour theory of value is an interesting alternative micro approach to the theory of prices because labour values can be viewed as *ex ante* values created in a capitalist production process before their long run equilibrium values (the production prices) are realized through the distribution or exchange process toward which market prices should converge. Of course, in a static general equilibrium approach, the distinction between *ex ante* and *ex post* values may seem irrelevant to those who attributes as much importance to the realization process as to the production process. But if one admits that production is logically anterior to distribution, which is viewpoint adopted in this paper, then *ex post* prices are sequentially determined or dependent of *ex ante* labour values. One of the main interest of the labour value theory is to explain that the origin of value is living labour or human capital, even if technology is far more advanced at the beginning of this new millennium than it was a century ago. Physical capital or dead labour transmits its value but it is labour power which creates all the new value added. It remains a very modern theory to explain productivity growth since the most updated explanation revolves around the quality of human capital.

The new set of assumptions for transforming the production costs presented here is based on

- I) a common general profit rate in labour value and money value, hence determined from a given labour value system and a given exploitation rate ingrained into the system;
- ii) a money wage level which will be determined simultaneously with prices.

This is quite different from the Samuelson or neo-Ricardian approach where the real wage rate is assumed exogenous and the profit rate is determined simultaneously with prices. This new solution will produce results which satisfy the macro constraints relating to the conservation of value and surplus-value and will maintain the interdependence between the labour value equation and the production price equation. Moreover, contrary to many Marxians who reject the general

equilibrium approach based on the linear theory of production³, our demonstration will be based on this general framework. Before entering into a more detailed discussion of the problem, one has to explain a little more about its origin and why it has caught the attention of so many economists over many generations.⁴

This paper will be divided into two parts: the first part will be a review of the literature on the transformation problem. Those who are already familiar with this subject are invited to skip this part and start immediately with the second part which will contain a numerical example of three solutions based on a three-sector model. In order to keep the mathematical presentation to a minimum, the algebra of the formal solution of four different approaches is presented in the appendix.

Part one: The literature on the transformation problem

1.1 The value-price problem

We will begin our review by recalling the labour theory of value of each of the three great classical economists (Smith, Ricardo and Marx) and by outlining some basic differences between them. We will proceed after to review more specifically the various attempts at the transformation problem.

«The value of any commodity, therefore, to the person who possesses it, and who means not to use or consume it himself, but to exchange it for other commodities, is equal to the quantity of labour which it enables him to purchase or command. Labour, therefore, is the real measure of the exchangeable value of all commodities» A. Smith (1776, ch.5, p. 31).

This statement is typical of Smith's command labour theory of value where he specifies that, although value originates in labour, the value of a commodity is not labour incorporated by the production process but is labour value gained through the exchange process. As it will be seen,

³ A. Freeman and A. Kliman (2001,2002) and, in general, all those who share the view of the Temporal Single System (TSS) approach are strong opponent of the static general equilibrium approach.

⁴ It is not our intention to make a thorough review of the debate on the transformation problem but simply to pick up some of the most interesting works which lead to the actual debate and to some new solutions in a static general equilibrium framework which will be presented in this paper. For a more complete coverage of the literature on the subject, the reader is invited to consult Dostaler (1978), Beaud and Dostaler (1993), Eatwell, Milgate and Newman (1987-90), Gill (1996), Laibman (1973), Mandel and Freeman (1984). For a review of the most recent literature of the non simultaneous (dynamic) approach, one should read Freeman, Kliman and Wells (2001).

although this view is more than two century year old, it has gained a new revival with the Temporal Single System school. Therefore, the natural price, around which the market price is fluctuating, is made of the constant capital (fixed and circulating) and the three component parts of value added: wages, profits and rents. Hence, there cannot be any discrepancy between value commanded and long term equilibrium price. Moreover, it implies that there must be some kind of macro equality between value added created by living or direct labour and value added distributed through exchange of commodities.

This macro link is however insufficient to express the connection between the production cost and the sale price. Smith quickly discarded the production approach in favor of the exchange approach in a capitalist economy. The fact that there is no discrepancy between exchange values and prices is probably one of the main reasons why economists consider Smith as the founder of modern microeconomic theory and, to some extent, the inspiration source of the renewal of the labour value theory.⁵ But how are production labour values connected with exchange values on a micro basis? This is a question that Smith answers only in the case of pre-capitalist society: prices are proportional to labour values since there is no rent to be paid and no profit to be distributed to a non-labourer.⁶

Ricardo challenged Smith's labour command theory of value by insisting that it is in the production process and not in the exchange process that value is created by living labour, although he never made the difference between labour and labour power as Marx will do. Like Smith and Marx, Ricardo accepted that some goods have values or prices without being the result of labour. Despite his insistence that labour values are created through the production process, Ricardo saw prices, which are formed through the exchange process, as mere approximations of labour values in a capitalist society:

« ...but I have already remarked that the effect on the relative prices of things, from a variation of profits, is comparatively slight; that by far the most important effects are produced by the varying quantities of labour required for production » (D. Ricardo, ch.1, section 6, p. 25).

⁵ Although it is not our intention to review in detail the Temporal Single System approach, there is a great similarity between Smith's labour command theory and the definition of the monetary equivalent labour time and the measurement of value of commodities which is entirely based in the exchange process in the TSS, (see in particular A. Freeman, 2001, Freeman-Kliman-Wells 2001). However, the TSS school emphasizes market prices instead of Smith's natural prices.

⁶ « As soon as the land of any country has become all private property, the landlords, like all other men, love to reap where they never sowed, and demand a rent even for its natural produce» (ch.6, p. 47).

Because the capital-labour ratio is different between industries, so will be the profit rates and the exchange value (prices) between the various industries, even if the same amount of labour power were employed everywhere. Therefore, prices will differ from values. Ricardo was also preoccupied to impose some form of macro constraint between the production process on one hand and the distribution or exchange process on the other, since the same amount of value added created in the production process by the labour power must be equal to the money value added in the exchange process.

Marx's theory of labour value is based on the notion of abstract labour or labour necessary in an efficient production process:

« A use-value, or useful article, therefore, has value only because human labour in the abstract has been embodied or materialised in it. How, then, is the magnitude of this value to be measured? Plainly, by the quantity of the value-creating substance, the labour, contained in the article...The labour-time socially necessary is that required to produce an article under the normal conditions of production, and with the average degree of skill and intensity prevalent at the time...We see then that which determines the magnitude of the value of any article is the amount of labour socially necessary for its production» (Marx, Volume I, Ch.1, p.38-39).

Since a useful article is produced, it is because there exists a demand for it, otherwise, it would remain a use-value without any exchange value. Therefore, Marx assumes in Volume I and II that exchange values are equal to prices and he makes an equality between the sum of values created in the production process and the sum of prices realized in the exchange process.

It is only in Volume III that he analyses more deeply the distribution process and introduces a discrepancy between values and prices. The title of Part 1 of Volume III « The Conversion of Surplus-Value into Profit and of the Rate of Surplus-Value into the Rate of Profit» is very telling by itself. The distinction between surplus-value and profit is, incidentally, an important one, since Ricardo always confused the two concepts. Surplus-value is the unnecessary (surplus) labour time produced by the labour power in the productive process while profit is a part of the surplus-value distributed to the owners of the means of production after the payment of interests and rents to other capitalists. This distribution of profits even includes profits earned in non-productive sectors. Marx took the problem where Ricardo had left it and brought an answer: Prices deviate from embodied labour values because, under a competitive capitalist regime, total value is created in

the production process but redistributed in the circulation process in such a way that capitalists can appropriate only the portion of surplus which will yield the same profit rate among the various owners of the total social capital. Indeed, As well outlined by Foley (1998), Marx was pursuing the following goals in presenting his solution to the transformation problem: society is organized into classes and the dominant or ruling class is the one that controls the distribution of surplus created in the productive process. The idea of « normal » profit or rent is replaced by the idea of exploitation of the labour power, and competition among capitalists leads them to a long-run equilibrium where they all get the same profit rate whatever their branch of activity.

Even if production costs are the same when valued in abstract labour or in money, prices will differ from values because profits will not be equal to surplus-values at the industry level, since the organic composition of capital is different in each industry. Only in the very special case of equal capital intensity would the profit rate be the same in abstract labour and in money and, hence, prices would be proportional to values.

The totality of value is created in the production process and must be preserved in the circulation or distribution process. There is not an iota of value or surplus-value created in the circulation process. That is why two macro constraints must be preserved between the labour value space and the monetary space. The first macro equality is the sum of labour values measuring gross output which must equal the sum of its monetary values (or more simply stated in the literature, the sum of values is equal to the sum of prices). This constraint is also essential for defining a numeraire, that is the monetary expression of one hour of labour. The second macro constraint is that the sum of surplus-values must equal the sum of profits (including interests and rents).

The problem of the transformation of value into production (or equilibrium) prices boils down to the calculation of such a set of prices which is compatible with an average general profit rate and a wage rate and, even if prices are not proportional to values, they will be determined in such a way as to satisfy the two macro constraints. The latter will preserve the interdependence between the labour value system and the price system. The ultimate goal is to explain what is behind prices. One could assimilate the macro links in the Marxian approach as analogous to the equilibrium condition of the neoclassical approach between (marginal) utility and price. Such links are essential conditions for the determination of long-run equilibrium production prices and show that human labour lies behind the money form of value.

Marx's solution to the transformation problem is based on the hypothesis of the non-transformation of the cost of production (constant and variable costs), whether they are calculated in labour value or in money value. Although the profit rate can be different from one sector to another when calculated in (abstract) labour value, the money profit rate in a competitive world must be the same in all sectors. Marx was well aware that the non transformation of production costs was a simplifying assumption and did not consider that his conclusion would be substantially changed if that assumption were modified⁷. His main conclusion is twofold: on a micro basis, prices differ from values, and this is why a labour value theory is relevant as an explanation because it can predict ex ante the expected production and redistribution of surplus-values (via the exploitation rate) in a capitalist society and give foundation to a price theory.⁸ But on a macro basis, there are two macro constraints which ought to be satisfied if the principle of value conservation is to be preserved. The first constraint is that the sum of values must be equal to the sum of prices, i.e. the aggregate gross production must be equal in both money and abstract labour. The second constraint is that the sum of profits must be equal to the sum of surplus-values. Since the transformation problem is a static analysis similar to a general calculable equilibrium approach⁹, the same amount of value must be preserved in both spaces when one chooses a general equivalent form for measuring prices. As rightfully pointed out by Foley (1998), Ricardo tried to solve this intriguing phenomenon by searching an explanation into an invariant standard of value for which distributional implications of the labour theory of value would hold exactly, but he failed to bring an adequate answer. This search for a precise standard of value will be pursued one century later by Sraffa (1960) and will give rise to the Neo-Ricardian school of economics.

In 1885, after Marx's death, Engels published the second volume of *Capital* and challenged his fellow economists in the Preface to hurry up in finding a proper solution to Ricardo's problem of

⁷ Marx (1967), in Book III, chapter 11, p. 165, says ... «Since the price of production may differ from the value of a commodity, it follows that the cost- price of a commodity containing this price of production of another commodity may also stand above or below that portion of its total value derived from the value of the means of production consumed by it.... Our present analysis does not necessitate a closer examination of this point».

⁸ One should not confuse long-run prices with current market prices which oscillate around these long-run prices.

⁹ The dynamic approach put forward by many other radical economists linked with the International Working Group on Value Theory, (the Freeman-Kliman school) is an attempt to develop an alternative dynamic approach to the static general equilibrium approach. I fear however that the reduction to a single equation system, the market price system, is an attempt to re-invent Adam Smith's command labour theory essentially based on exchange value instead of value created in the production process. Marx would have called that "vulgar economics".

establishing an adequate correspondence between embodied labour values and commodity prices, since Marx would give the answer when the third volume of *Capital* would be published a few years later. Needless to say economists like Böhm-Bawerk (1896), who was already a leader of the Austrian School on the marginal utility theory, were rather impatient to examine with a critical eye Marx's solution to the transformation problem. Böhm-Bawerk was the first to accuse Marx of inconsistency between Volume I of *Capital* where values seem to be always equal or proportional to prices and Volume III where it is now affirmed that it is no more the case. But the Böhm-Bawerk critique did not deliver a serious blow to Marx's approach. His critique of the equality of gross output between the labour space and the money space as a tautology was quickly dismissed by other more serious challengers like L. von Bortkiewicz (1907).

1.2 The origin of the neo-Ricardian solution

Bortkiewicz, who was an admirer of Walras' general equilibrium price system, wanted to formulate Marx's labour value and price system in a general equilibrium framework. Firstly, Bortkiewicz rightly noticed that Marx's transformation is concerned only with output prices and that input costs are not transformed even though Marx admitted himself that values and prices of inputs could differ. With a purely circulating capital model, where advanced capital is equal to capital expended, unless total costs are invariant through the transformation, the denominator of the average profit rate will be affected if costs are transformed. So, it is wrong to take the profit rate calculated from embodied labour values and apply it to the money space in order to deduct the corresponding production prices. Therefore, the profit rate has to be determined simultaneously with the price system. To achieve that, Bortkiewicz took Marx's simple reproduction system of Volume II of *Capital*¹⁰ which he presented into three sectors (means of production goods, wage goods and luxury goods) where values are expressed in social abstract labour units and attempted to transform these values into money prices by applying some transformation coefficients of value to price for each type of good. He had four endogenous variables (three price coefficients and the profit rate) for three equations. The implicit assumption which is made for the wage rate is that it is determined by a subsistence basket of goods¹¹ which is valued in the money space by the price of goods produced in the second sector,

¹⁰ The idea of starting with a simple reproduction model of three sectors was formulated first by Tougan-Baranovsky (1905) by inverting the transformation problem, i.e. start from a three sector model already expressed in production prices and transform it into a labour value model. Tougan-Baranovsky's conclusion is that none of Marx's macro constraints are satisfied and the average profit rate is different between the labour value space and the monetary space.

¹¹ It will be shown in the second part of this paper that the subsistence basket is equivalent to a real wage

the wage-good sector. However, it can be seen that the constancy of the real wage rate hypothesis is a major departure from Marx's approach because, even if the nominal wage rate is the same in labour value and in money, the real wage rate cannot be maintained equal in Marx's solution because the price of consumer good will differ from its labour value. Therefore the hypothesis of a subsistence basket as the basic determinant of wage is not Marx's hypothesis as part of the solution of the transformation problem. The hypothesis of a given subsistence basket of goods will become the standard assumption adopted by all neo-Ricardian solutions to the transformation problem up to the new solution proposed by Dumenil-Foley-Lipietz. Obviously, Bortkiewicz needed to fix a priori one of the four endogenous variables in order to get a unique solution.. He chose the proportion coefficient of the luxury good sector to be equal to one. In doing so, he assumed the equality between value and price for the third sector. But owing to the particularity of the simple reproduction model where the output (or supply) of each sector is equal to its demand and since all profits are spent on buying luxury goods, Marx's second constraint is satisfied, i.e. there is an equality between the sum of profits and the sum of the surplus-values. But the Bortkiewicz solution cannot satisfy Marx's first constraint, i.e. the sum of gross output in the value space is different from the gross output in the monetary space. Moreover, Bortkiewicz showed that the money rate of profit can be determined solely from the first two basic sectors,¹² the luxury good sector has no influence on the average profit rate! That was the first serious critique to Marx's construction between his value system and his price system . But was it a fatal blow? Many people, especially those identified with the general equilibrium approach, thought so. Even nowadays, there are many radical economists who reject the labour theory of value because they consider it flawed on that point. But, before examining other solutions based on different hypotheses which are supposed to avoid that flaw, we will continue the review of other main contributors.

Because Bortkiewicz's critique was not known to the English speaking world before Sweezy (1942,1949) mentioned the Bortkiewicz solution in his Volume *Theory of Capitalist Development* (1942) and published its translation in 1949, a long period of time elapsed before that issue was raised again. The first extension of Bortkiewicz's solution was brought by Winternitz (1948) who pointed out that it was not necessary to limit the case to a simple reproduction model: it could be extended to a more general model of growth of three sectors and the normalizing constraint could

rate which is the same for every worker if the exploitation rate is uniform in all industries.

¹² This is proved by expressing price coefficients in the relative form in the first two equations leaving two variables (the profit rate and the relative price coefficient) whose values are solved by these two equations.

be Marx's first macro constraint, i.e. the equality of total gross output between the labour value space and the monetary space. However, it is impossible to satisfy the second macro constraint, i.e. the sum of surplus-values cannot be equal to the sum of profits. Seton (1957), Okishio (1963) and Morishima and Seton (1961) were the first to generalize the Bortkiewicz solution to an n sector economy by using the input-output model. By imposing some additional constraints to the simple reproduction model, Seton showed that it is possible to satisfy Marx's two macro constraints, but Seton rejected this Marxian solution because the model becomes too restrictive and lacks of generality. Such a Marxian model would be valid (I) only in the case of simple reproduction, (ii) the structure of the whole economy must be the same as the structure of the third sector (the luxury good sector), (iii) the whole economy must be aggregated into three sectors, which implies that the aggregation rule within each sector is that all industries belonging to that sector must have the same value capital-labour ratio.

The decade of the 1970's was rich in publications of critics and defenders of Marx's solution and the dual system of value. The two most outspoken critics were Samuelson (1971) and Steedman (1977) and the main defenders were Morishima (1973), Okishio (1973), Shaikh (1977) and Morishima and Catephores (1978). The main critique made by Samuelson is that, even though there is no logical mistake in Marx's labour value system and its (Ricardian) price system, the two systems are totally independent since one of Marx's macro constraints cannot be satisfied.¹³ Therefore, the labour value system is redundant and should be dropped once for all. The same conclusion was reached by one of the most outspoken neo-Ricardians, Steedman, who questioned the fact that the embodied labour values can be derived from the same technology matrix of the means of production and subsistence which is used to calculate the prices and the average profit rate. So, there is no need to calculate two systems of values deducted from the same basic information system. Moreover, Steedman noted that, if the rate of profit calculated from the labour value space is not the same as the one calculated from the monetary space, then the total profit is not equal to total surplus-value. « Thus not only can one build the theory of profits and prices around the physical schema, rather than the value schema, but one is forced to do so », (Steedman, 1977, p.48). There are here two different reasons for rejecting the labour theory of value. The first reason is not valid if it can be demonstrated that the two systems are **interdependent**, because one could have three types of endogenous variables (labour value,

¹³ «...Profit-price equilibrium is determinable solely from the production coefficient specifying the required labour embodiments of the industries and from the minimum subsistence wage-good requirements. At no stage of the argument is there need for, or benefit from, utilizing the magnitudes ...of Volume I» (Samuelson, 1971, p. 419).

price and the profit rate) solved by a reduced-form system where the exogenous variables are the technology matrix, the output level and some given parameter of distribution.¹⁴ It would be a clumsy argument to say that, because endogenous variables have the same determinants, one of these variables is redundant and can be dropped. But, since the second argument - the independence of the labour value and the price system - is the one already put forward by Samuelson, Steedman simply repeats the same objection in a different way.

On the side of the defenders of the labour theory of value in the 1970's, there is the school of the iterative process: Marx's labour values are only the first approximation of the production prices which can be calculated from the technology matrix of the means of production and subsistence. Okishio (1973) and Shaikh (1977) are the two proponents of that method. This solution has some relation with Ricardo's intuition that labour values are only an approximation to the production prices. But it does not eliminate the stumbling block that one of Marx's constraints is not fulfilled.¹⁵

More interesting is the Morishima-Catephores' iterative solution (1978). The originality of this solution is that the average profit rate is first calculated from the technology matrix of the means of production and subsistence on the production side of the system instead of the monetary side as it is usually done. Morishima-Catephores use this average profit rate already determined from the physical system to impose it to the determination of the price system by defining a matrix which is simply the previous technology matrix times $(1 + r)$ where r is the average profit rate already calculated on the physical side. The beauty of this solution is that it gives a set of prices which is at last compatible with Marx's two macro constraints and the initial price system is equal to Marx's labour values. It is strange that Steedman's critique about the futility of the dual system has imposed itself much more strongly in the literature than the Morishima-Catephores' iterative solution which came out about the same time.

1.3 The new solutions of the 1980's

¹⁴ By definition, a reduced-form system is deduced from a structural system where the endogenous variables are all expressed in terms of the sole exogenous variables or parameters.

¹⁵ In fact, Shaikh (1981) attempts to give an answer to the discrepancy between total profits and total surplus-value. He explains that Marx did not take into consideration the leakage created by the capitalist consumption which appears in the circuit of revenue of capitalists. According to him, by integrating the circuit of capital and the circuit of capitalist revenue, the mystery of the discrepancy between total profit and total surplus-value is finally solved. This argument appears more as a « deus ex machina » than an adequate answer to the objection of the transformation problem raised since Bortkiewicz.

The decade of the 1980's was also rich in new approaches to Marx's transformation problem: the « new solution » proposed independently by Dumenil (1980, 1983) and Foley (1982, 1986) and a simultaneist production price system proposed by Wolff-Roberts-Callari (1982,1984). There is also new developments based on a dynamic approach: it is the Temporal Single System (TSS) approach, the main leaders of which are Freeman and Kliman (1998, 2001, 2002).

The Dumenil-Foley solution¹⁶ is based on the assumption that the wage rate is no more assumed to be equal to the value of a subsistence basket of goods. Its value is determined exogenously by the exploitation rate. Since the exploitation rate is assumed the same in the labour space and in the money space, it implies that the wage rate is fixed and equal in both spaces. Therefore, the labour cost is not subject to transformation. The non transformation of the labour cost is a serious limitation imposed to the solution of the transformation problem. Moreover, Dumenil-Foley insist strongly that the standardization constraint must choose value added instead of gross output value as the monetary expression of labour. This particular choice has the advantage of maintaining the equality between total profit and total surplus-value. The drawback however is, as outlined by Moseley (1993), that the average profit rate is different between the two spaces. A more detailed presentation of the DFL solution will follow in the second part of this paper.

Turning now to the simultaneist production price system of Wolff-Roberts-Callari (1982,1984), the originality is to postulate the equality of constant capital between the labour value and value in equilibrium or production prices, on the ground that, if constant capital is advanced in the production process, it has to be the same amount in the circulation process. This may look like an attempt to escape the transformation of costs, although this time it is the constant cost instead of the variable cost as it is the case in the DFL solution. But W-R-C reject this objection by breaking away from the classical Ricardian approach which postulates pre-given labour value from the physical production process. The over determination approach (influence of the French Marxian structuralist school lead by L. Althusser) commands that the circulation process has an as great influence (if not greater!) in determining the social content of labour value of a commodity than the production process. It is the rejection of the essentialist postulate of labour as a pre-given substance of value in the production process in favour of the existence of the value form (prices) in the exchange process.. It is the rejection of Ricardo's concept of labour value incorporated in the production of goods and a return to Smith's command labour theory in the exchange process.

¹⁶ This solution is sometime called the Dumenil-Foley-Lipietz solution since A. Lipietz published an article on the same topic in 1982. But according to a footnote in Foley (1998), it seems that the two original authors are Dumenil and Foley.

This is also the Benetti-Cartelier (1980) new interpretation at the turn of the 80's which adopts money as the direct expression of abstract labour in measuring value of a commodity. But they go even further by rejecting labour as the substance of value: the substance of value could be anything! Therefore, it is no more a useful concept.

In fact, the originality of the W-R-C construction is a neo-Ricardian production price equation, a labour value equation with constant capital evaluated in production prices and a living labour component for variable capital and surplus value. From a dimensional analysis viewpoint, one has to admit that money measures directly social abstract labour and has the same dimension as living labour. There is also a third relation, the average profit rate, defined in such a way that total profit equals total surplus value and the sum of gross labour value (derived from their labour value equation) is equal to the sum of gross value measured in production prices. This implies among other things that the wage rate is equal to the subsistence basket evaluated in production prices. It also implies that there is no more discrepancy between the average profit rate measured in labour value and in production prices. This is a good point which avoids the criticism made about the previous solutions. One also observes that their definition of labour value are directly dependent on (production) prices instead of the reverse as in the traditional Marxian approach. Labour values of commodities are determined ex post once production prices are known. The "hic" with this approach is that the labour value theory is no more useful as a theory explaining the price system since prices are required as determinants of labour values! Another difficulty with this over determined system is that one can isolate the production price system as a sub-system and determine the profit rate and the relative production prices as a typical neo-Ricardian solution as recommended by the authors. The only difference is in the determination of the nominal price level because of the choice of a different macro constraint for the measurement of gross value of production. If one admits that the choice of a macro constraint can be made in many ways, the W-R-C labour value concept fits one of them.

Although it is not our intention to review thoroughly the TSS (Temporal Single System) approach which has developed since the 90's, one has to look at the reasons for the rejection of the simultaneous dual approach to the transformation problem. Since 1994, A. Freeman and A. Kliman (2001,2002) have organized within the annual Eastern Economic Association a Mini-Conference on The International Working Group on Value Theory. Their condemnation of the dual system and the static general equilibrium approach is total. They consider that the static embodied labour theory approach must be scrapped and the problem of value and price must be

reformulated in a dynamic and non-equilibrium approach. This is a somewhat excessive reaction to Steedman's condemnation of the embodied labour value theory since Freeman and Mandel (1984) already strongly criticized Steedman (1977). They have been influenced by the critique of the essentialist neo-Ricardian approach made by Benetti-Cartelier and other French critical thinkers, as already mentioned in the W-R-C approach, and they have rejected simultaneity and the equilibrium production prices in favour of a dynamic market price approach. They have not however followed Benetti-Cartelier's rejection of labour as the substance of value. Outside the definition of a monetary equivalent of labour (MEL) either from the money value added or the money gross value divided by total living labour used to produce it, there is not much more to be said except that it is a neo-Smithian command labour value concept in the exchange process. The transformation problem is eliminated once for all since labour as a pre-given substance is a redundant concept. An interesting critique of the TSS approach appears in Dumenil-Levy (1998).

If one can now summarize this brief review of the literature on the transformation problem, the origin of which is rooted at the heart of Classical political economy, many attempts have been made to formulate an adequate answer. But, each time some critique has been made in outlining the independence between the labour value system and the production price system, because of the difficulty of closing the two systems which would make them interdependent instead of having one redundant. In most cases, this has led to the rejection of the labour value theory, in particular, the idea that labour expended in the production process, is the substance of value. A new approach will now be presented in the next section which will be based on the dual system in a general equilibrium framework which will fully transform constant and variable costs, keep total costs invariant, maintain the interdependence between the labour value equation and the price equation and satisfy Marx's two macro constraints. Even if the Morishima-Catephores' solution is one answer in that direction, our solution is far more simple: it has nothing to do with an iterative process. It is a final solution obtained in one calculation without any distinction between basic and luxury goods.

Part two: A new approach to the transformation of values into prices

2.1 Theoretical foundations

The justification for our particular choice of hypotheses is based on a return to Marx's approach instead of the neo-Ricardian approach to the transformation problem. It is also a return to the "substantialist" approach which postulates labour as the substance of value. The neo-Ricardian

approach leads to the independence of the two value spaces and, ultimately, to the rejection of the labour value theory. Our approach starts with the DFL main hypothesis: the exploitation rate is given as labour values are given by the technology matrix. Our plea for a new approach can be summarized by the following points.

i) Marx's aim was to demonstrate that the distribution of surplus-value (under the form of profit, interest and rent) is dependent on the average general profit rate which is determined in the first instance in the production process, since production is logically prior to distribution. The average general profit rate is the driving force which pushes capitalists to move in or out of an industry and it is only in the realization process that they will discover whether they fared better or less than the average. In a long run competitive equilibrium, firms fix their prices in order to get the same general profit rate. So entrepreneurs must know its value before determining their prices.

ii) There cannot exist two different general profit rates between the value and the money spaces in a static general equilibrium model where values and prices are interdependent. This is the main departure from the DFL approach.

iii) The calculation of the average profit rate in the labour value space by the ratio of the surplus-value to total capital (constant and variable capital) is logically prior to money profits and it can be done without any reference to monetary magnitudes. As outlined by Samuelson, Steedman and many other neo-Ricardians, labour values and the average profit rate can be determined from purely physical magnitudes which are the input-output coefficient matrix, the labour input vector and *the rate of exploitation which is a given data* and is at the origin of the distribution of the value added created by the labour power. Our difference with respect to them here is the rejection of a given subsistence basket as the determinant of the exploitation rate. The exploitation rate is exogenously determined as any other (input) coefficient by a given state of technical and social relations (or institutions) which are not necessarily inconsistent with the hypothesis of a subsistence basket of goods, but it is not inevitably so.

iv) This implies that, although the money exploitation rate and the money composition of capital are changed through transformation (as in the neo-Ricardian solution), total costs are invariant through transformation, since the equality between total surplus value and profit is maintained because of the constant average profit rate in both spaces.

v) The solution to the transformation problem must therefore be constructed on the interdependence between the two value spaces and, hence, reflect the prior determination of the average profit rate in the labour value space which is used as a constraint in the determination of the prices and the money wage rate.

vi) Although the wage rate is determined in the labour value space by its relation to a given exploitation rate, the income distributed in the monetary space is a certain sum of money which can be used to satisfy a flexible demand. Since the money wage rate is influenced by the profit rate which has among other determinants the given exploitation rate, the wage rate in the money space is dependent upon the wage rate pre-determined in the labour value space. The hypothesis of a flexible demand is also admitted in the DFL approach. But the transformation of all costs implies that the ex post money wage rate is different from its ex ante labour value, the ex post exploitation rate is different from its ex ante value and the ex post money value added is different from its ex ante labour value, which is the case in the neo-Ricardian solution but not in the DFL solution.

vii) This last hypothesis concerning the relative autonomy of the money wage is at the core of the new solution presented in this paper and represents a total reversal of position with respect to the neo-Ricardian solution, which has always assumed the existence of a rigid subsistence basket as part of the augmented matrix of technical coefficients in the value and the price equation and at the heart of the determination of the wage rate in both spaces.

viii) The substitution of the wage rate for the average profit rate as an endogenous variable in the production price equation allows for solving adequately the transformation problem since, with this new solution, all production costs are transformed (and not only the constant cost) and Marx's two macro equalities are preserved: one is used for expressing the monetary value of a unit of (abstract) labour and the other for maintaining the equality between total surplus labour value and total profit money value.

ix) The consequence of this choice is that the only equalities that are maintained between the two spaces are the average profit rate, the gross value of production, the total surplus or profit and total costs. That is what is required for maintaining the interdependence between labour values and prices, including the money wage rate without any redundancy of equation or constraint. In that respect, our solution is also very different from the neo-Ricardian solution where none of these equalities are maintained except the gross value of production.

x) Moreover, this solution shows that there is no more inconsistency between the linear production approach or the general equilibrium approach and the Marx approach. The dual system, which has been the stumbling block for many economists of various tendencies, is more relevant than ever.

In order to minimize the use of mathematics in the core of the text, the linear model of production, the labour value equation, the price equation, the average rate of profit and the

algebraic demonstration of each solution are put in the appendix. We will now present an empirical example of three solutions: the Marx solution, the neo-Ricardian solution and the new solution: the invariant profit rate solution.

2.2 Numerical example

In prior presentations, Loranger (1996, 1997, 1999), a numerical example was used with a two sector economy from a model that was invented with my students. Some small eccentricities were introduced such as two different rates of exploitation and wage rates in the labour value space. Some critics found that more confusing than anything else. So, in this numerical example, I will revert to the very classical model first formulated by Tougan-Baranosky (1905), and re-utilized by Bortkiewicz (1907) and Samuelson (1971). The value tableau contains figures associated with the case of simple reproduction (the zero growth model).

Table 1
A three sector model of simple reproduction.
(Units are in abstract labour time)

sector or department	constant capital(c)	variable capital(v)	surplus value(s)	total
I Production goods	225	90	60	375
II Basic goods	100	120	80	300
III Luxury goods	50	90	60	200
Total	375	300	200	875

As can be seen, the total input required for constant capital is equal to the total output of sector I. Similarly for the two other sectors or departments: the total value added or labour force required $(v+s) = (300+200)$ is equal to the goods consumed and produced (whether basic or luxury). The exploitation rate $e = s/v = 2/3$ in each sector. Note that the exploitation rate is given by the data matrix and also note that by reducing this three sector model to a two sector model, i.e. by aggregating the basic and luxury goods, the exploitation rate would still be the same and one could no more identify the consumer good sector to a subsistence sector. Therefore, the wage rate $\mu =$

$(1+e)^{-1} = 3/5 = .60$. Since the organic composition $\gamma_i = c_i /v_i$ is different for each sector, (i.e. 2.5, 5/6, 5/9), the profit rate earned by each department is different, (i.e. 4/21, 4/11, 3/7). The general average profit rate is $r = 200/675 = .2963$. In order to use the linear production model of input-output, it is necessary to formulate some hypothesis about the distribution of constant capital (c) among the three sectors. Since each c is defined by the following equation

$$c_i = a_{1i} \theta_1 x_i + a_{2i} \theta_2 x_i + a_{3i} \theta_3 x_i, \quad i = 1,2,3,$$

it is possible to determine only three a_{ji} coefficients out of a total of nine. Therefore, to make it easier for calculation, it will be assumed that only the three elements on the diagonal matrix A are different from 0. Of course, this particular choice does not seem very realist but the assumption of a different technical matrix would lead to a different computation and the basic conclusions would remain the same¹⁷. Moreover, it will be assumed that the output quantity vector is simply defined as $x' = (1000 \ 1000 \ 1000)$. Therefore, the other information necessary to the illustration of the new solution is summarized in table 2.

Table 2
Value and technical coefficients¹⁸

sector	output vector x	abstract labour value θ	capital input a_{ji}	labour input b
I	1000	0.375	0.60	0.15
II	1000	0.300	0.33	0.20
III	1000	0.200	0.25	0.15

A) The Marx solution

Marx's production price equation is

$$p = d(\theta A + \mu b), \text{ where } d = (1 + r) = 1.2963 \text{ and } \mu = 0.6.$$

¹⁷ Some critics have pointed out that a more realist hypothesis would be to assume that it is the first row elements of matrix A which are non zero instead of the diagonal elements. Maybe, but why should one ignore the possibility that some consumer goods also enter their production? A more complex specification of the technology matrix would necessitate more a priori assumptions but would not change the points that we want to outline except for different numerical results.

¹⁸ The calculation of the elements on the diagonal matrix is $a_{11} = 225/375 = .6$, $a_{22} = 100/300 = .33$, $a_{33} = 50/200 = .25$. The calculation of the elements of the b vector is simply the value added in each sector / 1000.

Hence,

$$p = 1.2963 \left[\begin{pmatrix} .375 & .30 & .20 \\ .33 & & \\ .25 & & \end{pmatrix} \begin{pmatrix} .60 \\ .33 \\ .25 \end{pmatrix} + .6 \begin{pmatrix} .15 & .20 & .15 \end{pmatrix} \right] = \begin{pmatrix} .408 & .285 & .182 \end{pmatrix}$$

Therefore, assuming that monetary equivalent of one unit of labour is unity, the gross value $px = 875$ is also equal to the gross value θx in the labour space. Moreover, the total cost is by definition $(\theta A + \mu b)x = [(.225 + .09) + (.10 + .12) + (.05 + .09)] 1000 = 675$. Hence, the sum of profits is $(875 - 675) = 200$ which is also the sum of surplus-values. It should be noted that, even with the no transformation of cost hypothesis, Marx's solution does not imply an equality of the real wage rate between the two spaces. Indeed, the measurement of the real wage rate in the labour value space is $\mu/\theta_2 = 0.6/0.3 = 2.0$ and in the money space is $w/p_2 = 0.6/0.285 = 2.11$. *Therefore, the hypothesis of a wage rate determined from a fixed subsistence basket of goods is not part of the Marx solution, although he alluded to that on many occasions.*

B) The invariant profit rate solution

From equations (18b) and (19) in the appendix, w and the price vector p are

$$(18b) \quad w = a / db(I - dA)^{-1}x$$

$$(19) \quad p = a b(I - dA)^{-1} / b(I - dA)^{-1}x.$$

Therefore, the following matrices have to be computed:

$$\begin{aligned} (I - dA)^{-1} &= \begin{vmatrix} 1 - 1.2963 (.60) & & \\ & 1 - 1.2963 (.33) & \\ & & 1 - 1.2963 (.25) \end{vmatrix}^{-1} \\ &= \begin{vmatrix} 4.500 & & \\ & 1.7475 & \\ & & 1.4794 \end{vmatrix} \end{aligned}$$

$$b(I - dA)^{-1} = \begin{pmatrix} .6750 & .3502 & .2219 \end{pmatrix}$$

$$b(I - dA)^{-1}x = 1247$$

$$w = a / db(I - dA)^{-1}x = 875 / 1.2963 (1247) = .5413$$

$$p = a b(I - dA)^{-1} / b(I - dA)^{-1}x = 875 \begin{pmatrix} .675 & .350 & .222 \end{pmatrix} / 1247 = \begin{pmatrix} .473 & .246 & .155 \end{pmatrix}$$

Hence,

$px = (.473 + .247 + .155) 1000 = 875$. Therefore, the gross value in both spaces is equal. The equality of profits and surplus-value is also verified. Indeed, the total surplus-value is

$$(1 - \mu)bx = 0.4(.15 + .20 + .15)1000 = 200.$$

The total profit is calculated from the following matrix product: $r(pA + wb)x$

$$pA = \begin{pmatrix} .473 & .247 & .155 \end{pmatrix} \begin{vmatrix} .60 & & \\ & .33 & \\ & & .25 \end{vmatrix} = \begin{pmatrix} .2838 & .0823 & .0388 \end{pmatrix}$$

$$wb = .54 \begin{pmatrix} .15 & .20 & .15 \end{pmatrix} = \begin{pmatrix} .081 & .108 & .081 \end{pmatrix}$$

$$r(pA + wb)x = .2963(.3648 + .1903 + .1198)1000 = 200.$$

The equality of total costs is also verified from equation (22):

$$(\theta Ax + \mu bx) = (.315 + .220 + .140)1000 = 675$$

$$(pAx + wbx) = (.365 + .190 + .120)1000 = 675.$$

C) The neo-Ricardian solution

From equation (13) in the appendix, the price solution is

$p\lambda = pM$ where $M = (A + cb)$ and $\lambda = 1/(1 + r)$. The column vector c is the subsistence basket which is the real wage $\mu/\theta_2 = .60/.30 = 2.0$. Since the wage good is found in sector 2 only, the first and third element of c are 0. Let $B = cb$. Hence,

$$B = \begin{pmatrix} 0 & (.15 & .20 & .15) \\ 2 & & & \\ 0 & & & \end{pmatrix} = \begin{vmatrix} 0 & 0 & 0 \\ .30 & .40 & .30 \\ 0 & 0 & 0 \end{vmatrix}$$

$$M = (A + B) = \begin{vmatrix} .60 & 0 & 0 \\ .30 & .73 & .30 \\ 0 & 0 & .25 \end{vmatrix}$$

$$|\lambda I - M| = (\lambda - .60)(\lambda - .73)(\lambda - .25) = 0.$$

Hence, the largest characteristic root is $\lambda = .73$ and $(1 + r) = \lambda^{-1} = 1.3698$. therefore, $r = .3698$.

The price equation is $p(.73 I - M) = 0$. Combined with the constraint $px = 875$, the solution is $p = (.5134 \ .2225 \ .1390)$. It is verified that $px = (.5134 + .2225 + .1390) 1000 = 875$. The wage rate is

$w = 2p_2 = 0.445$. The variable capital $v' = wbx = 0.445(500) = 222.5$. The constant capital is $c' = pAx = 416$. Total profit is $px - (c' + v') = 875 - (416 + 223) = 236$. The value added is 459. The exploitation rate is $e' = 236/223 = 1.058$. The composition of capital is $416/223 = 1.865$. As one can easily verify, all these calculated money values differ from their labour values, which is not surprising since there is an independence between the calculated money values and the given labour values. Table 3 summarizes the results of the three solutions.

Table 3
Comparative results of the three solutions

	Marx	Invariant profit rate	Neo-Ricardian
P_1	0.408	0.473	0.513
P_2	0.285	0.246	0.222
P_3	0.182	0.155	0.139
W= wage rate	0.60	0.541	0.445
R= profit rate	.2963	.2963	.3698
E= exploitation rate	0.667	0.741	1.058
$\Gamma = c/v$	1.25	1.491	1.865
$W/p_2 = \text{real wage rate}$	2.11	2.20	2.00

Conclusion

The main preoccupation of the classical economists was to explain price behaviour from the labour value theory. Ricardo rejected Smith's labour command theory in favour of his incorporated labour value theory which explains prices as approximations of labour values created in the production process. Marx wanted to complete Ricardo's approach by giving a more accurate general equilibrium solution to the interdependence between labour values and prices. Marx never advocated a return to Smith's labour command theory. But the interdependence link was under attack for more than a century, because it was shown that the consequence of transforming the input prices (which Marx neglected to do) would break that link. The failure to solve this puzzle adequately, coupled with the development of the Walrasian general equilibrium approach, caused the labour theory of value to fall into disrepute at the turn of this century. Our new solution is to re-establish the interdependence between labour value and prices and show that the Samuelson condemnation was not without appeal. Therefore, the labour theory of value can still be an interesting alternative explanation of prices in a static general equilibrium framework, because there is an alternative to the neo-Ricardian solution. The price to be paid is however the

rejection of the subsistence basket hypothesis, a price that even Marx himself did not hesitate to pay with his solution based on the non transformation of costs. The money wage rate is determined simultaneously with prices and, like any price, can diverge from its labour value. While the macro links between labour values and prices are maintained, the non equality of the wage rate between the two spaces will give a different money value for the exploitation rate, the capital-labour ratio and the value added. But this is not too high a price to be paid for the conservation of Marx's two macro constraints and transforming all types of costs since the neo-Ricardian solution also leads to the inequality of the same variables between the labour value and the money spaces. One cannot use a model for all types of purposes. The chief purpose here was to give an adequate answer to an objection raised more than a century ago and which had not received a satisfactory answer despite all the efforts made. It is obvious that this theoretical model cannot and will not answer many questions which can be brought in a dynamic approach, such as technological change and many other issues, such as negative values with a rectangular technology matrix, which are not considered here. But we do not agree with the Temporal Single System approach which is equivalent to the replacement of the transformation problem by a tautological definition of the labour value equation. Since all parameters calculated in the labour value space can be viewed as *ex ante* values and those calculated in the monetary space as *equilibrium* values toward which *ex post* market price converge, the strength of a theory or a model is not to rely only on identities but on behavioural assumptions which can be falsified in the real world. The fact that the wage rate can diverge from its expected value would surely have pleased Marx since he mentioned more than once that the money wage rate is not necessarily equal to its embodied labour value but is influenced by it. That is why the labour value theory is still a relevant theory to explain what can happen in the real world since *ex post* money values are dependent of *ex ante* labour values.

Appendix¹⁹

A.1 The linear production model

a) The physical (or primal) equation

Let the following linear production model

$$(1) \quad x = Ax + C$$

$$(2) \quad x_0 = bx$$

¹⁹ The algebra contained in this appendix has already been published in Loranger (1999).

x is an $n \times 1$ output vector, A is an $n \times n$ matrix of technical coefficients of type a_{ij} which measures the quantity of input i per unit of output j , C is an $n \times 1$ vector of final demand or value added in physical term, b is an $1 \times n$ vector of technical labour coefficients where each element b_j measures the quantity of living labour necessary per unit of output and x_0 is a scalar which measures the total quantity of living labour necessary for the production of x .²⁰ The solution of this system is

$$(3) x = (I - A)^{-1} C.$$

Substituting (3) in (2),

$$(2a) x_0 = b(I - A)^{-1} C = b(I + A + A^2 + \dots)C.$$

Hence, the total labour force employed for the production of output x is equal to value added created by direct and indirect labour.

b) The labour value (dual) equation

The dual in the labour value space (or abstract labour) is

$$(4) \theta = \theta A + b$$

where θ is an $1 \times n$ vector of commodity values measured in abstract labour units. Given the exploitation rate e defined and measured by the unpaid labour time over paid labour time, the wage rate μ (or the necessary labour in one hour of labour) is also known²¹, i.e. $\mu = (1+e)^{-1}$. Hence,

$$(5) \theta = \theta A + \mu b + (1-\mu)b.$$

The usual hypothesis accepted by neo-Ricardians (and some Marxians) is

$$(6) \mu = \theta c$$

where c is an $n \times 1$ vector of coefficients associated with the consumption goods of a subsistence basket. θc is therefore the value of the subsistence basket contained in one hour of abstract labour. *This is the hypothesis which is rejected in our approach.*

The solution to the dual is

$$(7) \theta = b(I - A)^{-1}.$$

The definition of an average general profit rate in the labour value space is the ratio of total surplus-value over capital as a whole. By post-multiplying (4) by x , we obtain the following global equation :

$$(8a) \theta x = \theta Ax + bx$$

or

$$(8b) \theta x = \theta Ax + \mu bx + (1-\mu)bx.$$

The average general profit rate is²²

$$(9) r = (1-\mu)bx / (\theta A + \mu b)x = (1-\mu)bx / b [(I-A)^{-1}A + \mu I]x.$$

As can be seen, this general average profit rate can be calculated from the technical parameters A , b , μ and from the output vector x . There is no need to refer to production prices or to other monetary values. Since the labour value space must be linked to the monetary value space, there is no justification for determining it again in the monetary space. **It must be imposed on the monetary space.** If redundancy exists between the average profit rate measured over the two value spaces, it is the profit rate determined in the monetary space which is redundant. This is the main error of interpretation in the neo-Ricardian solution and it is also the weakness of the « new solution » DFL.

²⁰This is a one period model of pure circulating capital, no joint production, and outputs emerge from production after inputs are committed.

²¹By definition, for each hour worked, there is $e = (\text{non necessary time})/(\text{necessary time}) = (1-\mu)/\mu$. Hence, $\mu = (1+e)^{-1}$.

²²The justification of an average profit rate is based on the existence of different sectoral profit rates within the labour value space, since the organic composition of capital differs from one sector to another.

c) The money value or price equation (second dual)

The production price equation with an average profit rate is

$$(10) \quad p = (1+r)(pA + wb)$$

This equation constitutes a second dual with respect to the primal eq(1). The issue here is why should one bother with two dual equations? Samuelson's answer is: erase one and be happy with the price equation only! The marxian answer is: eq(10) is dependent of eq(4)!

A.2 The three existing solutions

a) Marx's solution

Marx's solution is based on the following two hypotheses:

- i) value is preserved, i.e. $\theta x = px$. This is a very important assumption because it sets the monetary expression of one unit of labour equal to unity (or a constant if $\theta x = \beta px$ where β is the monetary equivalent of a labour unit);
- ii) the general profit rate is the same between the two spaces and preserves the equality between the sum of profits and the sum of surplus-value and the equality of the total costs of production between the two spaces.

Marx's production price equation is

$$(11) \quad px = d(pA + wb)x, \quad d = (1+r)$$

Since costs are equal and not transformed from the labour value space to the monetary space, one can directly substitute in (10) for the costs $p = \theta$ and $w = \mu$ and have:

$$(12) \quad p = d(\theta A + \mu b)$$

By substitution of (7) into (12), the price equation becomes

$$(12a) \quad p = db((I - A)^{-1}A + \mu I)$$

Hence, Marx's production prices are a function of (r, A, b, μ) .

b) The neo-Ricardian solution

The usual assumption made by the neo-Ricardians is that the wage rate is equal to the monetary value of the subsistence basket, i.e. $w = pc$. Substituting this hypothesis into the production price equation, we derive the following system :

$$(13) \quad p = (1+r)(pA + pcb) = (1+r)pM \quad \text{where } M = (A + B) \text{ and } B = cb.$$

$$(14) \quad px = \theta x = \text{constant.}$$

Setting $\lambda = (1+r)^{-1}$, under the hypothesis that p and M are semi-positive, the Frobenius theorem asserts the existence of one latent vector corresponding to the maximum value of the latent root λ^* . Given the additional macro constraint (14), which determines one of the prices (by fixing the monetary equivalent of one hour of labour), the price vector is entirely determined.

c) The Dumenil-Foley-Lipietz (DFL) solution

The money wage rate is assumed exogenous and equal to the embodied labour wage, which is equivalent to assume that the exploitation rate is given and not based on a subsistence basket. The DFL approach also contains another original feature: it standardizes the solution with respect to value added in the two spaces instead of using the gross value of production. The DFL solution is therefore built on the following system:

$$(10) \quad p = (1+r)(pA + wb)$$

$$(15) \quad px - pAx = \theta x - \theta Ax = bx$$

or

$$(16) \quad p(I - A)x = bx = x_0. \quad ^{23}$$

²³ This equation is used to isolate one particular price with respect to other (n-1) prices. The system (10) is solved by substitution after first eliminating the scalar (1+r) from each price equation and substituting the isolated price already obtained from (16). The solution is non linear with respect to one particular price.

For DFL, the choice of the value added instead of the gross value of production for the macro constraint is most important for the measurement of the monetary expression of one unit of living labour since $p(I - A)x/x_0 = 1$ or a constant. It should not be confused with the wage rate since the latter is the cost of reproducing one unit of labour. This solution has the great merit of preserving the equality between the sum of profits and the sum of surplus values because, with $w = \mu$ and given (15), it follows

$$(17) px - pAx - wbx = \theta x - \theta Ax - \mu bx.$$

The major flaw with this solution is that the general rate of profit is not preserved equal between the two spaces.

A.3 The invariant profit rate solution ²⁴

It is assumed that it is the profit rate and not the wage rate which is exogenous to the price system. Indeed, it is not necessary to preserve the hypothesis of the subsistence basket, because the labour force is a commodity as any other commodity, and its production price can differ from its value, whether or not the latter is equal to the labour value of a subsistence basket. The advantages of this new approach are numerous:

- i) Prices and wage differ from values, i.e. $p_i \neq \theta_i$, $\forall i = 1, \dots, n$ and $w \neq \mu$.
- ii) The two macro equalities are satisfied, because the simultaneous use of the gross value constraint (14) and the equality of the general average profit rate $r = r'$ (where r' is the profit rate in the monetary space) preserves the sum of surplus-values and the sum of profits between the two spaces.
- iii) If gross values and surplus-values (profits) are preserved between the two spaces, it follows that the sum of total costs is also maintained equal.

a) Formal solution

The production price equation is

$$(10) p = d(pA + wb), \quad d = 1 + r$$

$$(14) px = a, \quad a = \theta x.$$

After re-arranging (10),

$$(10a) p(I - dA) = dwb.$$

Post-multiplying by $(I - dA)^{-1}x$,

$$(10b) px = dwb(I - dA)^{-1}x.$$

Substituting (14) in (10b), w is calculated as

$$(18a) dwb(I - dA)^{-1}x = a$$

$$(18b) w = a / db(I - dA)^{-1}x.$$

Substituting (18b) in (10) the price vector p is

$$(19) p = a b(I - dA)^{-1} / b(I - dA)^{-1}x.$$

Therefore, in the monetary space, prices and wage can be specified by the following function:

$$(20) (p \ w) = f(r, \theta, A, b, x).$$

Interdependence between the two spaces is well illustrated by this function.

b) Verification of the macro constraints

The first constraint -preservation of gross value- is obvious since equality (14) is part of the solution. The preservation of the second constraint -equality between the sum of surplus-values and profits- is implicit with the constraint of a common general rate of profit between the two spaces. According to (17), the sum of surplus-values and profits are written respectively as

²⁴ This solution has already been presented on two occasions: International Conference on Politics and Languages of Contemporary Marxism, University of Massachusetts, Amherst, Dec. 5-8, 1996 and at the Mini Conference on Value Theory, Eastern Economic Association Convention, Washington, April 3-6 1997. Both papers are available on request (working papers 9625 and 9703). It has also been published in French in Loranger (1999).

	Labour value space	Monetary space
(17)	$\theta x - (\theta Ax + \mu bx)$	$px - (pAx + wbx)$
The profit rate is		
(21)	$r = [\theta x - (\theta Ax + \mu bx)] / (\theta Ax + \mu bx)$	$r' = [px - (pAx + wbx)] / (pAx + wbx)$
or	$1 + r = \theta x / (\theta Ax + \mu bx)$	$1 + r' = px / (pAx + wbx)$

Since $(1 + r) = (1 + r')$ and $\theta x = px$ by hypothesis, it follows

$$(22) \quad (\theta Ax + \mu bx) = (pAx + wbx)$$

that is (total costs in abstract labour) = (total costs in money).

From the substitution of (22) into (17), it follows that the sum of surplus-values is equal to the sum of profits. However, since $\mu \neq w$, labour value added will be different from money value added and the gap between the two value added can be measured by $(\mu - w)bx$ since the other component of value added measured by (17) is equal in the two spaces. It follows also that the money exploitation rate and the money value composition of capital ratio will differ from those measured in embodied labour value.

Bibliography

Beaud M. and Dostaler G. 1993. La pensée économique depuis Keynes, Paris, Seuil.

Benetti C. and Cartelier J. 1980. Marchands, salariat et capitalistes, paris, Maspero

Bortkiewicz L. von 1907. in P. Sweezy « On the Correction of Marx's Fundamental Theoretical Construction in the Third Volume of Capital » Clifton, A.M. Kelly, 1949.

Dostaler G. 1978. Valeur et prix. Histoire d'un débat, Grenoble, Montreal, Presses de l'Université de Grenoble et Presses de l'Université du Québec.

Dostaler G. and Lagueux M. 1985. Un échiquier centenaire, Paris-Sainte-Foy, La Découverte-Presses de l'Université du Québec.

Duménil G. 1980. De la valeur aux prix de production, Paris, Economica.

Duménil G. 1983. « Beyond the Transformation Riddle: a Labour Theory of Value », Science and Society, vol. 47, no 4, p. 427-450.

Dumenil G. , Levy D. 1998. « The Conservation of Value: A Rejoinder to Alan Freeman », Paper presented at the Mini-Conference of the International Working Group on Value Theory, Washington, 1997. Also published in Review of Radical Political Economics. (1998).

Eatwell J., Milgate M. and Newman P. (dir), 1987-1990. The New Palgrave. Marxian Economics, London, Norton.

Foley D. 1982. « The Value of Money, the Value of Labour Power and the Marxian Transformation Problem », Review of Radical Political Economics, vol. 14, no 2, p. 27-47.

Foley D. 1986. Understanding Capital. Marx's Economic Theory, Cambridge Mass., Harvard University Press.

Foley D. 1998. « Recent Development in the Labour Theory of Value », Review of Radical Political Economics, (1998).

- Freeman A. 1984. « The Logic of the Transformation Problem », in Mandel E. and Freeman A. (ed.) Ricardo, Marx, Sraffa, Thetford Press, London, pp 221-264.
- Freeman A. 1995. « Marx Without Equilibrium », Capital and Class, 56, Summer 1995, p. 49-90.
- Freeman A. 2001. "Money", in Critical Dictionary of Marxism, INKRIT, Berlin
- Freeman A., Carchedi G. (eds) 1995. Marx and Non-Equilibrium Economics, London, Elgar
- Freeman A. , Kliman A. 1998. « Simultaneous and Temporal Valuation Constrasted », Paper presented to the Marx International II Conference, Paris, Sept. 30-Oct. 3 1998.
- Freeman A. , Kliman A. , Wells P.J. 2001. The New Value Controversy, Cheltenham Elgar
- Freeman A., Kliman A. 2002. in L. Vasapello ed., An Old Myth: The Transformation of Values into Prices in Marx's Capital, Series in Critical Knowledge, University of Rome "La Sapienza", Rome
- Gill L. 1996. Fondements et limites du capitalisme, Montreal, Boréal.
- Laibman D. 1973. « Value and Prices of Production: the Political Economy of the Transformation Problem », Science and Society, vol. 4, p. 404-436.
- Lipietz A. 1982. « The So-Called Transformation Problem Revisited », Journal of Economic Theory, vol. 6, no 1, p. 59-88.
- Loranger J.G. 1996. « The Transformation Problem: an Alternative Solution with an Identical Aggregate Profit Rate in the Labour Value Space and the Monetary Space », Working paper 9625, Department of Economics, Université de Montréal.
- Loranger J.G. 1997. « The Wage Rate and the Profit Rate in the Price of Production Equation: a New Solution to an Old Problem », Working paper 9703, Department of Economics, Université de Montréal.
- Loranger J. G. 1999. « L'importance de l'égalité du taux de profit moyen dans la solution du problème de la transformation: une nouvelle approche d'équilibre général », Revue Économique, vol. 50, no.2, March 1999.
- Mandel E. and Freeman A. (dir), 1984. Ricardo, Marx, Sraffa, London, Verso.
- Marx K. 1967. Capital, Vol.I-III, New-York, International Publishers.
- Moseley F. (ed) 1993. Marx's Method in Capital. New-Jersey, Humanities Press.
- Moseley F. 1997. « The « New Solution » to the Transformation Problem: a Sympathetic Critique », Paper presented to the Mini-Conference of the International Working Group on Value Theory, Washington. April 1997.
- Morishima M. and Seton F. 1961. « Aggregation in Leontief Matrices and the Labour Theory of Value », Econometrica, vol. 29, no 2. p. 203-220.
- Morishima M. 1973. Marx's Economics, Cambridge, Cambridge University Press.
- Morishima M., Catephores G. 1978. « The transformation Problem: a Markovian Process », in Value, Exploitation and Growth, M. Morishima and G. Catephores (ed), McGraw-Hill, Chapter 6.
- Okishio N. 1963. « A Mathematical Note on Marxian Theorems », Weltwirtschaftliches Archiv, Bd. XCI.

- Okishio N. 1972. « On Marx's Production Prices » (in Japanese), Keizaigaku Kenkyu, 19, 1972, pp. 38-63
- Ricardo D. 1817. The Principles of Political Economy and Taxation, (with an Introduction by W. Fellner), Irwin Paperback, Illinois, 1963.
- Samuelson P. A. 1957. « Wages and Interest: a Modern Dissection of Marxian Economic Models », American Economic Review, vol.47, no 6, p. 884-912.
- Samuelson P.A. 1971. « Understanding the Marxian Notion of Exploitation: a Summary of the So-Called Transformation Problem Between Marxian Values and the Competitive Prices », Journal of Economic Literature, vol. 9, no 2, p. 399-431.
- Seton F. 1957. « The Transformation Problem » Review of Economic Studies, vol. 24, p. 149-160.
- Shaikh A. 1977. "Marx's Theory of Value and the Transformation Problem", in The Subtle Anatomy of Capitalism, J. Schwartz, ed., p. 106-139.
- Shaikh A. 1982. « Neo-Ricardian Economics: a Wealth of Algebra, a Poverty of Theory », Review of Radical Political Economics, vol. 14, no 2, p. 67-83.
- Smith A. 1776. An Inquiry into the Nature and Causes of the Wealth of Nations, (with an Introduction by B. Mazlish), Bobbs-Merrill, N.Y.
- Sraffa P. 1960. Production of Commodities by Means of Commodities, Cambridge, Cambridge University Press.
- Steedman I. 1977. Marx After Sraffa, London, New Left Volumes.
- Steedman I. (ed)1981. The Value Controversy, London, Verso and New Left Volumes.
- Sweezy P. M. (ed) 1949. Karl Marx and the Close of His System by Eugen Böhn-Bawerk and Böhn-Bawerk's Critism of Marx by Rudolf Hilferding, Clifton, A.M. Kelly.
- Tougan-Baranovsky M. 1905. Theoretische Grundlagen des Marxismus, Leipzig, Dunker and Humbolt.
- Winternitz J. 1948. « Value and Prices: a Solution of the So-Called Transformation Problem », Economic Journal, Vol. 58, pp. 276-280.
- Wolff R., Roberts B. et Callari A. 1982. « Marx's (not Ricardo) « Transformation Problem »: a Radical Conceptualization », History of Political Economy, 14, no 4, p.564-582.
- Woll R., Roberts B., Callari A. 1984. A Marxian Alternative to the Traditional Transformation Problem, Review of Radical Political Economics, vol.16, no. 2&3.