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The Return of the European Wage Phillips Curve

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Abstract

In this paper we compare the accelerationist Phillips curve to the New-Keynesian Wage Phillips curve in Euro Area countries which went through major swings in the unemployment rate in recent years. We find that the New-Keynesian wage Phillips curve signals cyclical fluctuations in unemployment more clearly and yields less pro-cyclical estimates of the NAWRU in four crisis-hit EU member states (Greece, Spain, Ireland and Portugal) than a traditional Phillips curve model, which may not treat price rigidities adequately. Slightly augmenting the NKP model by allowing for real wage rigidities further improves the extraction of a cyclical unemployment component.

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1. Introduction

Unemployment rates increased sharply in the EU in the wake of the crisis. The surge proved particularly persistent and estimates of the non-cyclical part of unemployment (e.g. the NAWRU) rose substantially, most notably in countries hard hit by the crisis, pointing to a lasting deterioration. These estimates could be considered too high, reflecting an overestimation of structural determinants of unemployment (see for instance Gerchert et al. 2015).

One reason behind this overestimation could be the need for a more adequate treatment of expected price inflation in times of large labour market adjustment, when price rigidity effects tend to play an important role. A Phillips curve specification, which could overcome this problem, is provided by Galí (2011). Galí (2011) revives the theoretical relationship between unemployment and wage inflation by deriving a Phillips curve based on a New-Keynesian model (NKP) with rational expectations and wage rigidities. Both models do not explicitly consider price adjustment frictions, but they differ concerning the assumptions they make about inflation expectations for wage setters. This paper argues that the implicit assumptions about inflation expectations entering the traditional model could bias the cyclical component of unemployment in a downward direction.

In contrast to the NKP model, the traditional Phillips curve model (TKP) is based on static or adaptative expectations and associates a decline in the growth rate of nominal unit labour costs with a positive unemployment gap, implicitly assuming that wage setters expect inflation to adjust quickly to a fall in the growth rate of nominal wages. A (low but) constant nominal wage growth then indicates that wage setters are intent on stabilising expected real wage growth, not displaying willingness to further adjust real wages. Only a deceleration of nominal wage growth (or nominal unit labour costs) signals such willingness. In contrast, the NKP uses real unit labour cost growth, assuming that wage setters are well informed about current price inflation. When nominal wages fall strongly and prices show some inertia, implications will differ across the two models and TKP assumptions may be strongly violated. To assess this risk of overestimation of the NAWRU, especially during volatile times, when relying on the TKP model, we examine whether an extended version of Galí's (2011) NKP model yields a lower NAWRU for four EU member states which were hit substantially by the economic crisis (Greece, Spain, Ireland and Portugal). We impose three extensions: (1) the NAWRU is assumed to be time-varying to be able to depict fluctuations in wage mark ups in particular on European labour markets, (2) we introduce adjustment costs based on Rotemberg (1982) instead of staggered wage setting based on Calvo (1983)[1] and (3) we introduce real wage rigidities. In practice we derive a (restricted) reduced form representation of the New Keynesian Phillips curve with and without real wage rigidity (see Blanchard and Galí (2007)), which allows us to analyse more clearly which indicator the NKP suggests for identifying the unemployment gap.

While it is well known that the TKP suggests the first difference of wage/NULC growth as unemployment gap indicator, we argue in this paper that the NKP is suggesting a variant of real unit labour cost growth as indicator for the unemployment gap. The focus on the respective indicators allows seeing more clearly the cyclical unemployment implications entailed by the two models.

In particular, we study two different scenarios for the NKP model, which we both compare to the TKP model:

- Scenario 1 (S1): Nominal wages are assumed to be sticky due to wage adjustment costs, where workers or trade unions aim at adjusting the nominal wage to price inflation and the actual change of labour productivity (baseline model).

[1] See Annex D for a comparison with Calvo pricing.

- Scenario 2 (S2): Workers adjust nominal wages in the short run to price inflation and to trend productivity growth, instead of actual productivity growth as suggested by S1. This allows for additional dynamics of the NKP Phillips curve, namely sluggish real wage adjustment (real wage rigidity).

We concentrate our analysis on four EU member states which were hit comparably hard by the crisis, namely Greece, Spain, Ireland and Portugal. In these countries the respective increases in both the NAWRU and the unemployment rate were among the largest in the EU member states. In particular, between 2008 and 2012 the unemployment rate in those countries increased within a range of 7 percentage points (Portugal) and 17 percentage points (Greece) (see Figure A.1 in Annex A). Similarly, in these countries the NAWRU increased between 2008 and 2012 ranging from 3 to 4 percentage points (see Figure A.2).[2] Notably, in Spain labour market developments were particularly dramatic with a rapid surge in unemployment in the aftermath of the crisis reaching a peak of over 26% in 2013.

The paper is organised as follows: in section 2 we describe the underlying theoretical models. Section 3 explains the estimation method and discusses econometric issues. In section 4 we present our empirical results and section 5 concludes.

2. THE THEORETICAL PHILLIPS CURVE MODEL

The *wage* Phillips curve stresses the existence of a link between short term unemployment fluctuations (i.e. cyclical unemployment) and wage inflation, postulating, in the traditional setup, that the acceleration or deceleration of unit labour cost is proportional to the unemployment gap. Using this relationship a "non-accelerating wage rate of unemployment" (NAWRU) can be determined. The precise functional form of the wage Phillips curve reflects a set of assumptions regarding the functioning of the labour market and the treatment of expectations.

More specifically, the Phillips curve describes the dynamics of wages in disequilibrium. For that purpose, certain adjustment frictions are introduced in the model. The most important adjustment friction is nominal wage rigidity, traditionally introduced by assuming that workers are slow in adjusting price inflation expectations. The specific choice made for such expectation schemes and related details (e.g. timing assumption for the setting of wages) represent important aspects of the implementation of the Phillips curve approach, as explained below.

2.1. EXTENDING GALÍ'S (2011) NEW-KEYNESIAN PHILLIPS (NKP) WAGE PHILLIPS CURVE

In this section we describe the model underlying the forward-looking wage Phillips-curve with adjustment costs in a standard New-Keynesian framework based on Galí (2011). The detailed derivation of the model is shown in Annex B. Note that there is a difference between the NKP shown here and the standard NKP curve shown in the literature, which relates to the modelling of trend unemployment fluctuations (NAWRU): In the model derived here trend unemployment (u_t^*) is determined by exogenous mark up and benefit (reservation wage) fluctuations, while in the standard NKP model, fluctuations in trend unemployment are determined by exogenous mark-up shocks. There are also differences concerning the modelling of the dynamics of the unemployment gap. As will be explained below, while in the standard NKP model the unemployment gap is proportional to the

[2] Bulgaria and Cyprus also witnessed large deterioration of their labour market conditions during the crisis but data availability is more limited in these countries, which also motivated our choice of the four abovementioned countries.

marginal rate of substitution between consumption and leisure, in the NKP model derived here, fluctuations of the unemployment gap could be modelled as persistent shocks to the wage equation and the labour demand equation (to the extent in which these are not perceived by wage setters).

We consider a representative household whose members offer different types of labour, indexed by i. These variants of labour are imperfectly substitutable by firms in production. The elasticity of substitution is denoted by the parameter θ. Firms can combine these different variants via a CES aggregator:

$$L_t = \left[\int_0^1 L_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}} \text{with } \theta > 1 \tag{2.1}$$

This yields a demand for labour of type i as:

$$L_t(i) = \left(\frac{W_t(i)}{W_t}\right)^{-\theta} L_t \tag{2.2}$$

Household's intertemporal utility is a function of current and expected consumption and leisure:[3]

$$V_t = E_t \sum_{j=0}^{\infty} \beta^j \left[\log\left(C_{t+j}\right) - \chi \int_0^1 \frac{\left(\left(\frac{W_{t+j}(i)}{W_{t+j}}\right)^{-\theta} L_{t+j}\right)^{\eta+1}}{\eta+1} di\right] \tag{2.3}$$

Where $E_t(.)$ denotes expectations conditional on time t information. Households face a budget constraint which is standard, apart from the fact that households bear wage adjustment costs which rise to the square as wage growth deviates from aggregate inflation and aggregate productivity growth:

$$BC_t \equiv B_t + C_t + \frac{\gamma}{2} \sum \left[\frac{w_t(i)}{\Pi_t w(i)_{t-1}} - 1\right]^2 \frac{W_t L_t}{P_t} = \int_0^1 \frac{W_t(i)}{P_t} L_t(i) di + (1 + r_{t-1}) B_{t-1} \tag{2.4}$$

Where an important variable, which determines the adjustment of wages to the unemployment gap, is the quadratic adjustment cost term:

$$AdjCost_t = \left[\frac{w_t(i)}{\Pi_t w(i)_{t-1}} - 1\right]^2 \tag{2.4.a}$$

Where Π_t denotes price inflation times trend growth in labour productivity.

[3] For expositional simplicity we do not write the utility function in expectations.

$$\Pi_t = \frac{P_t}{P_{t-1}} * \left(\frac{YL_t}{YL_{t-1}}\right)^T \qquad (2.4.b)$$

where P_t, YL_t, YL_t^T are the price level, labour productivity and trend labour productivity respectively. As mentioned above, nominal rigidities are introduced into this model by assuming that workers or trade unions have a preference for adjusting nominal wages to the actual inflation rate and actual (trend) growth rate of productivity. This adjustment is costly and is modelled by the adjustment cost function above.

The households' optimisation problem posed by the model above yields the following wage setting rule, where the real wage (W_t^r) is equal to the marginal rate of substitution between leisure and consumption, adjusted for a structural and cyclical mark up term

$$W_t^r = \frac{(1+mup_t^W)\chi L_t^\eta C_t}{\left(1-\frac{\gamma}{(\theta-1)}(\beta E_t\psi_{t+1}-\psi_t)\right)} \qquad (2.5)$$

As shown in Annex B one can derive the following version of the New Keynesian Phillips curve which postulates a link between the unemployment gap $(u_t - u_t^*)$ and real wage growth minus trend productivity and where $\pi_t^w, \pi_t^p, gyl_t^T$ are wage inflation, price inflation, and growth of the trend of labour productivity respectively

$$\pi_t^w - \pi_t^p - gyl_t^T = \beta(E_t\pi_{t+1}^w - E_t\pi_{t+1}^p - E_t gyl_{t+1}^T) - \frac{(\theta-1)}{\gamma}(u_t - u_t^*) \qquad (2.6)$$

For ease of exposition we define the ratio between real wage inflation and trend productivity

$$\psi_t = \pi_t^w - \pi_t^p - gyl_t^T \qquad (2.7)$$

And rewrite the NKP as follows

$$\psi_t = \beta E_t\psi_{t+1} - \frac{(\theta-1)}{\gamma}(u_t - u_t^*) \qquad (2.8)$$

In order to capture backward indexation, we can also write this equation as follows – yielding the hybrid Phillips curve:

$$\psi_t = \beta sf E_t\psi_{t+1} + (1-sf)\psi_{t-1} - \frac{(\theta-1)}{\gamma}(u_t - u_t^*) \qquad (2.9)$$

In Annex C we show how to obtain a backward solution of equation (2.9) using the method of undetermined coefficients and assuming the unemployment gap follows an AR(2) process:

$$\psi_t = \beta_0 \psi_{t-1} - \beta_1 (u_t - u_t^*) + \beta_2 (u_{t-1} - u_{t-1}^*) \tag{2.10}$$

– resulting from the specification of the adjustment cost function (2.4b). Notice that equation (2.10) comes close to the wage Phillips curve proposed by Galí, in particular the Phillips curve is characterised by the current and lagged unemployment gap. The presence of lagged unemployment (with a positive sign) results from the hump shaped nature of the unemployment gap. The major difference between the two formulations is that in our formulation current price inflation enters (as well as productivity growth or its trend).

2.1.1. Introducing real wage rigidities

Blanchard and Galí (2007) argue that adding real wage rigidities in a New-Keynesian Phillips curve model can capture the trade-off between stabilising inflation and stabilising the output gap, which the baseline New-Keynesian model does not capture. Another advantage is that it increases inflation inertia. As will be shown below, introducing real wage rigidity also helps to overcome an empirical problem the NKP Phillips curve is confronted with, namely that wage inflation responds negatively to the current period unemployment gap but positively to the lagged unemployment gap. This second effect is mitigated by the presence of real wage rigidity. Following Blanchard and Galí we assume that real wages exhibit a certain degree of inertia, determined by the parameter ϕ.

$$W_t^r = \left[\frac{(1+mup_t^W)\chi L_t^\eta C_t}{(1-\frac{\gamma}{(\theta-1)}(\beta E_t \psi_{t+1} - \psi_t)}\right]^{1-\phi} [W_{t-1}^r (1 + gYL^T{}_t)]^\phi \tag{2.5'}$$

In Annex B we derive the forward-looking Phillips curve with real wage rigidities as:

$$\psi_t = \frac{\beta}{(1+\frac{(\theta-1)}{\gamma}\frac{\phi}{1-\phi})} E_t \psi_{t+1} - \frac{\eta(\theta-1)}{\gamma(1+\frac{(\theta-1)}{\gamma}\frac{\phi}{1-\phi})}(u_t - u_t^*) \tag{2.8'}$$

Similarly to the above we can write down the hybrid form (2.9') and determine the coefficients of the backward solution (2.10') by the method of undetermined coefficients as:

$$\psi_t = \frac{\beta}{(1+\frac{(\theta-1)}{\gamma}\frac{\phi}{1-\phi})} sf E_t \psi_{t+1} + (1-sf)\psi_{t-1} - \frac{\eta(\theta-1)}{\gamma(1+\frac{(\theta-1)}{\gamma}\frac{\phi}{1-\phi})}(u_t - u_t^*) \tag{2.9'}$$

$$\psi_t = \beta_0^r \psi_{t-1} - \beta_1^r (u_t - u_t^*) + \beta_2^r (u_{t-1} - u_{t-1}^*) \tag{2.10'}$$

with

$$\psi_t = \pi_t^w - \pi_t^p - gyl_t^T$$

This Phillips curve with real wage rigidities has exactly the same form as the Phillips curve with nominal wage rigidities only. However, while in the case with nominal rigidities the discount factor β is assumed to take a value smaller but close to one, the discount factor in the Phillips curve with real wage rigidities is $\beta^r = \frac{\beta}{(1+\frac{(\theta-1)}{\gamma}\frac{\phi}{1-\phi})}$ and can take any value arbitrarily close to zero depending on the parameters θ, γ and most importantly ϕ. Therefore, the coefficient on the unemployment gap in the Phillips curve denoted as $\beta_2^r = \frac{\beta^r sf\alpha_2}{1-\beta^r sf\beta_0}\beta_1$ (see equation C.14c' in Annex C) also depends on the parameters θ, γ and ϕ and can take any positive value. Note that if real wage rigidities term ϕ moves to its upper bound of one, the discount factor β^r and the coefficient β_2^r take values going towards 0.

2.2. COMPARING THE (TRADITIONAL) KEYNESIAN AND THE NEW KEYNESIAN PHILLIPS CURVE

In this section we highlight that different assumptions across the NKP and the TKP models imply that these models rely on different labour cost indicators to identify labour market slack. In turn, this has implications on the estimation of equilibrium unemployment rates and unemployment gaps.

The wage Phillips curve postulates a relationship between the *expected* real unit labour cost growth rate and the unemployment gap. In the TKP model expected inflation is proxied using lagged growth rate of nominal unit labour costs while productivity is commonly assumed to be known. This set up yields the traditional "accelerationist" Phillips curve form that features a relationship between the second difference of NULC and the unemployment gap. In the traditional Phillips curve literature equilibrium unemployment rate is then defined as the rate consistent with a non-accelerating rate of wage (or NULC) growth (NAWRU).

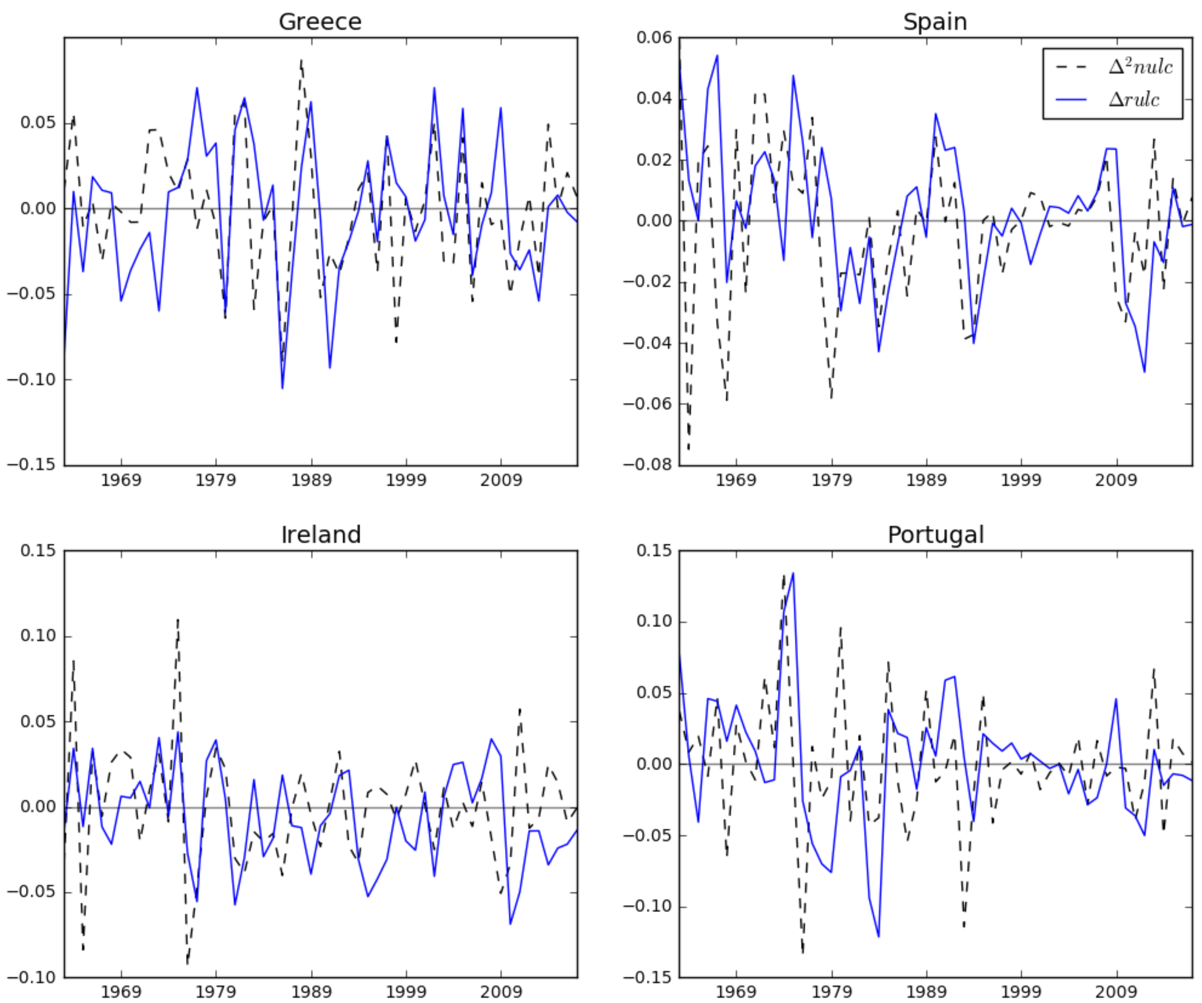
A particular timing assumption in the NKP model yields a fundamental difference with respect to the TKP model. Wage contracts are set during period t, while assuming wage setters have access to information about economic conditions during that period. In particular, the current price level (and inflation) is assumed to be known. This implies that the NKP can rely on the actual rather than the expected current real unit labour cost to identify the unemployment gap. Note that details of the model still require agents to form expectations on inflation for t+1, as wage contracts formed in t extend to the next period. In practice, expectations about inflation and productivity in period t+1 enter the wage decisions, relying on rational expectations. The latter entails that the relationship between the labour cost indicator and the unemployment gap features lags of those variables in the NKP model (see Annex C for further details).

As the TKP and the NKP rely on different indicators to identify labour market slack, they may yield different estimates of structural unemployment and the unemployment gap if those indicator post different developments. The graph below compares the two indicators, showing broadly similar developments in general but also highlighting occasions in which their pattern differs. During such episodes, the assessment of degree of labour market slack may differ notably across the two models.

Focusing on the recent past, clear differences across the two indicators are discernible in all four countries depicted. The real unit labour cost growth rate tended to signal more labour market slack in the midst of the great financial crisis than the second difference of nominal unit labour cost. In particular, the former posted a more protracted tendency to remain below the zero line. A particularly large gap between the two indicators is observable in the case of Spain in the aftermath of the crisis.

Noteworthy, the second difference of nominal unit labour cost showed a clear tendency to revert quickly and to the zero line and even turn decisively positive in the aftermath of the crisis. As such, it pointed to a much quicker closing of the unemployment gap compared to the indicator used by the NKP model. In a context of still high unemployment, such quick closing of the unemployment gap entails a substantial increase in the level of the TKP-based structural unemployment estimates for those countries.

Figure 2.1: **Comparing the TKP and NKP indicators used to identify the unemployment gap**



Source: AMECO database, Autumn 2015 vintage.

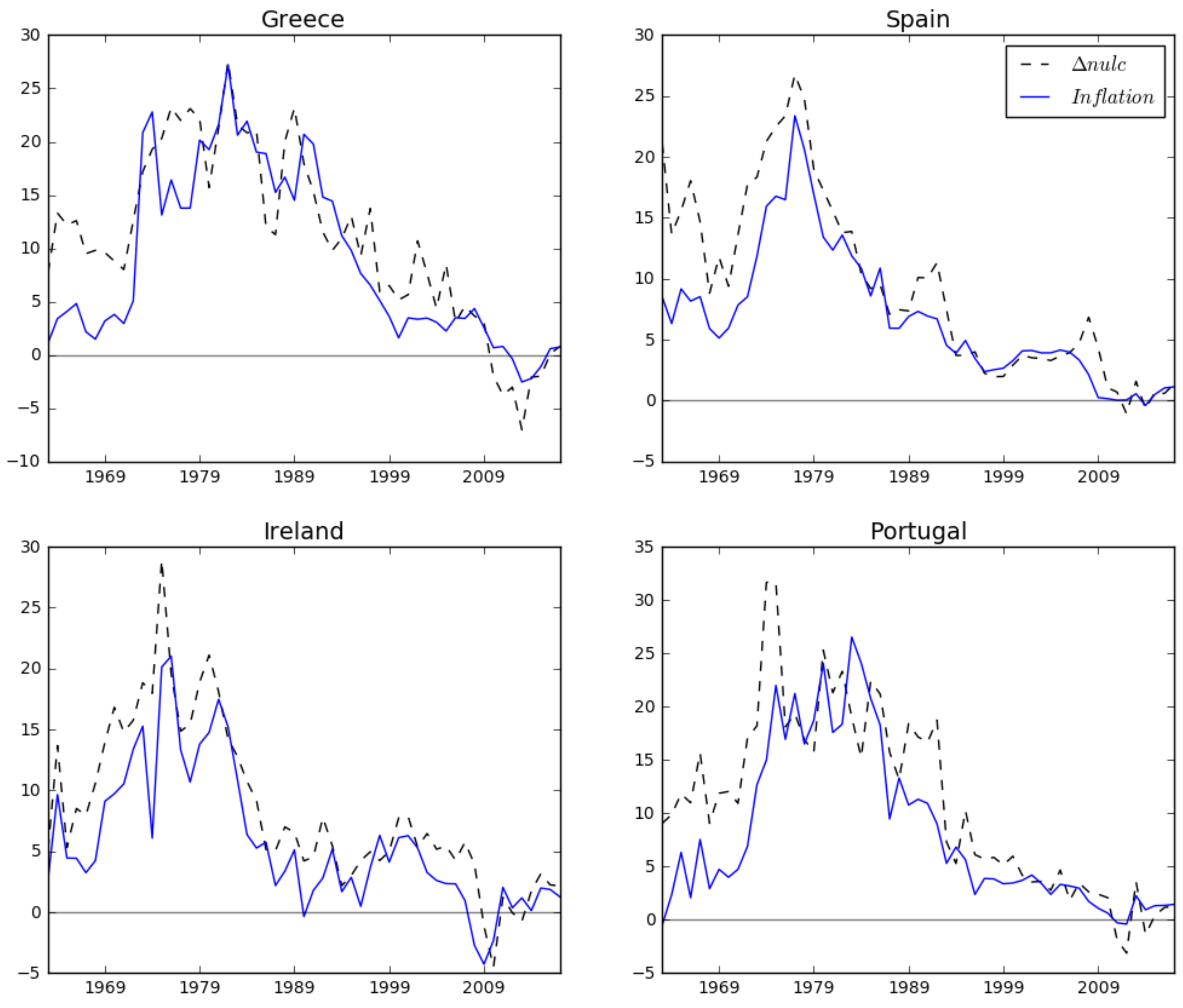
Note: $\Delta^2 nulc$ stands for second difference of nominal unit labour cost; $\Delta rulc$ stands for real unit labour cost growth.

What is causing the gap between those two indicators? In the context of the Phillips curve framework, a key factor to bear in mind is the fact that the use of the second difference of nominal unit labour cost in the TKP model results from the assumption that inflation can be well approximated by nominal unit labour cost growth developments. The validity of this assumption is reviewed in the graph below.

Over the recent past, inflation and nominal unit labour cost growth have not moved in lockstep in the countries depicted. This suggests that the fundamental assumption embedded in the TKP model was not verified. In turn, this reveals a risk of bias when relying on this model to identify the unemployment gap. Specifically, in the case of Ireland and Spain, wage setters that relied on nominal unit labour cost to forecast inflation in the context of the TKP model tended to overestimate inflation developments, around year 2009. The same is observed for Ireland and Portugal in the run up to the crisis. In the context of the NKP model, such forecast error is avoided, as wage setters are assumed to access current information about inflation.

In sum, divergences between inflation and nominal unit labour cost are an important factor driving the gap between the indicators used by each model. Such divergences drive a different labour market slack assessment across the two models, as evidenced in the next section.

Figure 2.2: **Highlighting violation of underlying TKP assumption**



Source: AMECO database, Autumn 2015 vintage.

Note: $\Delta nulc$ stands for nominal unit labour cost growth; $inflation$ stands for GDP deflator growth.

3. ESTIMATION OF THE NEW-KEYNESIAN PHILLIPS CURVE

By now, the empirics of the NKP spans a vast literature dating back to early seminal contributions by Roberts (1995), Fuhrer and Moore (1995), Galí and Gertler (1999), Galí et al. (2001) and Sbordone (2002). Early results (see Galí et al. (2001)) also pointed to a good fit for the euro area (even better than for the US). NKP estimation however presents a number of challenges (e.g. Mavroeidis, 2005, 2006; Stock et al., 2002; Rudd and Whelan, 2005; Bardsen et al., 2004). Yet, Galí et al. (2005) provide evidence of robustness of early results.

More recent contributions also tend to confirm empirical relevance of the NKP model for the EU (e.g. Jondeau and Bihan (2005), Paloviita (2006), Rumler (2007), Hondroyiannis et al. (2008), Vogel (2008), Tillmann (2009), Alstadheim (2013), Lopez-Perez (2016)). Noteworthy aspects of recent results include signs of notable heterogeneity in the degree of price rigidity across countries (i.e. Rumler (2007)), substantial change in NAIRU levels over time (i.e. Vogel (2008)) and notable uncertainty of point estimates in NKP models (i.e. Tillmann).

Overall, while robustness is still assessed, the use of NKP for the EU is well documented. A recent study illustrates well this state of affairs. Mazumder (2012) recognises that by now "a majority of macroeconomists who have studied inflation dynamics in Europe argue that the NKP provides a good way to describe changes in the price level from the 1970". Yet, the author critiques the use of labour income share as a proxy for real marginal cost.

The empirical work presented here contributes to this empirical literature by checking performance of a particular form of the NKP model, namely the wage-NKP. The analysis also stresses the importance of checking performance under unusual circumstances. In particular, the NKP appears to outperform the TKP in crisis times.

3.1. DATA

We use data on the harmonised unemployment rate, wage inflation, consumer price inflation and labour productivity growth from the AMECO database[4] and the ECFIN European Forecast Autumn 2015[5]. The sample reaches from 1963 to 2017. For the estimation of the NKP with real-wage rigidities and wage adjustment based on trend labour productivity growth (S2), we employ a Hodrick-Prescott filter to obtain trend labour productivity growth with a smoothing parameter of 100. Results are shown in Figure A.3 in Annex E. The resulting series are used to compute the wage indicator in this scenario and the Kalman filter approach is used to estimate the NAWRU.

3.2. ECONOMETRIC APPROACH

The NAWRU is determined by an unobserved component model, which is estimated by a bi-variate Kalman filter using ECFIN's GAP software (Planas and Rossi 2009). This econometric model include the Phillips curve specified in equation (2.16) and (2.16`) respectively as well as an equation that assumes a second order auto-regressive process for the cyclical term of the unemployment rate (the

[4] http://ec.europa.eu/economy_finance/db_indicators/ameco/index_en.htm
[5] http://ec.europa.eu/economy_finance/eu/forecasts/index_en.htm

unemployment gap) and a second order random walk for the trend component (the NAWRU). The econometric equation for the TKP[6] and the NKP take the respective forms:

$$\Delta\pi_t^w - \Delta gyl_t = -\beta(u_t - u_t^*) + \varepsilon_t^{TKPC} \tag{3.1}$$

$$\pi_t^w - \pi_t^p - gyl_t = \beta_0(\pi_{t-1}^w - \pi_{t-1}^p - gyl_{t-1}) - \beta_1(u_t - u_t^*) + \beta_2(u_{t-1} - u_{t-1}^*) + \varepsilon_t^{NKPC} \tag{3.2}$$

The model for unemployment takes the form:

$$u_t = c_t + n_t \tag{3.3}$$

$$c_t = \delta_1 c_{t-1} + \delta_2 c_{t-2} + \varepsilon_t^c \tag{3.4}$$

$$n_t = \gamma_{t-1} + n_{t-1} + \varepsilon_t^n \tag{3.5}$$

$$\gamma_{t-1} = \gamma_{t-2} + \varepsilon_t^\gamma \tag{3.6}$$

For a more detailed description of the estimation method of the NAWRU applied here see Havik et al. (2014) and Planas and Rossi (2009).

4. EMPIRICAL RESULTS

In this section we compare NAWRU estimates based on the NKP model to those based on the TKP model. For the NKP model we study both the baseline scenario (S1) and the scenario with real wage rigidities (RWR) (S2). Key insights of this comparison are that the NKP model yields less pro-cyclical NAWRU estimates during severe recessions and that the fit of the wage Phillips curve improves as well as the significance of the link between labour market slack and labour cost developments when relying on the NKP approach.

Figures 4.1–4.4 show that NAWRUs based on the NKP model are slightly less pro-cyclical than those based on the TKP model, around the time of the recent crisis, in the case of Greece, Ireland and most notably Spain. In the case of Portugal, it is less apparent that NKP-based NAWRUs are less pro-cyclical. Nevertheless, for that country the NKP model appears to yield estimates for the NAWRU that are, overall, smoother.

Tables at the bottom of Figures 4.1–4.4 show that the fit of the wage Phillips curve improves when relying on the NKP model in all cases, except for Greece. Moreover, the t-statistic of the coefficient on the unemployment gap (β_1) is higher in absolute terms in the NKP model in all cases, suggesting that this model identifies a stronger link between labour market slack and labour cost developments.

Adding real-wage rigidities to the NKP model, by allowing wages to adjust to trend rather than actual labour productivity (i.e. scenario S2), the t-statistic on the β_1 and the fit of the wage Phillips curve further improves in all cases. In particular in Ireland, the results improve notably and the NAWRU displays a less pro-cyclical pattern as recent declines in the labour cost indicator are better tracked by the model than in the case of the TKP or the NKP without real wage rigidities. Noteworthy, the labour

[6] For a derivation of the TKP, see for example Blanchard and Katz (1999).

cost indicator in the NKP model with real wage rigidities may be non-stationary, as one of its components is a trend variable. Annex F reports unit root test results which however indicate that the Null hypothesis of a unit root can be rejected in all cases.

To build intuition, note that the NAWRU estimates based on the NKP and the TKP model differ when nominal wages fall strongly while price development display some inertia, a situation which characterised developments in Spain, for instance, in the aftermath of the crisis. Under such circumstances, real unit labour cost growth (the NKP labour cost indicator) declines more strongly (and persistently) than the change in the nominal unit labour cost (the TKP labour cost indicator). As such, the NKP model identifies a larger (and more persistent) unemployment gap and posts a less pro-cyclical NAWRU. To illustrate, note that in Spain, after 2009, the change in the nominal unit labour cost rapidly reverted back to zero, while real unit labour cost growth posted a more protracted negative dynamic, pointing at larger and more persistent slack in the Spanish labour market. In turn, the NKP based NAWRU estimate for Spain reached 22% by 2015, compared to 26.4% for the TKP based estimate.

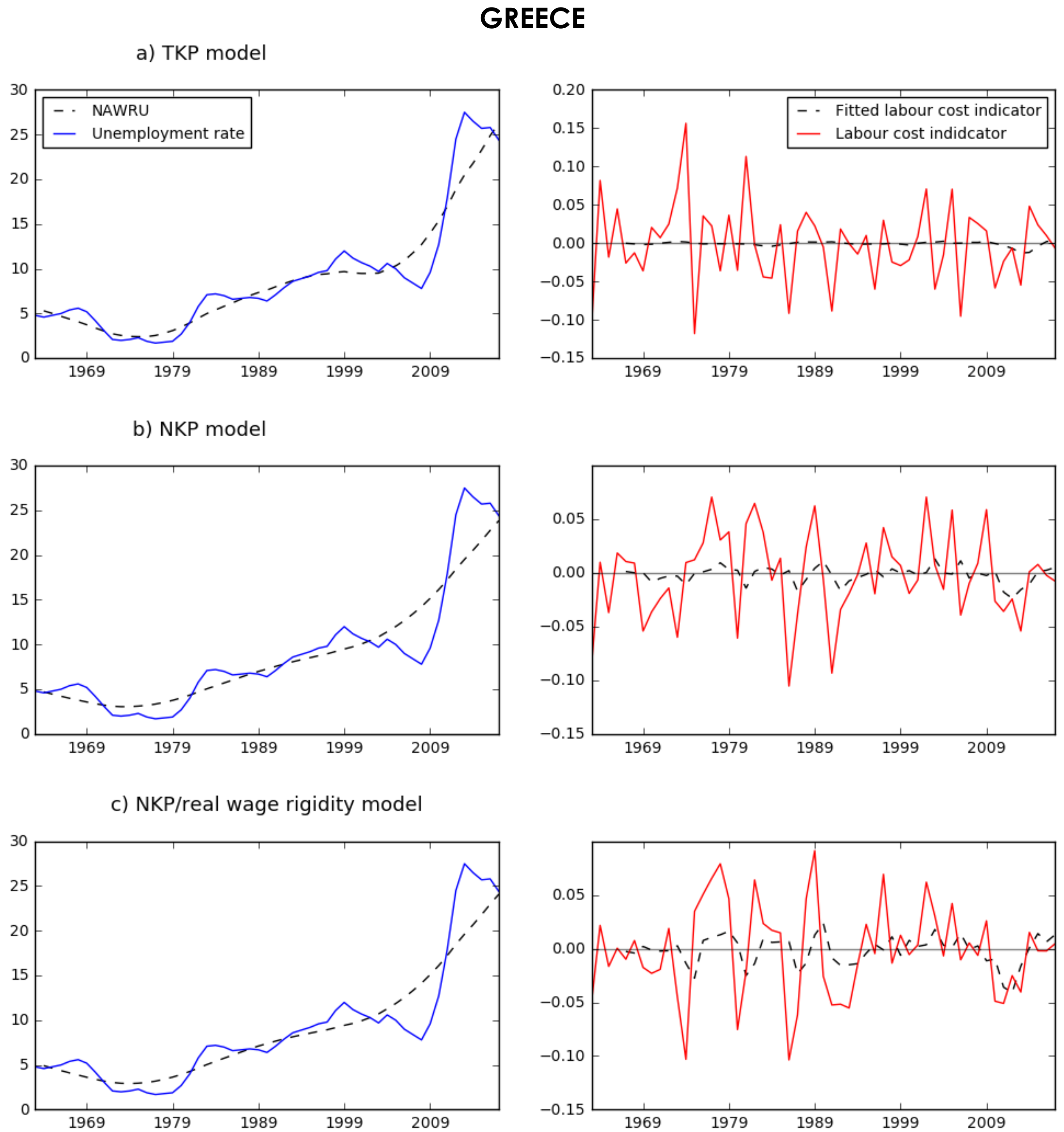
Figure 5 highlights comparison across the TKP and the NKP in terms of unemployment gap. As TKP based NAWRUs tend to be excessively pro-cyclical in times of crisis, that model also yields narrower unemployment gaps under such circumstances. This is the case for all countries, except Portugal. Yet when adding the real wage rigidity to the NKP model (i.e. scenario S2) the same is also observed for Portugal. The latter suggests that adding rigidities may further improve accuracy of trend-cycle decomposition, especially in volatile times where such rigidities become more binding.

Overall, as the two labour cost indicators appear to diverge only occasionally, the choice of the model bears consequences only during volatile times. In all countries, the two models tended to yield similar NAWRUs before the crisis. This suggests the two labour cost indicators differ notably only in times of large labour market adjustments. As such, the use of the NKP approach provides some insurance against the risk of reporting upwardly biased NAWRU estimates in times of crisis while adding more rigidities such as real wage rigidities, may provide additional robustness against, as results for Portugal suggest.

In sum, evidence reported in this section suggests that, compared to the TKP, the NKP yields a better fit for the wage Phillips curve and a stronger link between the labour slack and the labour cost indicator and offers some insurance against risks of NAWRU over-estimation in times of crisis. These results suggest that the NKP model (with real wage rigidities) provides a valuable alternative to the commonly used TKP model for analysis of the labour market in terms of trend-cycle decomposition.

Note that to assess the merit of switching to the NKP model for real-time analysis would require checking the degree to which its NAWRU estimates are prone to revisions. While the NKP appears to be better at “getting the story right” than the TKP, if it were prone to larger NAWRU revisions than the TKP this would undermine its use in real-time. Checking this is beyond the scope of this paper. It would require assessing whether and at what horizon NKP based NAWRU estimates are more unstable than TKP based ones. Whether the benefits the NKP model yields in terms of accuracy of the trend-cycle decomposition are at the expense of relative stability of NAWRU estimates is an issue worth investigating in future work to assess its use for real-time analysis.

Figure 4.1: **NAWRU and fit of the Phillips curve—TKP versus NKP**[7]

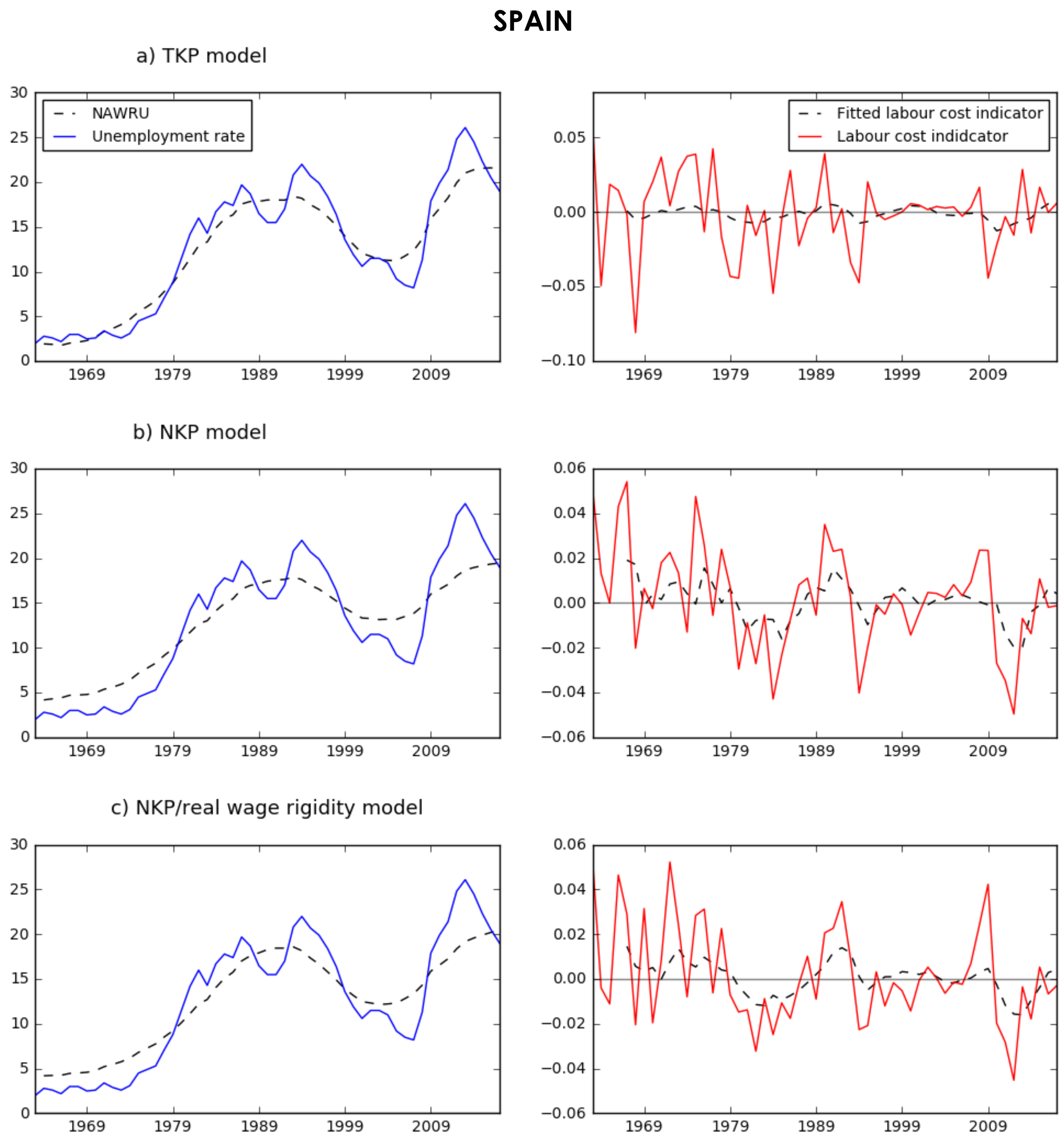


Notes: Labour cost indicator across the models: (a) 2nd difference of NULC; (b) 1st difference of RULC; (c) 1st difference of RULC computed using *trend* productivity.

	TKP	NKP	NKP-RWR
Coefficient estimates (t-statistics in brackets) (**) if restricted UB or LB if (upper or lower) bound reached	β_1: -0.20 (-0.6052)	β_1: -0.35 (-0.8696) β_2: 0.28 (**)	β_1: -0.56 (-1.2354) β_2: 0.57 (1.2790)
R2	0.1263	0.1150	0.1223

[7] Note that the variance bounds for Greece for the TKP model were taken from the Winter 2014 forecasting exercise as it was the last exercise based on the TKP model.

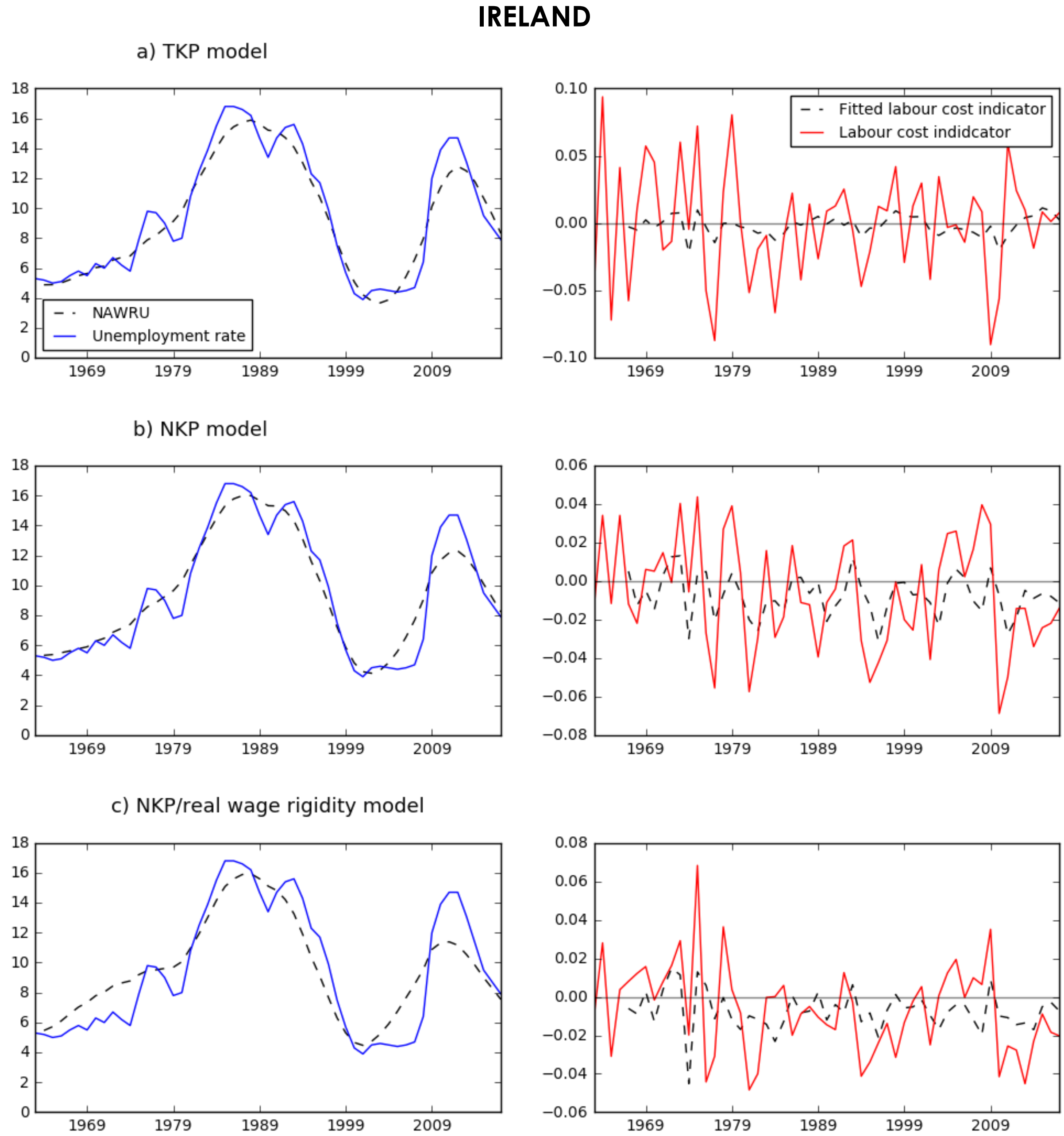
Figure 4.2: **NAWRU and fit of the Phillips curve—TKP versus NKP**



Notes: Labour cost indicator across the models: (a) 2nd difference of NULC; (b) 1st difference of RULC; (c) 1st difference of RULC computed using *trend* productivity.

	TKP	NKP	NKP-RWR
Coefficient estimates	β_1: -0.29	β_1: -0.36	β_1: -0.28
(t-statistics in brackets)	(-1.7412)	(-2.1699)	(-2.3826)
(**) if restricted			
UB or LB if (upper or lower) bound reached		β_2: 0.19 (**)	β_2: 0.00 (LB)
R2	0.1821	0.2268	0.2148

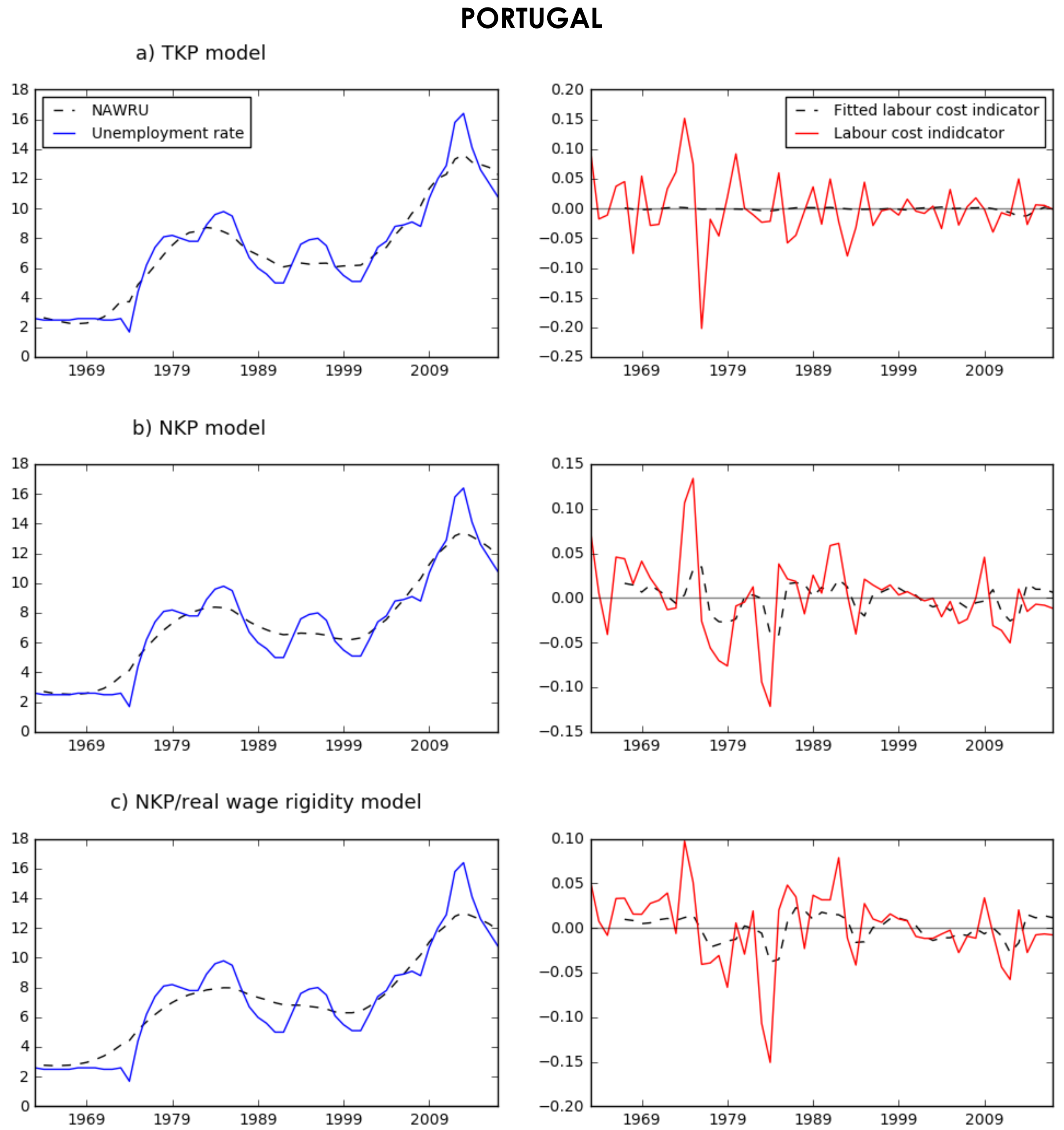
Figure 4.3: **NAWRU and fit of the Phillips curve—TKP versus NKP**



Notes: Labour cost indicator across the models: (a) 2nd difference of NULC; (b) 1st difference of RULC; (c) 1st difference of RULC computed using *trend* productivity.

	TKP	NKP	NKP-RWR
Coefficient estimates (t-statistics in brackets) (**) if restricted	β_1: -0.74 (-1.5215)	β_1: -0.87 (-2.0743)	β_1: -0.59 (-2.7605)
UB or LB if (upper or lower) bound reached		β_2: 0.43 (**)	β_2: 0.00 (LB)
R2	0.1527	0.1795	0.2426

Figure 4.4: **NAWRU and fit of the Phillips curve—TKP versus NKP**



Notes: Labour cost indicator across the models: (a) 2nd difference of NULC; (b) 1st difference of RULC; (c) 1st difference of RULC computed using *trend* productivity.

	TKP	NKP	NKP-RWR
Coefficient estimates	β1:-0.89	β1:-1.37	β1: -1.64
(t-statistics in brackets)	(-1.4059)	(-1.9435)	(-2.3959)
(**) if restricted			
UB or LB if (upper or lower) bound reached		β2: 0.74 (**)	β2: 0.79 (1.1623)
R2	0.0872	0.1874	0.1400

Figure 4.5: **Are unemployment gaps bigger with NKP than TKP, in the recent recession?**

a) **Unemployment gaps**: TKP *versus* TKP estimates

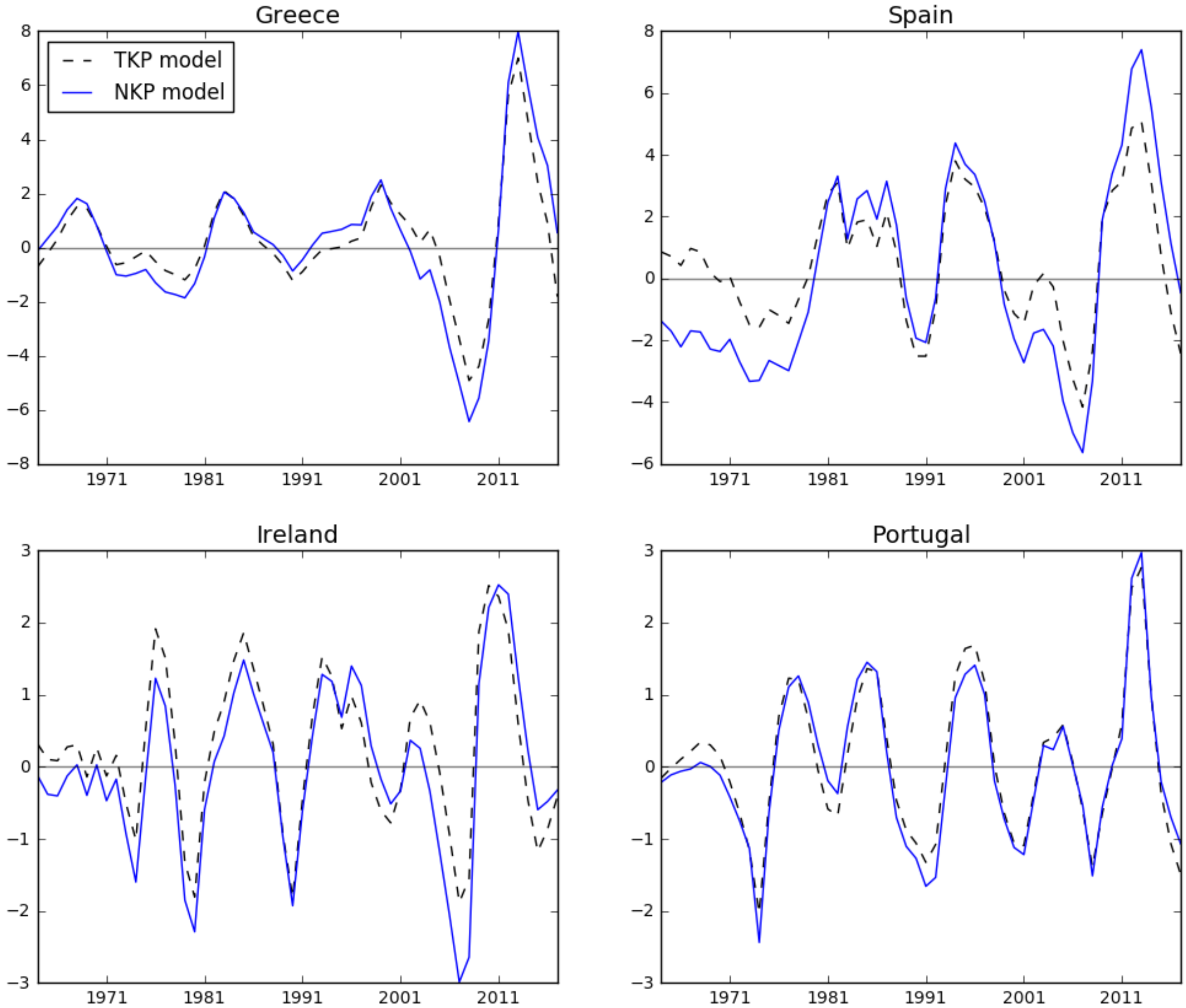
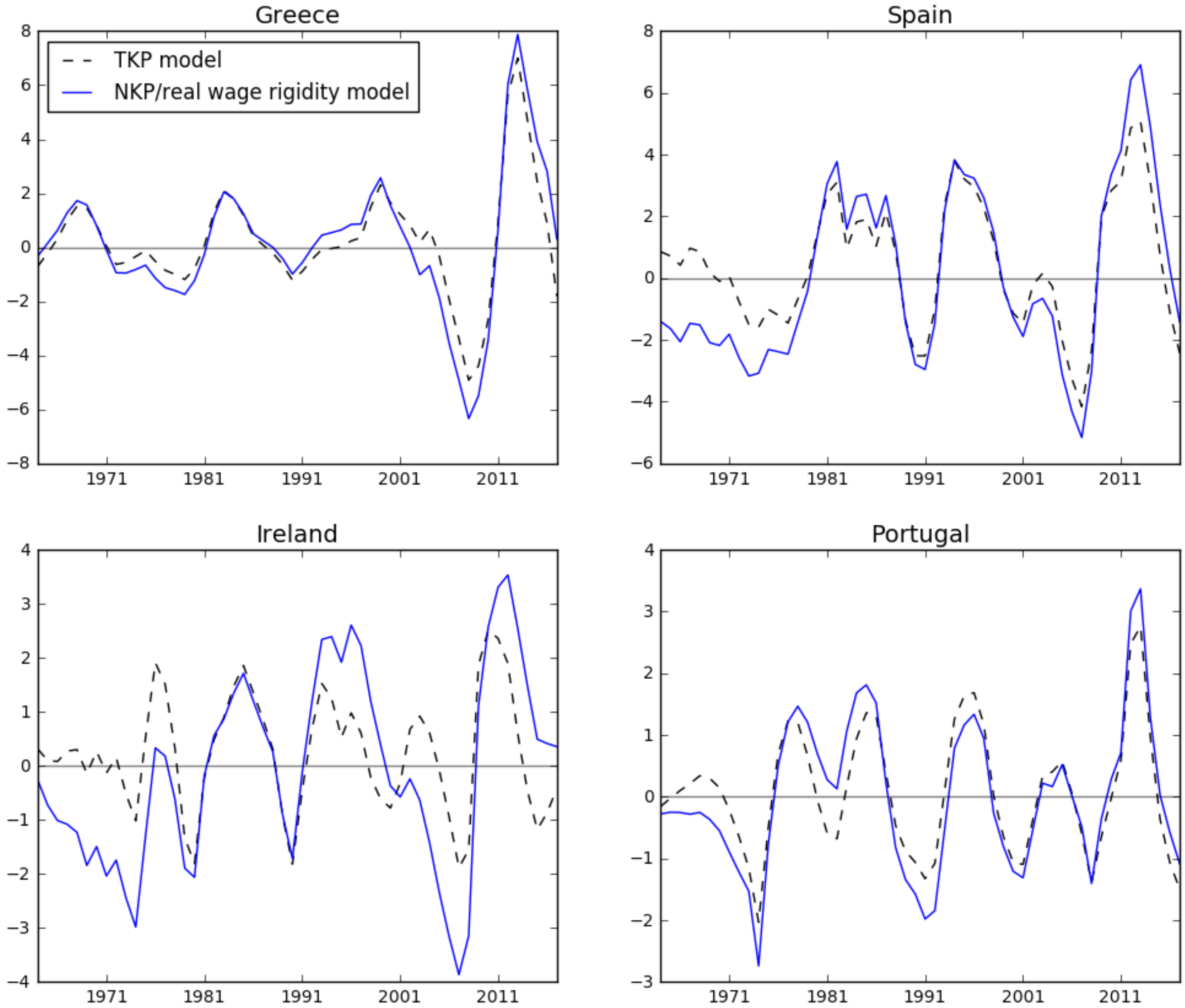


Figure 4.5 (continued)

b) **Unemployment gaps**: NKP/real wage rigidity *versus* TKP estimates



5. CONCLUSION

In this paper we argue that non-cyclical unemployment estimates based on a traditional (accelerationist) Phillips curve risk being too pro-cyclical, due to inadequate treatment of price expectations, an issue likely to matter particularly in times of large labour market adjustments, such as around crisis episodes. In turn, we argue that the New-Keynesian Phillips curve provides a remedy, by providing better treatment for price rigidities, which, as we show, can be inferred from theory.

To assess this empirically, we compare non-cyclical unemployment estimates based, respectively, on the accelerationist Phillips curve and the New-Keynesian Wage Phillips curve. We provide estimates for four crisis-hit EU member states (Greece, Spain, Ireland and Portugal) that underwent major swings in the unemployment rate, in recent years. Our results confirm that the New-Keynesian Wage Phillips curve yields less pro-cyclical estimates of the non-cyclical part of unemployment for those countries during the crisis. The empirical fit also improves when using the New-Keynesian Wage Phillips curve. Additionally, augmenting the New-Keynesian Wage Phillips curve to reflect the impact of real wage rigidities further improves the fit, pointing at further improvement in the extraction of the cyclical component of unemployment.

The practical interpretation of these results is that the change in nominal unit labour cost growth, the indicator suggested by the accelerationist Phillips curve to identify the unemployment gap, provides a poor signal in volatile times. Instead, the real unit labour cost growth, which is the indicator suggested by the New-Keynesian Wage Phillips curve, is a better indicator of labour market slack, under such circumstances.

Our analysis generally contributes to the vast literature documenting the empirical relevance the New-Keynesian (Wage) Phillips curve, with particular focus on the euro area. This analysis points at the importance of accounting properly for various rigidities that, in particular, drive price and wage developments. In times of large labour market adjustment, adequately accounting for those aspects appears key to support Phillips curve based trend-cycle decomposition of unemployment developments. Evidence that accounting for real wage rigidities further improves results, confirm the merit of seeking to model additional rigidities in frameworks that underpin empirical Phillips curve specifications. Both theoretical and empirical results presented here suggest that nominal and real wage rigidities are not the only sources of variation of the equilibrium unemployment rate. In particular, our New-Keynesian Wage Phillips curve estimates remain noisy, pointing at the need to further enrich the underlying model (e.g. labour demand frictions). Recalling the important theoretical and empirical differences across alternative Phillips curve models, as highlighted in this paper, is also warranted when debating the performance of model from a practical perspective.

From a policy perspective, the illustrated risk of reporting biased NAWRU estimates when relying on traditional Phillips curve approaches is unsettling. Evidence suggests the bias would be most severe in times of crisis, when accurate assessment is arguably most needed. In the EU context, NAWRU estimates also feed into potential output calculations which, in turn, affect computation of cyclically-adjusted fiscal variables used to underpin the EU fiscal surveillance framework. Hence, the reporting of biased NAWRU estimates has far reaching implications from a policy perspective, pointing at the merit of further investigating the apparent merit of relying on NKP models featuring all relevant rigidities, rather than relying on the commonly used TKP approach.

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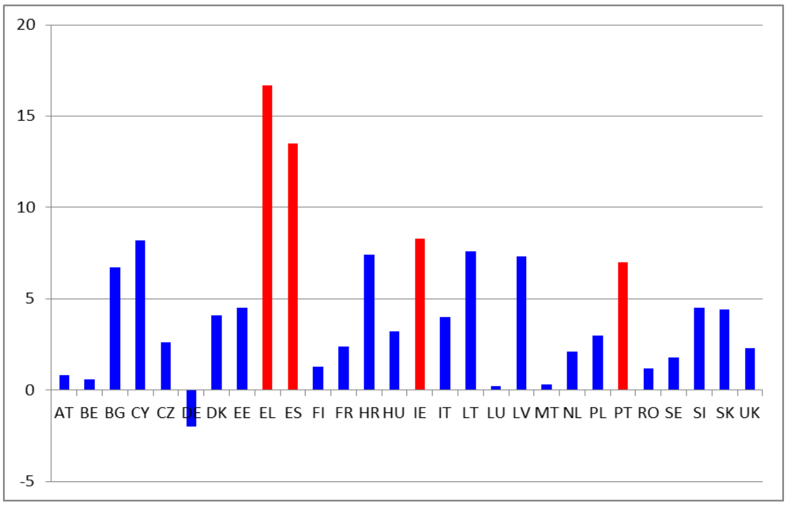
7. ANNEX

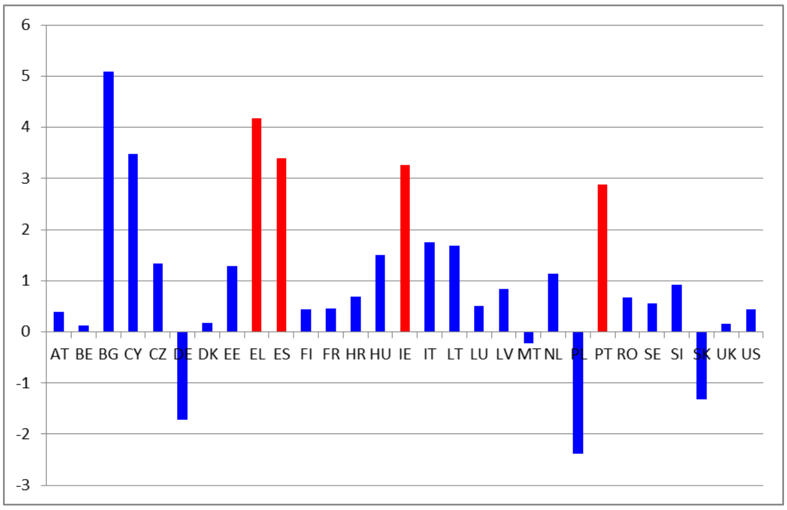
ANNEX A: UNEMPLOYMENT RATES AND NAWRUS ACROSS EU MEMBER STATES

Figure A.1: **Percentage point change between 2008 and 2012 in the unemployment rate across EU member states**



Source: AMECO database, Autumn 2015 vintage.

Figure A.2: **Percentage point change between 2008 and 2012 in the NAWRU across EU member states**



Source: AMECO database, Autumn 2015 vintage.

ANNEX B: DETAILED DERIVATION OF THE PHILLIPS CURVE

Nominal rigidities

To solve the optimisation problem in section 2, define the Lagrangian as:

$$\mathcal{L}_t \equiv E_t(\sum_{j=0}^{\infty} \beta^j U_{t+j} - \sum_{j=0}^{\infty} \beta^j \lambda_{t+j} BC_{t+j}) \tag{B.1}$$

The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial B_t} = 0 \Leftrightarrow \lambda_t = E_t \lambda_{t+1} \beta (1 + r_t) \tag{B.2}$$

$$\frac{\partial \mathcal{L}}{\partial C_t} = 0 \Leftrightarrow u_{C_t} = \lambda_t = \frac{1}{C_t} \tag{B.3}$$

$$\frac{\partial \mathcal{L}}{\partial w_t(i)} = 0 \Leftrightarrow u_{L_t(i)}(-\theta) W_t(i)^{-\theta-1} \frac{L_t}{W_t{}^{-\theta}} + \lambda_t \frac{(1-\theta) W_t(i)^{-\theta}}{P_t} \frac{L_t}{w_t{}^{-\theta}} - \lambda_t \gamma \left[\frac{W_t(i)}{\Pi_t W(i)_{t-1}} - 1\right] \frac{w_t L_t}{p_t} \left(\frac{1}{\Pi_t w(i)_{t-1}}\right) + E_t \left(\lambda_{t+1} \beta \gamma \left[\frac{w_{t+1}(i)}{\Pi_{t+1} w(i)_t} - 1\right] \frac{w_{t+1} L_{t+1}}{p_{t+1}} \left(\frac{w_{t+1}(i)}{\Pi_{t+1} (w(i)_t)^2}\right)\right) = 0$$

(B.4a)

Assume symmetry: $W_t = W_t(i)$ and therefore $L_t = L_t(i)$

$$u_{L_t}(-\theta) \frac{L_t}{W_t} + \lambda_t \frac{(1-\theta) L_t}{P_t} - \lambda_t \gamma \left[\frac{W_t}{\Pi_t W_{t-1}} - 1\right] \frac{W_t L_t}{P_t} \left(\frac{1}{\Pi_t W_{t-1}}\right) + E_t \left(\lambda_{t+1} \beta \gamma \left[\frac{w_{t+1}}{\Pi_{t+1} w_t} - 1\right] \frac{w_{t+1} L_{t+1}}{p_{t+1}} \left(\frac{w_{t+1}}{\Pi_{t+1} w_t{}^2}\right)\right) = 0 \tag{B.4.b}$$

Rearranging yields:[8]

$$-u_{L_t}(-\theta) = \lambda_t \frac{W_t}{P_t} \left((1-\theta) - \gamma \left[\frac{W_t}{\Pi_t W_{t-1}} - 1\right] \left(\frac{W_t}{\Pi_t W_{t-1}}\right) + E_t \left(\frac{\beta \lambda_{t+1}}{\lambda_t} \gamma \left[\frac{W_{t+1}}{\Pi_{t+1} W_t} - 1\right] \left(\frac{W_{t+1}}{\Pi_{t+1} W_t}\right)\right)\right) \tag{B.4.c}$$

Equation (B.4.c) is not linear in adjustment costs so we perform a first order Taylor approximation of $\gamma \left[\frac{W_t}{\Pi_t W_{t-1}} - 1\right] \left(\frac{W_t}{\Pi_t W_{t-1}}\right)$ around the steady state.

To this purpose we define

$\Psi_t = \frac{W_t}{\Pi_t W_{t-1}}$ and consequently $f(\Psi_t) = \gamma(\Psi_t - 1)\Psi_t$

In the steady state we have

[8] Assuming $\frac{W_{t+1} L_{t+1}}{P_{t+1} L_t} \approx \frac{W_t}{P_t}$.

$\frac{W_t}{\Pi_t W_{t-1}} = \frac{W_{t+1}}{\Pi_t W_t} = 1$ and $\Pi_t = \Pi = 1$ and consequently $\Psi = 1$

The Taylor approximation of $f(\Psi_t)$ is

$$f(\Psi_t) \approx f(\Psi) + f'(\Psi)(\Psi_t - \Psi)$$

with

$$f'(\Psi_\mathrm{t}) = \gamma\Psi_t + \gamma(\Psi_\mathrm{t} - 1) = \gamma(2\Psi_\mathrm{t} - 1) \text{ and } f'(\Psi) = \gamma$$

So

$$f(\Psi_t) \approx \gamma(\Psi_t - 1)$$

Then, using $u_{L_t} = -\chi L_t^\eta$, equations (B.2-B.3) and the Taylor approximation above and assuming a constant discount factor $\beta = \frac{1}{1+r_t}$ or no consumption fluctuations $E_t \frac{\lambda_t}{\lambda_{t+1}} = E_t \frac{C_{t+1}}{C_t} = 1$, we can rewrite equation (B.4.c) as

$$\chi L_t^\eta(-\theta) = \frac{1}{C_t}\frac{W_t}{P_t}\left((1-\theta) - \gamma(\Psi_t - 1) + \beta\gamma(E_t\Psi_{t+1} - 1)\right) \qquad \text{(B.4.d)}$$

Simplifying and defining $\psi_t = \Psi_t - 1$, we can rewrite equation (2.8.e) as

$$\chi L_t^\eta(-\theta) = \frac{1}{C_t}\frac{W_t}{P_t}\left((1-\theta) - \gamma\psi_t + \beta\gamma E_t\psi_{t+1}\right)$$

Further, rearranging and defining $(1 + mup_t^w) = \frac{\theta}{\theta-1} = \frac{(-\theta)}{(1-\theta)}$ we can write

$$\frac{(-\theta)\chi L_t^\eta C_t}{(1-\theta)} = \frac{W_t}{P_t}\left(1 + \frac{\gamma}{(1-\theta)}(\beta E_t\psi_{t+1} - \psi_t)\right)$$

$$\frac{(-\theta)\chi L_t^\eta C_t}{(1-\theta)} = \frac{W_t}{P_t}\left(1 - \frac{\gamma}{(\theta-1)}(\beta E_t\psi_{t+1} - \psi_t)\right)$$

Define $W_t^r = \frac{W_t}{P_t}$ and rewrite

$$W_t^r = \frac{(1+mup_t^w)\chi L_t^\eta C_t}{\left(1-\frac{\gamma}{(\theta-1)}(\beta E_t\psi_{t+1}-\psi_t)\right)} \qquad \text{(B.4.d)}$$

Or alternatively as labour supply equation

$$L_t = \left(\frac{W_t^r\left(1-\frac{\gamma}{(\theta-1)}(\beta E_t\psi_{t+1}-\psi_t)\right)}{\chi C_t(1+mup_t^w)}\right)^{1/\eta} \qquad \text{(B.4.e)}$$

This relationship can also be used to determine the labour force, namely as the number of workers which are indifferent between working and not working

$$LF_t = \left(\frac{W_t^r}{\chi C_t}\right)^{1/\eta} \qquad \text{(B.5)}$$

The cyclically adjusted employment rate (the employment rate in the absence of wage adjustment costs) is given by

$$LCA_t = \left(\frac{W_t^r}{\chi C_t(1+mup_t^W)}\right)^{1/\eta} \qquad \text{(B.6)}$$

And the cyclically adjusted unemployment rate is given by

$$\frac{LF_t}{LCA_t} = (1+u_t^*) = (1+mup_t^W)^{\frac{1}{\eta}} \qquad \text{(B.7)}$$

The actual unemployment rate

$$\frac{LF_t}{L_t} = (1+u_t\ ) = \left(\frac{(1+mup_t^W)}{\left(1-\frac{\gamma}{(\theta-1)}(\beta E_t\psi_{t+1}-\psi_t)\right)}\right)^{\frac{1}{\eta}} \qquad \text{(B.8)}$$[9]

Substituting equation (B.7) into (B.8) yields

$$(1+u_t\ ) = (1+u_t^*)\left(\frac{1}{\left(1-\frac{\gamma}{(\theta-1)}(\beta E_t\psi_{t+1}-\psi_t)\right)}\right)^{\frac{1}{\eta}} \qquad \text{(B.9)}$$

$$1-\frac{\gamma}{(\theta-1)}(\beta E_t\psi_{t+1}-\psi_t) = \left[\frac{(1+u_t)}{(1+u_t^*)}\right]^{-\eta}$$

Taking logs yields

$$-\frac{\gamma}{(\theta-1)}(\beta E_t\psi_{t+1}-\psi_t) = -\eta\ln\left[\frac{(1+u_t)}{(1+u_t^*)}\right]$$

$$-\frac{\gamma}{(\theta-1)}(\beta E_t\psi_{t+1}-\psi_t) = -\eta(\ln(1+u_t)-\ln(1+u_t^*))$$

$$-\frac{\gamma}{(\theta-1)}(\beta E_t\psi_{t+1}-\psi_t) = -\eta(u_t-u_t^*)$$

$$\psi_t = \beta E_t\psi_{t+1} - \frac{\eta(\theta-1)}{\gamma}(u_t-u_t^*) \qquad \text{(B.10)}$$

Real wage rigidities

Rewrite equation (B.4.e) by adding a term reflecting dependence on past real wages (real wage rigidity) – in addition to labour and consumption, wages are adjusting sluggishly to past real wages (corrected for the productivity growth trend) – as below. Note that in this section we assume that $\psi_t = \pi_t^w - \pi_t^p - gyl_t^T$.

$$W_t^r = \left[\frac{\chi L_t^\eta C_t(1+mup_t^W)}{(1-\frac{\gamma}{(\theta-1)}(\beta E_t\psi_{t+1}-\psi_t)}\right]^{1-\phi}[W_{t-1}^r(1+gYL^T{}_t)]^\phi \qquad \text{(B.4.d')}$$

[9] In approximation around the steady state this expression can also be written as $\frac{LF_t}{L_t} = (1+u_t\ ) = \left(\frac{1}{\left(1-\frac{\gamma}{(\theta-1)}(\beta E_t\psi_{t+1}-\psi_t)\right)(1-mup_t^W)}\right)^{\frac{1}{\eta}}$

Rearranging and solving for L_t yields

$$W_t^{r^{\frac{1}{1-\phi}}} W_{t-1}^{r^{\frac{-\phi}{1-\phi}}} (1+gYL^T{}_t)^{\frac{-\phi}{1-\phi}} = \frac{\chi L_t^{\eta} C_t (1+mup_t^W)}{(1-\frac{\gamma}{(\theta-1)}(\beta E_t \psi_{t+1} - \psi_t))}$$

$$L_t = \left[W_t^{r^{\frac{1}{1-\phi}}} W_{t-1}^{r^{\frac{-\phi}{1-\phi}}} (1+gYL^T{}_t)^{\frac{-\phi}{1-\phi}} \frac{(1-\frac{\gamma}{(\theta-1)}(\beta E_t \psi_{t+1} - \psi_t))}{\chi C_t (1+mup_t^W)} \right]^{1/\eta} \qquad \text{(B.4.e')}$$

Divide equation (B.5) by (B.4e') to determine the unemployment rate as a function of those who are unemployed due to frictions in the labour market and full employment under now frictions:

$$\frac{LF_t}{L_t} = (1+u_t) = \left[\frac{\frac{W_t^r}{\chi C_t}}{\frac{W_t^{r^{\frac{1}{1-\phi}}} W_{t-1}^{r^{\frac{-\phi}{1-\phi}}} (1+gYL^T{}_t)^{\frac{-\phi}{1-\phi}} (1-\frac{\gamma}{(\theta-1)}(\beta E_t \psi_{t+1} - \psi_t))}{\chi C_t (1+mup_t^W)}} \right]^{1/\eta}$$

$$\frac{LF_t}{L_t} = \left[\frac{(1+mup_t^W)}{W_t^{r^{\frac{\phi}{1-\phi}}} W_{t-1}^{r^{\frac{-\phi}{1-\phi}}} (1+gYL^T{}_t)^{\frac{-\phi}{1-\phi}} (1-\frac{\gamma}{(\theta-1)}(\beta E_t \psi_{t+1} - \psi_t))} \right]^{1/\eta}$$

$$\frac{LF_t}{L_t} = \left[\frac{(1+mup_t^W)}{\left[\frac{W_t^r}{(1+gYL^T{}_t)W_{t-1}^r}\right]^{\frac{\phi}{1-\phi}} (1-\frac{\gamma}{(\theta-1)}(\beta E_t \psi_{t+1} - \psi_t))} \right]^{1/\eta} \qquad \text{(B.8')}$$

Where

$$\frac{W_t^r}{(1+gYL^T{}_t)W_{t-1}^r} = \frac{\frac{W_t}{P_t}}{\left(\frac{W_{t-1}}{P_{t-1}}\right)\left(\frac{YL_t}{YL_{t-1}}\right)^T} = \frac{\frac{W_t}{P_t}\left(\frac{L_t}{Y_t}\right)^T}{\left(\frac{W_{t-1}}{P_{t-1}}\right)\left(\frac{L_{t-1}}{Y_{t-1}}\right)^T} = 1 + \left(\pi_t^w - \pi_t^p - gyl_t^T\right) = 1 + \psi_t$$

Using equation (B.7) we can write equation (B.8') as

$$\frac{(1+u_t)}{(1+u_t^*)} = \left[\frac{1}{[1+\psi_t]^{\frac{\phi}{1-\phi}} (1-\frac{\gamma}{(\theta-1)}(\beta E_t \psi_{t+1} - \psi_t))} \right]^{1/\eta} \qquad \text{(B.9')}$$

$$1 - \frac{\gamma}{(\theta-1)}(\beta E_t \psi_{t+1} - \psi_t) = \left[\frac{(1+u_t)}{(1+u_t^*)}\right]^{-\eta} \frac{1}{[1+\psi_t]^{\frac{\phi}{1-\phi}}}$$

$$1 - \frac{\gamma}{(\theta-1)}(\beta E_t \psi_{t+1} - \psi_t) = \left[\frac{(1+u_t)}{(1+u_t^*)}\right]^{-\eta} \frac{1}{[1+\psi_t]^{\frac{\phi}{1-\phi}}}$$

Taking logs yields

$$-\frac{\gamma}{(\theta-1)}(\beta E_t \psi_{t+1} - \psi_t) = -\eta \ln\left[\frac{(1+u_t)}{(1+u_t^*)}\right] + \ln\left[[1+\psi_t]^{\frac{-\phi}{1-\phi}}\right]$$

$$-\frac{\gamma}{(\theta-1)}(\beta E_t\psi_{t+1}-\psi_t)=-\eta(\ln(1+u_t)-\ln(1+u_t^*))-\frac{\phi}{1-\phi}\ln(1+\psi_t)$$

$$-\frac{\gamma}{(\theta-1)}(\beta E_t\psi_{t+1}-\psi_t)=-\eta(u_t-u_t^*)-\frac{\phi}{1-\phi}\psi_t$$

$$\psi_t=\beta E_t\psi_{t+1}-\frac{(\theta-1)}{\gamma}\frac{\phi}{1-\phi}\psi_t-\frac{\eta(\theta-1)}{\gamma}(u_t-u_t^*)$$

$$(1+\frac{(\theta-1)}{\gamma}\frac{\phi}{1-\phi})\psi_t=\beta E_t\psi_{t+1}-\frac{\eta(\theta-1)}{\gamma}(u_t-u_t^*)$$

Based on equation (B.9') we can rewrite the Phillips curve as

$$\psi_t=\frac{\beta}{(1+\frac{(\theta-1)}{\gamma}\frac{\phi}{1-\phi})}E_t\psi_{t+1}-\frac{\eta(\theta-1)}{\gamma(1+\frac{(\theta-1)}{\gamma}\frac{\phi}{1-\phi})}(u_t-u_t^*) \qquad \text{(B.10')}$$

ANNEX C: BACKWARD SOLUTION FOR THE HYBRID PHILLIPS CURVE

Assuming the unemployment gap follows an AR(2) process, a backward solution can be obtained using the method of undetermined coefficients.

First, postulate that:

- The unemployment gap is defined as the unemployment rate minus the NAWRU: $(u_t - u_t^*) = \hat{u}_t$
- The unemployment gap follows the AR(2) process $\hat{u}_t = \alpha_1\hat{u}_{t-1} + \alpha_2\hat{u}_{t-2}$ with $\alpha_1 > 1$ and $\alpha_2 < 0$ as the unemployment gap is a cyclical process

We write the backward solution of the Phillips curve (equation B.10) we are looking for as:

$$\psi_t = \beta_0\psi_{t-1} + \beta_1\hat{u}_t + \beta_2\hat{u}_{t-1} \tag{C.11}$$

The equations above imply:

$$E_t\hat{u}_{t+1} = \alpha_1\hat{u}_t + \alpha_2\hat{u}_{t-1} \tag{C.12}$$

$$E_t\psi_{t+1} = \beta_0\psi_t + \beta_1(\alpha_1\hat{u}_t + \alpha_2\hat{u}_{t-1}) + \beta_2\hat{u}_t \tag{C.13}$$

Inserting equation (C.11) in equation (C.13)

$$E_t\psi_{t+1} = \beta_0(\beta_0\psi_{t-1} + \beta_1\hat{u}_t + \beta_2\hat{u}_{t-1}) + \beta_1(\alpha_1\hat{u}_t + \alpha_2\hat{u}_{t-1}) + \beta_2\hat{u}_t \tag{C.13'}$$

Equivalently (rewriting equation (C.13')):

$$E_t\psi_{t+1} = \beta_0^2\psi_{t-1} + (\beta_0\beta_1 + \beta_1\alpha_1 + \beta_2)\hat{u}_t + (\beta_0\beta_2 + \beta_1\alpha_2)\hat{u}_{t-1} \tag{C.13''}$$

Equating coefficients (equation (C.11) = (B.10) with (C.13'') replacing ψ_{t+1}):

$$\beta_0\psi_{t-1} + \beta_1\hat{u}_t + \beta_2\hat{u}_{t-1} = \beta sf[\beta_0^2\psi_{t-1} + (\beta_0\beta_1 + \beta_1\alpha_1 + \beta_2)\hat{u}_t + (\beta_0\beta_2 + \beta_1\alpha_2)\hat{u}_{t-1}] + (1 - sf)\psi_{t-1} - \delta\hat{u}_t \tag{C.14}$$

Equation (C.14) implies that:

$$\beta_0 = \beta sf\beta_0^2 + (1 - sf) \tag{C.14a}$$

$$\beta_1 = \beta sf(\beta_0\beta_1 + \beta_1\alpha_1 + \beta_2) - \delta \tag{C.14b}$$

$$\beta_2 = \beta sf(\beta_0\beta_2 - \beta_1\alpha_2) \tag{C.14c}$$

Or equivalently:

$$\beta_0 = \frac{1-\sqrt{1-4\beta sf(1-sf)}}{2\beta sf} \tag{C.14a'}$$

$$\beta_1 = 1/\beta_0 \frac{\delta(1-sf)\beta}{\beta s(\beta(1-sf)\alpha_2+\beta(1-sf))-\beta(1-sf)(1-\alpha_1^2)+\alpha_2/2(\alpha_1-\beta sf)} \tag{C.14b'}$$

$$\beta_2 = \frac{\beta s f \alpha_2}{1-\beta s f \beta_0}\beta_1 \qquad \text{(C.14c')}$$

with $\alpha_1 > 0$ and $\alpha_2 < 0$

Note, from (C.14a) one can derive the share of forward looking wage setters from the estimate of β_0.

Note also that (C.14c') implies that the lagged unemployment gap has a positive effect on the wage indicator in period t. This is due to the cyclicality of the unemployment gap. A negative unemployment gap in t-1 is signalling a positive gap in t+1 and since wages are set in period t with an expectation about cyclical conditions in t+1, a negative unemployment gap in t+1 predicts a wage increase in period t.

How does real wage rigidity modify the backward solution?

The wage Phillips curve with and without real wage rigidity differs essentially by the coefficient of the wage expectation term. Without real wage rigidity this term is equal to the discount factor and is constrained to a value slightly below 1.

$$\psi_t = \beta E_t \psi_{t+1} - \frac{\eta(\theta-1)}{\gamma}(u_t - u_t^*) \qquad \text{(C.15)}$$

In contrast, with real wage rigidity, the coefficient in front of the wage expectation term can become arbitrarily small as the degree of real wage rigidity becomes large ($\phi \rightarrow 1$).

$$\psi_t = \frac{\beta}{(1+\frac{(\theta-1)}{\gamma}\frac{\phi}{1-\phi})} E_t \psi_{t+1} - \frac{\eta(\theta-1)}{\gamma(1+\frac{(\theta-1)}{\gamma}\frac{\phi}{1-\phi})}(u_t - u_t^*)$$

This relaxes the parameter constraint (C.14c') to

$$\beta_2 = \frac{\beta^* s f \alpha_2}{1-\beta s f \beta_0}\beta_1 \qquad \text{where } \beta^* = \frac{\beta}{(1+\frac{(\theta-1)}{\gamma}\frac{\phi}{1-\phi})} \qquad \text{(C.14c'')}$$

And β^* becomes a free parameter with $0 < \beta^* < 1$. Thus, real wage rigidity imposes the constraint

$$\beta_2 > 0$$

on the backward solution.

ANNEX D: COMPARISON WITH WAGE PHILLIPS CURVE DERIVED UNDER CALVO WAGE SETTING

J. Galí (2011) presented a paper where he derived a wage Phillips curve in a Calvo wage setting framework. The Calvo framework is attractive since it allows to microfound the wage setting process in terms of (expected) contract length and specific wage indexation schemes (for those wage setters which are not able to renegotiate wages in the current period).

Galí makes a number of assumptions, which need to be taken into account when comparing his Phillips curve to ours. Galí assumes that, for workers that cannot optimise in the current period, wages will only be indexed to average productivity growth. Moreover, he assumes a constant NAWRU (for the US).

Otherwise, as shown in the equation below, he applies a standard wage indexation rule that features inflation. That is, wages (for non-optimisers) are indexed to (a weighted average of) inflation in the previous period and the average inflation (i.e. $\overline{\pi}$):[10]

$$E_t w_{t+k} = E_t w_{t+k-1} + E_t \gamma \pi_{t+k-1} + (1-\gamma)\bar{\pi} + \overline{\Delta yl} \qquad \text{(D.1)}$$

As shown below, this yields a similar specification as the one we obtained with quadratic wage adjustment costs. Especially for γ close to one, the wage indicator based on the standard Calvo model comes close to the growth rate of real unit labour cost.

$$\Delta w_t - \gamma(\pi_{t-1}) = \frac{1}{1+r_t} E_t(\Delta w_{t+1} - \gamma \pi_t) - \lambda_w [u_t - u^*] \qquad \text{(D.2)}$$

That is, an identical model to ours can be obtained in the Galí's set up by assuming the following updating scheme:

$$E_t w_{t+k} = E_t w_{t+k-1} + E_t(\pi_{t+k} + \Delta yl_{t+k}) \qquad \text{(D.3)}$$

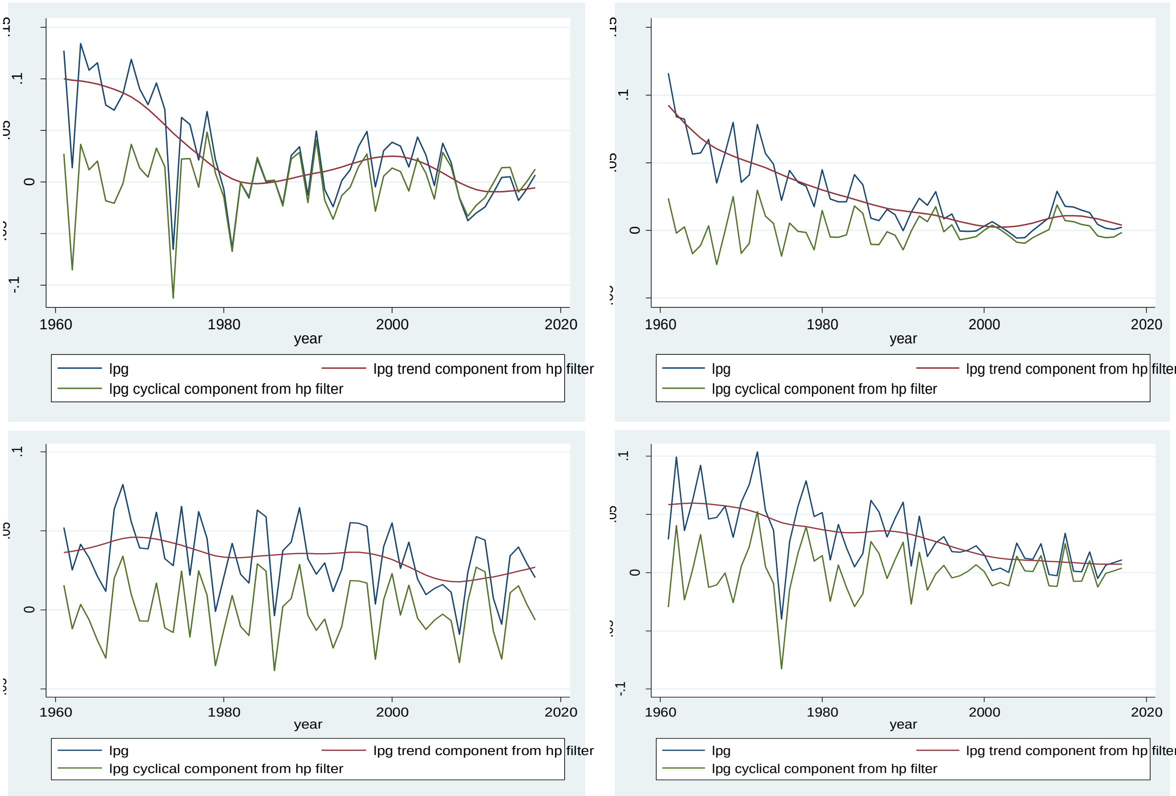
This would imply the following NKP specification in the Galí's framework, which is closest to ours (although our set up also relaxes the assumption of a constant NAWRU:

$$\Delta w_t - (\pi_t + yl_t) = \frac{1}{1+r_t} E_t(\Delta w_{t+1} - (\pi_{t+1} + \Delta yl_{t+1})) - \lambda_w [u_t - u^*] \qquad \text{(D.4)}$$

[10] Note that in order to bring our set up closer to Galí´s model we assume same indexation scheme for labour productivity.

ANNEX E: HP-FILTERED LABOUR PRODUCTIVITY GROWTH SERIES

Figure A.3: **HP filtered labour productivity growth series for Greece, Spain, Ireland and Portugal**



ANNEX F: UNIT ROOT TESTS OF WAGE INDICATORS

GREECE

Null Hypothesis: EL_DWST has a unit root

Exogenous: Constant

Lag Length: 1 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-5.658358	0.0000
Test critical values:	1% level	-3.560019	
	5% level	-2.917650	
	10% level	-2.596689	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: EL_DWST has a unit root

Exogenous: Constant, Linear Trend

Lag Length: 1 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-5.600697	0.0001
Test critical values:	1% level	-4.140858	
	5% level	-3.496960	
	10% level	-3.177579	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: EL_DWST has a unit root

Exogenous: None

Lag Length: 1 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-5.707337	0.0000
Test critical values:	1% level	-2.609324	
	5% level	-1.947119	
	10% level	-1.612867	

*MacKinnon (1996) one-sided p-values.

SPAIN

Null Hypothesis: ES_DWST has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-6.091440	0.0000
Test critical values:	1% level	-3.557472	
	5% level	-2.916566	
	10% level	-2.596116	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: ES_DWST has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-6.285612	0.0000
Test critical values:	1% level	-4.137279	
	5% level	-3.495295	
	10% level	-3.176618	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: ES_DWST has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-6.159692	0.0000
Test critical values:	1% level	-2.608490	
	5% level	-1.946996	
	10% level	-1.612934	

*MacKinnon (1996) one-sided p-values.

IRELAND

Null Hypothesis: IE_DWST has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-6.586388	0.0000
Test critical values:	1% level	-3.557472	
	5% level	-2.916566	
	10% level	-2.596116	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: IE_DWST has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-7.027301	0.0000
Test critical values:	1% level	-4.137279	
	5% level	-3.495295	
	10% level	-3.176618	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: IE_DWST has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-6.192221	0.0000
Test critical values:	1% level	-2.608490	
	5% level	-1.946996	
	10% level	-1.612934	

*MacKinnon (1996) one-sided p-values.

PORTUGAL

Null Hypothesis: PT_DWST has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-5.367395	0.0000
Test critical values:	1% level	-3.557472	
	5% level	-2.916566	
	10% level	-2.596116	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: PT_DWST has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-5.418609	0.0002
Test critical values:	1% level	-4.137279	
	5% level	-3.495295	
	10% level	-3.176618	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: PT_DWST has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-5.422067	0.0000
Test critical values:	1% level	-2.608490	
	5% level	-1.946996	
	10% level	-1.612934	

*MacKinnon (1996) one-sided p-values.