Disentangling Global Value Chains[*]

JOB MARKET PAPER

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First version: August 12, 2016
This version: October 12, 2017

Abstract

I present a global value chain (GVC) framework that incorporates the fact that intermediate input suppliers produce specialized inputs that are only compatible with specific downstream uses. This feature is confirmed by firm-level data and is at odds with the current approach which assumes that all products, within narrowly defined industries, utilize the same intermediate inputs. I show how this framework can use partial snapshots of the overall world supply chain picture in order to obtain more accurate GVC estimates. For example, firm-level data reveals that Mexican manufacturing exports to U.S. consumers utilize relatively more U.S. inputs than exports to other countries. Disciplining GVC estimates with this knowledge yields that 27% of this $118bn flow is U.S. value-added while ignoring this fact implies a share of only 17.6%. This has serious implications for the ongoing renegotiation of NAFTA. I also show that current data limitations impede the use of the influential sufficient statistics approach used to quantify the welfare gains from trade in models with specialized inputs linkages.

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1   Introduction

As you read this paper, the North American Free Trade Agreement (NAFTA) is being renegotiated for the first time since its inception in 1994.[1] The stakes could not be higher. Canada, Mexico and the U.S. account for over a quarter of world GDP, trade more than one trillion dollars amongst themselves annually, and form one of the most integrated regional blocks.

At the same time, the specter of protectionism is at its strongest since the original signing of NAFTA and the risks of supply chain disruptions triggered by an increase in trade barriers have not been lost on many of the leading experts. Major news outlets, CEOs, concerned research institutions, and scholars have repeatedly warned about the potential losses to be incurred in such a scenario.[2] Furthermore, despite the negative rhetoric emanating from the current administration, even the U.S. Trade Representative has acknowledged the risks by stating "Our objective is to, first of all, do no harm" (Committee on Ways and Means 2017).

Supply chain disruption is highly costly because modern supply chains feature specialized inputs linkages, where intermediate input suppliers customize their goods to be compatible with only specific downstream uses. For example, the lithium battery supplier in Apple's famously long iPod supply chain manufactures it exactly to the size of the metal frame while the screen supplier ensures that the touch, color, and dimming capabilities are in line with Apple's iOS software (Linden et al. 2011). Today, this form of input compatibility is ubiquitous (Rauch 1999, Nunn 2007, Antràs and Staiger 2012, Antràs and Chor 2013).

In contrast, these linkages have yet to be incorporated into the workhorse model for estimating global value chains (GVCs, henceforth). The latter, defined as the aggregate value of supply chain flows across all products produced throughout a particular sequence of locations, need to be estimated since the universe of product-level supply chain data is not available. The current approach shuts down the specialized inputs dimension and instead constructs GVCs by imposing the ironclad assumption that all goods, within a given industry, utilize the exact same inputs. Since GVCs underlie the statistics proxying regional integration, this suggests that these shortcomings might be seriously misguiding the current trade policy debate.

I develop a new GVC measurement framework that is consistent with a class of structural specialized inputs models and fully characterizes GVCs with a set of primitives that can be measured directly with firm-level data. Importantly, the framework is more versatile than any specific model since it requires imposing no particular microfoundation and instead assimilates the fact that these primitives take the same values in equilibrium across the whole class of models. Specifically, the primitives require linking the sources of input purchases with

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[1]President Donald Trump provided Congress with the 90 day notice required prior to beginning negotiations on May 18, 2017. The first round of negotiations began on August 16, 2017. Current U.S. fast-track law, under which the U.S. Congress forgoes its constitutional right to amend a treaty and which is widely understood as a necessary condition for finalizing a trade agreement with the U.S., expires on June 30, 2018. It is likely that the U.S. will try to finalize an agreement by this date, though the President could request a three year extension.

[2]See the reports Wilson (2011), U.S. Chamber of Commerce (2015), Dziczek et al. (2016), Blanchard (2017), and news articles/blogposts Sandbu (2017), Parilla (2017), Amiti et al. (2017), Murphy (2017) and Donnan (2017).

the destination and use of exports. For example, I leverage Mexican import-export shipment data to obtain information about the cross-border supply chains underlying the Mexican primitives and this reveals that U.S. inputs are used relatively more for exports to the U.S. than in exports to other countries. I illustrate this for the vehicle industry in Figure 1.1, which shows that the U.S. accounts for a colossal 74% of the foreign inputs embedded in Mexican vehicles sold to U.S. consumers but for only 18% of the inputs of those sold to German consumers.[3]

In practice, only partial snapshots of the universe of supply chain data are available and so I embed the measurement framework in a quadratic program that delivers point estimates for the universe of primitives needed for characterizing GVCs. Specifically, I show how to combine rich micro-level data with aggregate bilateral trade data in order to obtain estimates of the overall GVC picture in two steps. First, I impose a set of linear constraints that ensure that the estimated primitives are in line with the observable bilateral trade data and such that they represent an internally consistent system. These ensure that the primitives implement a class of structural specialized inputs models. Second, I show how the quadratic objective function can be used by a researcher to incorporate additional sources of data or specific priors over the complex supply chain flows present in today's global economy. In particular, I map the Mexican microdata directly into the objective function in order to ensure that the GVCs that cross through Mexico reflect the empirical regularities exemplified in Figure 1.1.

Armed with this framework, I then study the degree of integration within the NAFTA region, as proxied by Mexico-U.S. trade. While computing the input shares in Figure 1.1 is a purely data-driven exercise, GVC estimates are essential for studying integration since decomposing the sources of value-added in Mexican exports requires tracing value across all upstream stages of production. I obtain my GVC estimates by combining the Mexican microdata together with bilateral trade data from the World Input-Output Database (WIOD).

My main empirical result is that U.S. value-added accounts for 27% of the $118bn of Mexican manufactures purchased by U.S. consumers and contrasts with the current benchmark estimate of 17.6%. Integration is particulary deep for Mexico's main export sector, the motor vehicle industry, where U.S. value-added accounts for 38% of the $35bn of exports to U.S. consumers (17% in the benchmark). The wedge between these estimates is driven by the relatively higher share of U.S. inputs in exports to the U.S. that is ignored when the specialized inputs channel is not taken into account since this implies assuming that Mexico uses the same distribution of inputs for all of its exports.

The most useful feature of this estimation approach is that it is easily adaptable and can incorporate additional information in a practical manner. While supply chain data is rarely publicly available, many researchers have access to partial snapshots of the overall supply chains underlying global trade that are extremely informative about how intermediate inputs

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[3]The source is a confidential government-owned database reporting the universe of manufacturing import and export shipments. I obtained overall input shares by assuming that, within a firm, every dollar of exports utilizes the exact same content of imports. Though this might not be true in multi-product firms, there is little one can do to address this issue since within-firm data is not available (see Manova and Zhang 2012).
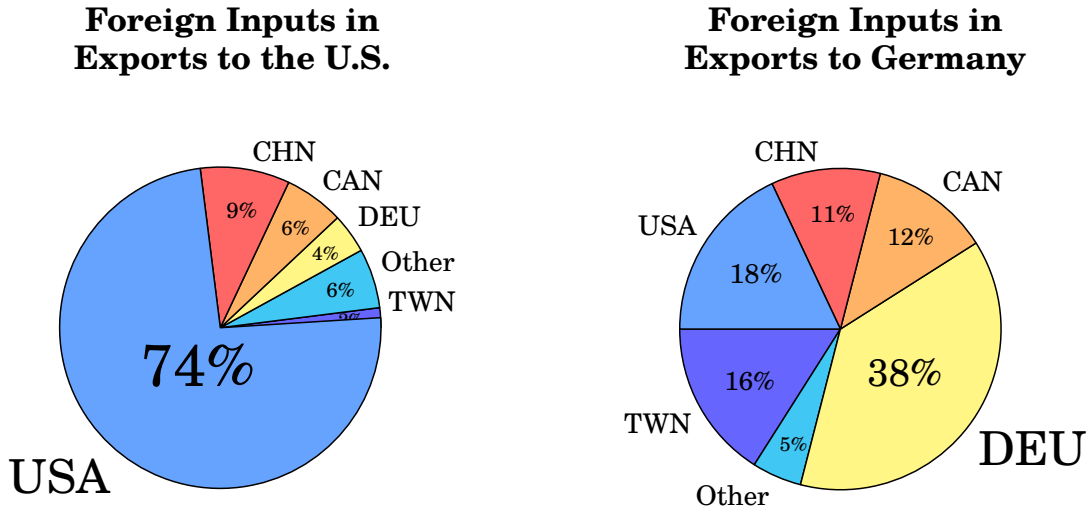
Figure 1.1: Distribution of Sources of Foreign Inputs Used in Mexican Final Good Vehicle Exports to Consumers in Various Markets: Charts are constructed using the universe of Mexican firm-level import/export shipment data for 2014 (see footnote 3).

are used. The tools I propose can be readily applied by those with access to specific sets of data or knowledge. From a methodological standpoint, this is the paper's key contribution.

Furthermore, this approach nests the current benchmark which I refer to as the roundabout approach since it is microfounded by a production function in which the exact same technology is used to produce the intermediate inputs that intermediate inputs themselves require. This property is convenient since it implies that bilateral intermediate input trade shares define the use of inputs at any stage of production and thus bilateral data fully, and uniquely, characterize GVCs. More generally, this property defines the measurement framework of Input-Output analysis (I-O, henceforth), as originally developed by Leontief (1941). This fact has justified the use of I-O analysis as a GVC estimation framework since it delivers the same estimates as any structural roundabout model that perfectly fits bilateral data.

The downside of the roundabout approach is that it is sharply at odds with the heterogeneity in Figure 1.1. In reality, the aggregate use of intermediate inputs varies depending on the destination of exports since it reflects the range of specialized inputs embedded in the set of products sold to each country.[4] For example, when Mexico exports Ford Fiestas to the U.S. it uses different inputs than when it exports Volkswagen Beetles to Germany since each vehicle has its own technological requirements and since both Ford and Volkswagen have separate supply chains. Models without specialized inputs cannot incorporate this variation since they

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[4]Similar empirical patterns have been found before. Within industry exports may vary across destinations due to quality (Bastos and Silva 2010, Brambilla et al. 2012, Brambilla and Porto 2016, Ding 2017), trade regime (Dean et al. 2011, Koopman et al. 2012), or credit constraints (Manova and Yu 2016). Meanwhile, there is also significant variation on the import side (Schott 2004, Hanson et al. 2005) and it is well-known that importing firms tend to also be exporters (Bernard et al. 2007). These facts suggest that the use of imports depends on the destination of production. See Manova and Zhang (2012), Fieler et al. (2017), Bernard et al. (2017a), Bernard et al. (2017b).

assume that both vehicles utilize the exact same inputs.[5,6]

Ultimately, the specialized inputs critique could be addressed with I-O analysis if bilateral data were further disaggregated to the product level since it is (probably) accurate to assume that each Ford Fiesta and each Volkswagen Beetle is built with the same intermediates regardless of where they are sold to. In practice, this is currently infeasible and is best illustrated by noting that the current state-of-the-art data, the WIOD, has only 56 sectors (20 in manufactures) and required a massive collaborative effort (see Timmer et al. 2015). Specialized inputs structural models dictate that the number of industrial categories required to overcome this issue is within the order of millions of sectors (i.e. products) per country. In any case, the empirical GVC literature uses data at this level of aggregation.

The specialized inputs channel thus paints a radically different GVC picture and has major implications on trade policy since it suggests that the potential risks of supply chain disruption are even more dire.[7] Furthermore, I will show that running a proper counterfactual in a structural specialized inputs model is very hard to do, given current data limitations, unless one is willing to impose heroic assumptions. Thus, measuring GVCs more accurately by leveraging firm-level data is especially valuable for the policy debate because it enriches our knowledge about the context itself and this is particularly significant in scenarios where the specialized inputs dimension is first order as is in the NAFTA region. Hence, if anything, fears of supply chain disruptions stoked by GVC statistics that underplayed NAFTA integration should be heightened once it is revealed that integration is actually much deeper.[8]

I kickoff the paper in Section 2 by setting the stage and discussing why different theories of intermediate input use imply sharply different GVC estimates. Specifically, a theory is required in order to provide a lens for interpreting whatever data is available given that the lack of the universe of supply chain data prevents this from being a purely data-driven exercise. The current benchmark consists in interpreting bilateral data through the lens of the roundabout model and has far-reaching implications since the latter is just a knife-edge case of a continuum of specialized inputs models that match bilateral trade data. Indeed, specialized

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[5]History provides examples where this was accurate: During the 1800's the U.S. imported pig iron for a wide variety of uses (iron sheets, rails, bars, nails, wire, etc.) and not for a specific product (see Irwin 2000).

[6]More formally, there is a single set of GVC estimates that can be derived from a given bilateral trade database utilizing roundabout models and there is zero scope for a researcher to further discipline these estimates. Thus, additional microdata is, in a sense, useless. There is, however, a literature that shows how additional data can be used to break up this bilateral data into more disaggregate flows and then apply a roundabout framework to the new database (Batten 1982, Golan et al. 1994, Canning and Wang 2005, Dean et al. 2011). This is not what this paper is about; here I develop new tools for deriving different GVC estimates using the original database.

[7]Koopman et al. (2010) and De La Cruz et al. (2011) made a similar point by splitting Mexican bilateral trade flows across processing and non-processing trade. Though an useful approach, it is not practical nor easily replicable since disaggregating bilateral trade data is often extremely challenging. In contrast, the tools I propose can be generally applied to incorporate any type of extra information in a straightforward manner.

[8]Several studies have argued that supply chains magnify the potential losses from protectionism. Yi (2003), Yi (2010), Antràs and de Gortari (2017), and Lee and Yi (2017) show that this is quantitatively true in general equilibrium models. Kremer (1993) argues that this "can paralyze production by preventing bottleneck sectors from being bypassed." Barrot and Sauvagnat (2016), Boehm et al. (2016), and Carvalho et al. (2016) use natural disasters to empirically measure the costs of supply chain disruption. For counterfactuals on NAFTA, see Caliendo and Parro (2015) and Head and Mayer (2017).

inputs models are consistent with shares of U.S. value in Mexican exports to the U.S. as low as 3% and as high as 40% whereas the roundabout model necessarily implies 17.6%. Hence, incorporating firm-level data is crucial for obtaining an accurate point estimate.

I then devote Section 3 to developing the specialized inputs measurement framework that can handle the degrees of freedom arising from there being a whole class of models consistent with the same bilateral data. I do this by defining GVC objects themselves as the basic building blocks and this lets me recast any theory of intermediate inputs as a guidebook for tracing the upstream input purchases of any sequence of production. Section 4 develops the optimization framework that shapes GVC estimates with supply chain information.

Finally, I close the paper in Section 5 by deriving the tools for counterfactual analysis in a fully-fledged multi-country, multi-sector, multi-stage, Ricardian general equilibrium model with international trade barriers and variety-specific input linkages based on the one sector model of Antràs and de Gortari (2017). The main innovation is that, with multiple sectors, specialized inputs linkages occur not only as pure snakes but also as spiders (see Baldwin and Venables 2013). The first theoretical result is that the model's GVCs are consistent with the measurement framework developed in Sections 3 and 4 and thus provides a microfounded justification for its use, much like roundabout models justify the use of I-O analysis. Secondly, I derive a formula for computing the welfare gains from trade conditional on the knowledge of some sufficient statistics and key elasticities. This is useful because the model's direct use is limited since it depends on a large number of (unknown) parameters.

Unfortunately, current data limitations make it hard to implement the sufficient statistics formula properly in practice. More specifically, the sufficient statistics literature based on roundabout models (Dekle et al. 2007, Arkolakis et al. 2012) claims that "micro-foundations are not particularly important for determining a trade model's macro-economic implications" (Allen et al. 2017) since the change in welfare depends on the change in domestic expenditures. However, the sufficiency of bilateral data depends crucially on the assumption of roundabout production and is thus deeply related to the sufficiency of bilateral data for GVC estimation. In contrast, the required sufficient statistic in specialized inputs models is the expenditure share on goods produced through entirely domestic supply chains and which is a datapoint that is rarely collected by statistical agencies.[9] Thus, the microfoundation is everything since these statistics can only be obtained structurally, and, though theoretically appealing, this renders the sufficient statistics approach an empirically elusive one until we obtain richer data.

This paper fits into a new literature that estimates GVCs through the lens of specialized inputs models. Specifically, intermediate inputs account for two-thirds of world trade and have been widely studied albeit mainly through roundabout production models, as exemplified by the structural gravity literature (see Krugman and Venables 1995, Eaton and Kortum 2002, Balistreri et al. 2011, di Giovanni and Levchenko 2013, Bems 2014, Caliendo and Parro 2015,

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[9]The standard formula does not apply since the partial elasticity of relative imports from two sources depends on trade costs with third countries and so the macro-level restriction "the import demand system is CES" in Arkolakis et al. 2012 does not hold. Section 5 discusses this in detail while Appendix Section B.6 presents formal proof.

Ossa 2015, and Allen et al. 2017). This class of models justifies I-O analysis since it delivers the same prescription for constructing GVC flows. Hence, the empirical GVC literature is entirely based on I-O analysis and has focused primarily on vertical specialization (Hummels et al. 2001, Johnson and Noguera 2012), tracing value (Koopman et al. 2014, Wang et al. 2013), and downstreamness (Antràs et al. 2012, Fally 2012, Antràs and Chor 2013), though many other classical international economics questions have also been addressed such as the factor content of trade (Trefler and Zhu 2010), value-added exchange rates (Bems and Johnson 2017), international inflation spillovers (Auer et al. 2017), and business cycle synchronization (di Giovanni and Levchenko 2010, Johnson 2014, Duval et al. 2016, di Giovanni et al. 2017).

Though the international trade field has shifted profoundly over the last two decades to an approach emphasizing firm heterogeneity (Melitz and Redding 2012), this has had little effect on the empirics of GVCs. There is, however, a small but increasingly important literature of structural models with specialized inputs as in Yi (2010), Costinot et al. (2012), Antràs and Chor (2013), Fally and Hillberry (2016), Johnson and Moxnes (2016), and Antràs and de Gortari (2017). This paper is the GVC estimation analog to the latter since its measurement framework is consistent with the GVCs they deliver (whereas I-O analysis is not).

In terms of numerical work, the quadratic programming approach follows a long tradition of exploiting linearity in order to solve for high-scale optimization problems in economics. Samuelson (1952) is a major inspiration in that it utilized linear programming (see Dorfman et al. 1958) to ask: How can bilateral exports be determined if we only observe aggregate exports? This paper tackles a corollary: How can GVCs be determined if we only observe bilateral exports? Relatedly, the field of regional science has occasionally used these tools for the data reconciliation process of building I-O tables when parts of the data are not observed (Harrigan and Buchanan 1984, Canning and Wang 2005, Miller and Blair 2009).

Finally, the roundabout approach has been enormously influential beyond trade. Samuelson (1951) provided the key insight that this measurement framework is consistent with the equilibrium of a constant returns to scale production economy. Subsequently, intermediate inputs have been widely incorporated in the form of roundabout production in the macroeconomics literature following the seminal I-O models of Domar (1961), Hulten (1978), and Long and Plosser (1983) to study business cycles (Basu 1995), growth (Jones 2011), misallocation (Jones 2013, Bigio and La'O 2016, Caliendo et al. 2017), aggregate fluctuations (Acemoglu et al. 2012, Carvalho and Gabaix 2013, Carvalho 2014, di Giovanni et al. 2014, Baqaee 2014, Baqaee and Farhi 2017), and development accounting (Bartelme and Gorodnichenko 2015, Cuñat and Zymek 2017). While Leontief (1941) is the bedrock of this literature, it could be extended to incorporate more complex production networks featuring specialized inputs linkages.

2 The Hunt for GVCs: The Challenge

I begin by framing the challenge of estimating GVCs with bilateral trade flows in the presence of specialized inputs linkages. Specifically, I illustrate, through the lens of toy models, that bilateral trade data can be explained by a wide range of supply chain models and that the literature has so far focused on the special case of roundabout production.

2.1 Observable Data

Let $\mathcal{J}$ be the set of countries of the world and suppose that the world economy is such that there is a single good, called widgets, that can be produced and traded. Luckily, we observe the bilateral flow of widgets with $X(j',j)$ being the aggregate sales from j' to j of widgets used as intermediate inputs and with $F(j',j)$ being the aggregate sales of j' to j of widgets that are consumed as final goods. Gross output $Y(j)$ equals aggregate widget sales and gross domestic product $GDP(j)$ equals the former minus intermediate input purchases

$$Y(j) = \sum_{j'\in\mathcal{J}} X(j,j') + \sum_{j'\in\mathcal{J}} F(j,j'), \qquad GDP(j) = Y(j) - \sum_{j'\in\mathcal{J}} X(j',j).$$

I refer to the collection of $X(j',j)$ and $F(j',j)$ as the world input-output table (WIOT).

2.2 A Toy Roundabout Production Model

I begin by focusing on a very stylized model in which I make the following simplifying assumptions: (i) technology is Cobb-Douglas, (ii) market structure is perfect competition, (iii) labor in each country is normalized to one, and (iv) preferences are such that country j spends a share $\alpha_{j'j}$ of its income on widgets from j'. Since production features constant returns to scale it is useful to work directly with prices (the dual). Country j sells widgets at unit price

$$p_j = \left(w_j\right)^{\beta_j} \left(\prod_{j'\in\mathcal{J}} \left(p_{j'}\right)^{\pi_{j'j}}\right)^{1-\beta_j}, \tag{1}$$

where w_j is the wage (or GDP), β_j is value-added share, and $\sum_{j'\in\mathcal{J}} \pi_{j'j} = 1$.

The final assumption is that of roundabout production where intermediate inputs require intermediates produced with the exact same technology. That is, widgets are produced with widgets and of each dollar of widget production in j a share $1 - \beta_j$ is spent on widgets used as intermediate inputs and a share $\pi_{j'j}$ of that on widgets from source country j'. For now, I assume that the shares $\pi_{j'j}$ are a set of numbers fixed by nature but I will later show that the equations that characterize the model's equilibrium are isomorphic to the class of structural models that deliver gravity and that vary mainly as to how $\pi_{j'j}$ is microfounded. A key implication of roundabout production is that the share of inputs from j' required to produce a widget in j is independent of where j sells its own output to.

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The equilibrium can be characterized by mapping it into WIOT terms and defining a fixed point that pins down wages. Specifically, intermediate input sales between j' and j can be found by noting that gross output is given by w_j/β_j, that a share $1-\beta_j$ is spent on intermediate inputs, and that a share $\pi_{j'j}$ of that expenditure is spend on inputs from j'. Meanwhile, final good sales between j' and j are determined by the share of income that j spends on these goods. The WIOT is thus

$$\hat{X}(j',j) = \pi_{j'j}(1-\beta_j)\frac{w_j}{\beta_j}, \qquad \hat{F}(j',j) = \alpha_{j'j}w_j, \tag{2}$$

where I have added a hat to indicate that these variables correspond to the simulated model and not to data.[10] Wages are pinned down by equating income to value-added production[11]

$$w_j = \sum_{j'\in\mathcal{J}}\left(\hat{X}(j,j') + \hat{F}(j,j')\right) - \sum_{j'\in\mathcal{J}}\hat{X}(j',j). \tag{3}$$

The equilibrium depends on the following free parameters: $\pi_{j'j}$, $\alpha_{j'j}$, and β_j. There is a single parameterization such that the simulated model replicates the observable data. To see this, let the relative shares of input and final good sourcing discipline $\pi_{j'j}$ and $\alpha_{j'j}$, and let the value-added to gross output ratio inform the choice of β_j. That is

$$\pi_{j'j} = \frac{X(j',j)}{\sum_{i'\in\mathcal{J}}X(i',j)}, \qquad \alpha_{j'j} = \frac{F(j',j)}{\sum_{i'\in\mathcal{J}}F(i',j)}, \qquad \beta_j = \frac{GDP(j)}{Y(j)}. \tag{4}$$

It is straightforward to check that the simulated WIOT equals the data, i.e. $\hat{X}(j',j) = X(j',j)$ and $\hat{F}(j',j) = F(j',j)$ for all $j',j \in \mathcal{J}$, and that this parameterization is unique.

Roundabout production models are important because they provide a structural justification for the computation of GVC related empirical statistics. To see this, let me focus on measures of value-added trade as studied recently by Johnson and Noguera (2012) who show that value-added and gross trade balances differ substantially and by Koopman et al. (2014) who show how to quantify double-counting in gross exports by explicitly taking into account the fact that value crosses borders multiple times. The common feature across these statistics is that none of them are directly observable in bilateral trade flows and require a model for interpreting the data since value needs to be traced up the GVC.

The theory of roundabout production provides a lens for interpreting bilateral trade flows and deriving value-added measures. Define $\tilde{\pi} \equiv \pi_{j'j}(1-\beta_j)$ as the share of j's output value spent on inputs from j'. The share of value-added produced in country j' in every dollar of country j's output can be computed by summing up the production of value across all upstream

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[10]Trade imbalances can be incorporated using the tools developed in Dekle et al. (2007) by defining final good flows as as $\hat{F}(j',j) = \alpha_{j'j}(w_j - D_j)$, with D_j the dollar value deficit (if negative) or surplus (if positive).

[11]Note that this fixed point applies also in microfounded models in which π_{ij} and α_{ij} depend on wages.

stages of production and across all sequences that deliver inputs to country j as

$$\hat{V}\left(j',j\right) = \beta_{j'}1_{[j'=j]} + \beta_{j'}\tilde{\pi}_{j'j} + \beta_{j'}\sum_{i\in\mathcal{J}}\tilde{\pi}_{j'i}\tilde{\pi}_{ij} + \beta_{j'}\sum_{i'\in\mathcal{J}}\tilde{\pi}_{j'i'}\sum_{i\in\mathcal{J}}\tilde{\pi}_{i'i}\tilde{\pi}_{ij} + \dots$$

The first term represents domestic value added directly into j's production and thus appears only if $j' = j$, the second term represents value added by j' directly into intermediates used by j, so on and so forth. This decomposition can be written compactly by defining $\hat{V} = \left[\hat{V}\left(j',j\right)\right]$ and $\tilde{\pi} = \left[\pi_{j'j}\left(1-\beta_j\right)\right]$ as matrices of size $|\mathcal{J}| \times |\mathcal{J}|$, and $\beta = \left[\beta_j\right]$ as a vector of size $|\mathcal{J}| \times 1$. The decomposition of value-added by source country equals

$$\hat{V} \equiv \operatorname{diag}\{\beta\}\left[\mathbb{I} - \tilde{\pi}\right]^{-1}, \tag{5}$$

and clearly $\sum_{j'\in\mathcal{J}}\hat{V}\left(j',j\right) = 1$ for all $j \in \mathcal{J}$.

Note that the decomposition in equation (5) does not depend on the specific microfoundation underlying $\pi_{j'j}$ in practice. For example, the equilibrium of this model looks exactly as the one-sector version of the seminal Ricardian model of Eaton and Kortum (2002), with the exception that that model is more restrictive since it imposes $\pi_{j'j} = \alpha_{j'j}$. More generally, tracing value in almost any model that features gravity, as defined broadly in Costinot and Rodríguez-Clare (2015) and Head and Mayer (2015) and including Armington trade models (Bems 2014) where each country produces a differentiated variety or extensions of imperfect competition models á la Melitz (2003) to incorporate intermediate input trade (Balistreri et al. 2011), can be done using equation (5). Though different microstructures imply different mappings of deep structural parameters and general equilibrium variables to final good trade flows $\hat{F}$, intermediate input expenditure shares $\tilde{\pi}$, and value-added shares β, all models have the same implications over GVCs since they are all parameterized with the goal of replicating the same WIOT data in equilibrium.[12] Importantly, this means that the Cobb-Douglas and perfect competition assumptions imposed in equation (1) have no influence on how value is traced in equilibrium.

Another way to say this is that (most) of the structural models that incorporate intermediate inputs through roundabout production are part of the class of models consistent with the general accounting framework of Input-Output analysis.[13] Indeed, the empirical GVC literature defines GVC statistics directly with I-O analysis and assumes the existence of technical coefficients denoting the share of inputs from j' needed to produce a dollar of output in j and

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[12]The importance of the word equilibrium here is crucial since different models can obviously differ substantially as to which variables adjust in order to achieve such equilibrium or may provide different counterfactual implications when shocked. But in equilibrium, if two structural roundabout production models deliver the exact same WIOT flows then the decomposition of value-added trade will also match. In practice, models often incorporate so much structure that they cannot fit the data perfectly and thus may deliver different estimates for value-added trade. However, when these differences arise in equilibrium they are entirely due to the lack of fit and not because these different models have distinct implications over these statistics.

[13]Imperfect competition models with fixed costs can be tricky depending on how one maps the latter into bilateral trade flows. When one assumes that fixed costs are capital investment, or final good consumption by the firm more broadly, then these models are consistent with IO analysis. However, matters can be complicated when fixed costs are thought of as intermediate inputs and their unit costs differ from marginal costs.

which are defined directly with data as proportional to the share of aggregate intermediate input purchases

$$a\left(j'|j\right) \equiv \frac{X\left(j',j\right)}{Y\left(j\right)} = \frac{X\left(j',j\right)}{\frac{1}{1-\beta_j}\sum_{i'\in\mathcal{J}}X\left(i',j\right)}, \qquad \Rightarrow a\left(j'|j\right) = \pi_{j'j}\left(1-\beta_j\right). \tag{6}$$

The second part shows formally how the above structural roundabout model is consistent with I-O analysis since it replicates bilateral trade data.

Hence, the empirical GVC literature is correctly specified under I-O analysis as long as researchers believe that the data generating process underlying international trade flows is consistent with a roundabout production model. Most importantly, the precise model is irrelevant since the empirical GVC literature cares not about counterfactuals but about descriptive economics and in equilibrium all of these models are observationally equivalent. This can be exemplified with the decomposition of value-added by source which I-O analysis defines as

$$\mathbf{V} \equiv \mathrm{diag}\{\boldsymbol{\beta}\}\left[\mathbb{I}-\mathbf{A}\right]^{-1}, \tag{7}$$

where $\mathbf{A} = \left[a\left(j'|j\right)\right]$ is the matrix of technical coefficients and where $\left[\mathbb{I}-\mathbf{A}\right]^{-1}$ is commonly known as the Leontief inverse matrix. Clearly $\hat{V}$ in equation (5) exactly matches $\mathbf{V}$.

A corollary of this discussion is that actually, with regard to GVC estimation, there are no degrees of freedom in the class of roundabout production models. That is, I-O analysis is fully characterized by WIOT data flows.[14] Since (most) models are special cases of I-O analysis then there is a unique set of values that GVC statistics can take in any given dataset if one believes in this type of microstructure. Naturally, the implications of counterfactual analysis on GVC statistics will be different across models but this will be driven entirely by what happens in the counterfactual equilibrium since the baseline is common. That said, there is an important ongoing debate arguing on one side that a wide range of models deliver the exact same counterfactual analysis (Arkolakis et al. 2012), and proxied by the change in aggregate domestic expenditures, while the other side has argued that the margins of adjustment actually matter (Melitz and Redding 2015). Regardless, this debate is largely driven by the roundabout production assumption since the the sufficiency of bilateral trade flows for welfare occurs for the exact same reason that this data is sufficient for estimating GVCs. However, both statements are only true in the absence of trade in specialized inputs and I will argue in Section 5 that this is a somewhat moot debate since in reality these forces are ubiquitous.

I summarize the discussion in this section with two statements.

Fact 2.1. *GVC statistics from roundabout production models and I-O analysis are equivalent.*

Fact 2.2. *GVC statistics from I-O analysis are uniquely determined by bilateral trade data.*

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[14]Note that equation (7) depends on value-added shares that depend on GDP and gross output, the Leontief matrix that depends on intermediate input flows and gross output, and on final good flows.

2.3 A Toy Specialized Inputs Model

In reality, countries use different inputs when producing exports for different destinations, even within narrowly defined sectors, since countries sell different varieties to different locations (see Figure 1.1 and footnote 4). This empirical fact cannot be incorporated into roundabout production models since they are characterized by input shares $\pi_{j'j}$ that are independent of where output is sold to (this is also true in multi-sector versions). However, a more general model with specialized inputs linkages can incorporate this variation since it is designed precisely for this purpose. I exemplify this with the following modification: assume that now the price of country j's widget varies depending on the market i to which it is exported to

$$
p_{j,i} = \left(w_j\right)^{\beta_j} \left(\prod_{j' \in \mathcal{J}} \left(p_{j',j}\right)^{\pi_{j'j,i}}\right)^{1-\beta_j}.
$$

Country j sells one unit of its widget in market i at $p_{j,i}$ and the variation across destinations is driven by variation in the use of intermediate inputs. Specifically, I assume that country j spends a share $\pi_{j'j,i}$ on inputs from j' when selling to i and the input prices which country j itself faces are given by $p_{j',j}$ since each source country j' uses its own specific supply chain for producing widgets sold in market j. Hence, the variation in $p_{j,i}$ across export markets is driven by some (currently unspecified) variation in the use of intermediate inputs.[15] Note that this model nests the roundabout production model, which arises whenever $\pi_{j'j,i}$ is constant across all destination markets $i \in \mathcal{J}$ for all input-output pairs $j', j \in \mathcal{J}$.

In terms of interpretation, the literal reading is that country j sets different prices in different export markets because it builds the same good differently. This could happen as consequence of trade policy in that rules of origin require goods to have a certain amount of regional content in order to have access to zero or lower tariffs. Alternatively, a more useful interpretation is that countries actually produce a continuum of varieties of widgets and they sell different sets of widgets to different markets. In a world with roundabout production this has no bearing over input shares since the production of each variety of widgets uses the same intermediate inputs. In contrast, in a world with specialized input linkages each widget variety requires a specific set of input shares. Thus, the aggregate share of inputs that country j uses from a specific source i will vary depending on the destination of its exports because it reflects the overall distribution of supply chains used to produce the widgets sold in each market. In reality, this distribution may be reflection of a diverse set of economics forces such as compatibility or multinationals and will be explored further in Section 5.

The equilibrium of this model can be characterized as before in that $\hat{F}(j', j)$ is given by the same term as in equation (2) and the fixed point in equation (3) also applies. However, deriving

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[15]This discussion refers to country j setting different prices at the dock (FOB, free-on-board prices) when exporting to different locations. Of course, CIF (cost, insurance and freight) prices will differ across locations even in roundabout production models when importer-exporter-specific iceberg trade costs are included though this has no consequence on the relative use of intermediate inputs since trade costs are proportional to output.

bilateral intermediate input flows is more intricate since gross output cannot be used to derive aggregate input flows given that the use of inputs depends on the downstream use of output. Specifically, let $\tilde{\pi}_{j'j,i} \equiv \pi_{j'j,i}\left(1 - \beta_j\right)$ be the expenditure on inputs from j' for every dollar of j exports to i. Overall intermediate input trade between j' and j can be obtained by summing up the use of intermediates at every upstream stage of production

$$\hat{X}\left(j',j\right) = \sum_{i\in\mathcal{J}} \tilde{\pi}_{j'j,i}\hat{F}\left(j,i\right) + \sum_{i'\in\mathcal{J}} \tilde{\pi}_{j'j,i'}\sum_{i\in\mathcal{J}}\tilde{\pi}_{ji',i}\hat{F}\left(i',i\right) + \sum_{i''\in\mathcal{J}}\tilde{\pi}_{j'j,i''}\sum_{i'\in\mathcal{J}}\tilde{\pi}_{ji'',i'}\sum_{i\in\mathcal{J}}\tilde{\pi}_{i''i',i}\hat{F}\left(i',i\right) + \dots \tag{8}$$

The first term represents intermediate inputs from j' used by j to produce final goods, the second term represents intermediate inputs from j' used by j to produce intermediate inputs that are embedded directly into final goods, so on and so forth. Though an infinite sum, it is straightforward to show that this can be computed exactly with the tools of linear algebra.[16] Crucially, note that here, in contrast to the roundabout production model, the pattern of intermediate input purchases is determined by the patterns of final demand.

The specialized inputs model depends on the following free parameters: $\pi_{j'j,i}$, $\alpha_{j'j}$, and β_j. In contrast to the roundabout toy model, there is a continuum of parameterizations that replicate WIOT data. To see this, first note that $\alpha_{j'j}$ and β_j can be mapped as before into final good shares and the GDP to gross output ratios as in equation (4). In contrast, input shares $\pi_{j'j,i}$ are hard to discipline because $\hat{X}\left(j',j\right)$ is a construct that depends on the whole set of $\left\{\pi_{j'j,i}\right\}_{j',j,i\in\mathcal{J}^3}$ but in which there is no direct way to reverse engineering a specific $\pi_{j'j,i}$ from the flow $X\left(j',j\right)$. It is only when input shares $\pi_{j'j,i}$ are constant across all destinations $i \in \mathcal{J}$ that the data $X\left(j',j\right)$ provides a unique mapping into these variables. Indeed this is what happens in roundabout production models (i.e. equation 4). Another way to put this is that bilateral intermediate input trade flows are a set of $|\mathcal{J}|^2$ numbers while $\pi_{j'j,i}$ are a set of $|\mathcal{J}|^3$ numbers. The difference between $|\mathcal{J}|^3$ and $|\mathcal{J}|^2$ drives the degrees of freedom that imply a continuum of parameterizations for $\pi_{j'j,i}$ that replicate the observable data. Among these, the literature has focused exclusively on the set of GVCs in which $\pi_{j'j,i}$ is constant across all $i \in \mathcal{J}$ and in which input shares have only $|\mathcal{J}|^2$ degrees of freedom.

Conceptually, the observable data is an aggregate of a rich micro-level data generating

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[16] Let $\hat{F}$ be a vector of size $|\mathcal{J}|^2 \times 1$ of the elements $\hat{F}\left(j',j\right)$ ordered first along the first dimension and then along the second dimension, let $\tilde{\pi}_{j'j} = \left[\tilde{\pi}_{j'j,1}, \dots, \tilde{\pi}_{j'j,|\mathcal{J}|}\right]$ be a vector of size $1 \times |\mathcal{J}|$, and let $\tilde{\pi}$ be the stacked up version of these vectors ordered analogously to $\hat{F}$ so that it is of size $|\mathcal{J}|^2 \times |\mathcal{J}|$. Define the auxiliary matrix $\tilde{\Pi} = \left[\mathbf{1}_{|\mathcal{J}|\times 1} \otimes \mathbb{I}_{|\mathcal{J}|\times|\mathcal{J}|}\right] * \tilde{\pi}$, with $\otimes$ the Kronecker product and $*$ the Khatri-Rao product (Kronecker at the row level), of size $|\mathcal{J}|^2 \times |\mathcal{J}|^2$. Then

$$\hat{X} = \tilde{\Pi}\left[\mathbb{I}_{|\mathcal{J}|^2\times|\mathcal{J}|^2} - \tilde{\Pi}\right]^{-1}\hat{F},$$

where $\hat{X} = \left[\hat{X}\left(j',j\right)\right]$ is a vector of size $|\mathcal{J}|^2 \times 1$. The decomposition of value-added by source country is now conditional on where output is sold to and given by

$$\hat{V} \equiv \left[\mathrm{diag}\left\{\beta\right\} \otimes \mathbf{1}_{1\times|\mathcal{J}|}\right]\left[\mathbb{I}_{|\mathcal{J}|^2\times|\mathcal{J}|^2} - \tilde{\Pi}\right]^{-1}, \tag{9}$$

where $\hat{V} = \left[\hat{V}\left(j',j,i\right)\right]$ is the size $|\mathcal{J}| \times |\mathcal{J}|^2$ matrix of value-added shares from j' in every dollar of sales from j to i.

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process and the degrees of freedom reflect the fact that part of this richness is eliminating through aggregation. The special case of roundabout production is the one case in which no information is lost through aggregation. While this is not testable with WIOT data it can be tested using micro-level firm data and in practice, at least for Mexico, it is firmly rejected (see Figure 3). Furthermore, while different parameterizations of $\pi_{j'j,i}$ have no implication on aggregate measures such as GDP or bilateral trade (by construction), they do have enormous consequence on GVC related statistics. For example, suppose there are two sets of parameterizations that replicate WIOT data but in one $\pi_{ChinaChina,US}$ is very low and in another it is very high. It is then likely that the U.S. consumes more Chinese value-added in the former case since China's exports to the U.S. contain more Chinese inputs than in the latter.

In sum, the main takeaway from the specialized inputs model is that neither Fact 2.1 nor Fact 2.2 hold any longer. First, this model is not equivalent to I-O analysis since input shares are not independent of where output is sold to. Second, now there are degrees of freedom in that there is a continuum of parameterizations that rationalize the same WIOT data.

Fact 2.3. *GVC statistics from specialized inputs models are not equivalent to those from I-O analysis nor uniquely determined by bilateral trade data.*

2.4 The Perils of Roundabout Production Models

I now show that the specialized inputs distinction has quantitative bite. Specifically, I simulate 1,000 specialized inputs models, while constraining each to replicate the same WIOT, and show that their GVCs vary substantially. I use the World Input-Output Database (WIOD) for the year 2014, the state-of-the-art WIOT dataset (see Timmer et al. 2015 and Timmer et al. 2016), which contains data for $|\mathcal{J}| = 44$ countries. In order to focus on GVC statistics related to manufactures I aggregate the data's industrial dimension into two aggregate manufacturing and rest of the economy sectors and apply two-sector versions of the above toy models.

Figure 2.1 plots the histogram for the joint distribution of the share of U.S. value-added in Mexican final good manufacturing exports to the U.S. itself and to the rest of the world across all simulations. More specifically, for each simulation I compute a set of $\left\{\pi_{j'j,i}\right\}_{j',j,i\in\mathcal{J}}$ such that the simulated WIOT exactly matches the data, i.e. $\hat{X}(j',j) = X(j',j)$ and $\hat{F}(j',j) = F(j',j)$ for all $j', j, \in \mathcal{J}$, and compute the statistics of interest with equation (9). The key takeaway from the histogram is not the distribution of values but rather the range given that I plot a random (non-representative) set of 1,000 parameterized models.[17] The solid black lines represent the benchmark shares of 17.6% and 13.1%, in exports to the U.S. and the rest of world, respectively,

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[17]Obtaining a uniform sample is a computationally hard problem. Specifically, the specialized inputs toy model can be mathematically defined as a system of linear inequalities describing a convex polytope. However, the dimensionality of the polytope prevents me from obtaining a uniform sample. To put this into perspective, obtaining the vertices of the convex polytope is a much simpler problem that is also very challenging. I applied the Lexicographic Reverse Search algorithm of Avis and Fukuda (1992) and found that in the simplest case with $|\mathcal{J}| = 2$ countries and a single sector the null space describing the polytope is of size $X \times X$ and had X vertices. Solving for a more dimensional problem with $|\mathcal{J}| = 4$ countries and a null space of dimension $X \times X$ delivered X vertices and took hours to compute. The null space of the $|\mathcal{J}| = 44$ example with two sectors is of dimension $X \times X$.
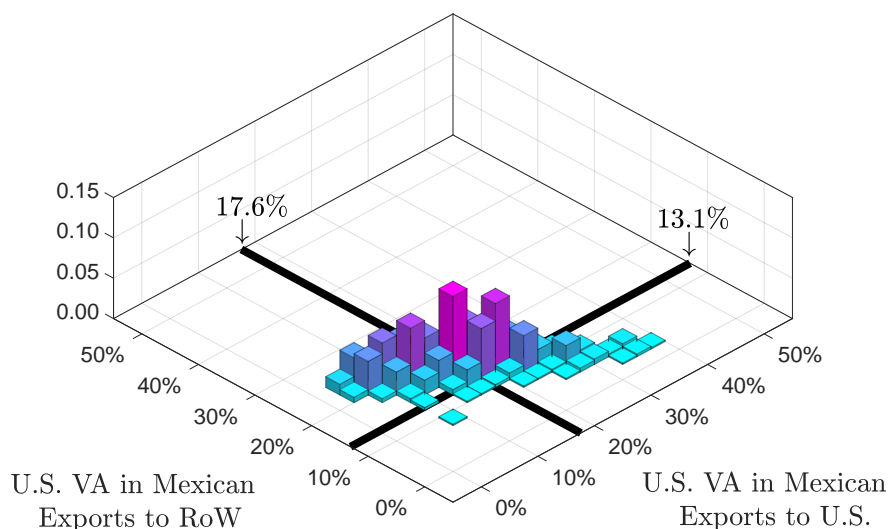
Figure 2.1: Share of U.S. Value-Added in Mexican Manufacturing Final Good Exports to the U.S. and to the Rest of World in 2014 Using WIOD Data: The histogram plots the joint distribution of these statistics across 1,000 simulations of the specialized inputs model using the WIOD data with two aggregate sectors (manufactures and agriculture+services). The solid black lines indicate the shares computed with the roundabout production model or directly with I-O analysis in the full database with 56 sectors.

as backed out by the roundabout model with equation (5) or directly with I-O analysis as in equation (7) using the fully disaggregated WIOD with 56 sectors per country.[18]

There is one key takeaway: The statistics for value-added trade vary substantially relative to the roundabout values. This is important because all of the simulations are observationally equivalent and thus any could represent the true data generating process underlying the observed WIOD data. Hence, the debate that international trade economists have been participating in over the last decade or so may have been led seriously astray by Leontief. To exemplify this, the statistic of U.S. value-added in Mexican exports to the U.S. has received a lot of recent attention given the ongoing renegotiation of NAFTA and has been used as a proxy of supply chain integration between the two countries. Figure 2.1 reveals the perils of basing policy on the roundabout model, which implies a share for U.S. value-added of 17.6%, since actually this statistic may be as low as 3% indicating a much lesser degree of integration or as high as 40% indicating a much deeper degree of integration.

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[18]Note that the share of U.S. value-added in Mexican exports is the same regardless of where these are sold to at the most disaggregate industrial level, but the share may differ at the aggregate manufacturing level because Mexico sells relatively more to the U.S. in those industries in which the share of U.S. value-added is highest. Alternatively, recomputing these shares on the aggregated two-sector WIOD delivers a common share of 14.7% to any location and the mismeasurement relative to 17.6% and 13.1% reflects the industry aggregation bias.

2.5 Moving Beyond Roundabout Production: Specialized Inputs

In a nutshell, the empirical GVC literature has mismeasured GVCs because it has taken roundabout models at face value and these have failed to fully internalize the decades-old critique of aggregation bias in I-O analysis. The latter can be explained as occurring in two flavors.

First, the classical critique is that input shares vary across sectors so that I-O analysis is misspecified if the data is not disaggregated across narrow industrial categories.[19] The roundabout production literature has internalized this critique and moved towards multi-sector models in which intermediate input shares across sectors, say vehicles and electronics, may vary. Nonetheless, I show in Appendix Section C that this issue may still be prevalent in practice by taking the domestic U.S. I-O tables at the 6-digit NAICS level (with 237 manufacturing codes) and assuming that in reality we only have access to the data at the aggregate 3-digit level (with 19 manufacturing codes). I show that within each 3-digit category there is substantial variation in the input shares across the more disaggregate 6-digit sectors and this implies that one can at the very least claim that I-O analysis is misspecified at the 3-digit level. This exercise is relevant since the WIOD has only 20 manufacturing sectors.

Second, the roundabout production literature has not internalized the aggregation bias ocurring even within narrowly defined industrial categories and driven by the rise of specialized inputs linkages. The previous toy models were intended to convince the reader that this bias is important both in theory and practice and that new tools need to be developed in order to estimate GVCs that can take into account the heterogeneity present in firm-level data such as in Figure 1.1. Doing this requires heavier machinery for the simple reason that bilateral trade data does not uniquely characterize GVCs in specialized inputs models.

The next two sections present a toolbox for estimating GVCs when both biases are present. Specifically, one can think about the industry aggregation bias as moving from one-sector models as in Eaton and Kortum (2002) to multiple sectors as in Caliendo and Parro (2015). For GVC estimation purposes this implies using a more disaggregate WIOT with multiple sectors per country and applying I-O analysis as the estimation framework. Meanwhile, the specialized inputs aggregation bias requires moving beyond roundabout production as in Eaton and Kortum (2002) to a world with specialized input linkages as in Antràs and de Gortari (2017). In terms of GVC estimation this implies developing a new measurement framework that goes beyond I-O analysis and that can estimate GVCs within a class of specialized inputs models.

3 Measurement Framework: Specialized Inputs

Trade economists have long focused on bilateral trade flows as the basic units of analysis and this view is sufficient for studying GVCs in a roundabout production world. However, these tools are inadequate in the presence of specialized inputs linkages and the literature's focus on the former has prevented the development of a unified framework that can be reconciled

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[19]See Leontief (1949), Hatanaka (1952), McManus (1956a), and McManus (1956b).

with richer theories of intermediate input trade. This section resolves this issue.

3.1 GVC Definition

I introduce new notation that centers attention on GVCs as the central objects of interest and which resolves the limitations of using bilateral trade variables as the basic building blocks. I define $\mathcal{G}\left(\cdot\right)$ as the key GVC object denoting the dollar value flow through a specific ordered set of country-sectors all the way to final consumption. More specifically, in a single sector world let $\mathcal{J}$ be the set of countries so that for $j, j', j'' \in \mathcal{J}$ the object $\mathcal{G}\left(j', j\right)$ denotes the dollar value that j' sells to j, and which the latter uses for final consumption, while $\mathcal{G}\left(j'', j', j\right)$ is the dollar value that j'' sells to j' which j' uses as inputs for goods then sold as final consumption to j. In general, a GVC may be specified through an arbitrary number of nodes so that I will add a superscript N indicating the dimension of $\mathcal{G}^N\left(\cdot\right)$; i.e. N is the number of nodes previous to final consumption that are specified, and throughout I will use j^n to denote the nth node from final consumption. That is, instead of $\mathcal{G}\left(j', j\right)$ I will write $\mathcal{G}^1\left(j^1, j^0\right)$ and instead of $\mathcal{G}\left(j'', j', j\right)$ I will write $\mathcal{G}^2\left(j^2, j^1, j^0\right)$. In a single-sector world $j^n \in \mathcal{J} \; \forall n$ and the n is only meant to indicate the dimension for which country j^n is relevant.

The extension to a multi-sector world is immediate. Let $\mathcal{K}$ be the set of sectors and $\mathcal{S} = \mathcal{J} \times \mathcal{K}$ be the set of country-sectors. GVCs can be defined in the most general way as follows.

Definition 3.1. For any length $N \in \mathbb{Z}^+$, $\mathcal{G}^N : \mathcal{S}^N \times \mathcal{J} \to \mathbb{R}^+$ is the function describing truncated GVC flows leading to final consumption in countries in $\mathcal{J}$ through a sequence of N upstream stages of production given by an element of $\mathcal{S}^N$.

A generic GVC is then $\mathcal{G}^N\left(s^N, \ldots, s^1, j\right)$ and I will refer to the elements of a country-sector pair as $s^n = \{j^n, k^n\}$ with j^n the country and k^n the sector of s^n. As before $s^n \in \mathcal{S} \; \forall n$ and the n is only meant to indicate the dimension of $\mathcal{G}^N\left(\cdot\right)$ for which s^n is relevant. Examples are useful for fixing ideas: a flow of length $N = 1$ could be $\mathcal{G}^1\left(s^1, j\right) = \mathcal{G}^1\left(\{\text{Mexico,cars}\}, \text{U.S.}\right)$, the sales of Mexican cars to U.S. consumers, while a flow of length $N = 2$ could be $\mathcal{G}^2\left(s^2, s^1, j\right) = \mathcal{G}^2\left(\{\text{U.S.,car parts}\}, \{\text{Mexico,cars}\}, \text{U.S.}\right)$, the sales of U.S. car parts in the form of intermediate inputs that are used exclusively by the Mexican car industry to produce final goods sold to U.S. consumers. Analogously for any $N \in \mathbb{Z}^+$ and any sequence of production in $\mathcal{S}^N$ that produces a final good eventually sold to consumers in some country in $\mathcal{J}$.

I now explain why the word *truncated* appears in Definition 3.1.

Assumption 3.2. *Let* $\beta : \mathcal{S} \to (0, 1)$ *be the value-added share such that for every dollar produced in* $s \in \mathcal{S}$ *a share* $1 - \beta\left(s\right)$ *is spent on upstream inputs.*

Since $\beta\left(s\right) < 1$, all production processes necessarily require intermediate inputs and thus GVCs are of infinite length. The object $\mathcal{G}^N\left(\cdot\right)$ is a truncated GVC because it only specifies the flow through N nodes of production even though its most upstream node, s^N, also uses inputs

and the full GVC is characterized by an infinite number of nodes of production. A natural accounting relation that should hold in the previous example is that

$$\sum_{t \in \mathcal{S}} \mathcal{G}^2\left(t, \{\text{Mexico,cars}\}, \text{U.S.}\right) = \left(1 - \beta\left(\{\text{Mexico,cars}\}\right)\right)\mathcal{G}^1\left(\{\text{Mexico,cars}\}, \text{U.S.}\right).$$

On the right-hand side, $\mathcal{G}^1\left(\cdot\right)$ indicates the dollar value of Mexican cars sold to the U.S. and $\left(1 - \beta\left(\cdot\right)\right)$ imputes the value of its aggregate intermediate input requirements by removing the value added at this node. Meanwhile, $\mathcal{G}^2\left(\{\text{U.S.,car parts}\}, \{\text{Mexico,cars}\}, \text{U.S.}\right)$ is only one of many possible input suppliers to the right-hand side so that the aggregation across all possible input sources $t \in \mathcal{S}$ yields aggregate input sales to the downstream sequence on the right-hand side. More generally, the following accounting relation need always hold

$$\sum_{t \in \mathcal{S}} \mathcal{G}^{N+1}\left(t, s^N, \ldots, s^1, j\right) = \left(1 - \beta\left(s^N\right)\right)\mathcal{G}^N\left(s^N, \ldots, s^1, j\right). \tag{10}$$

That is, the aggregate intermediate input purchases of any sequence $s^N \to \cdots \to s^1 \to j$ of any length $N \in \mathbb{Z}^+$, as denoted on the right-hand side, must equal the aggregate intermediate input sales from all upstream suppliers in $t \in \mathcal{S}$ as denoted on the left-hand side.

3.2 Relation to Observable Data

Needless to say, $\mathcal{G}^N\left(\cdot\right)$ is unobserved. Before discussing how $\mathcal{G}^N\left(\cdot\right)$ can be estimated I show how these variables map into the data we do observe. Throughout this paper I will assume that the data is available in WIOT format such that $X\left(t, s\right)$ equals aggregate bilateral intermediate input sales from t to s while $F\left(t, j\right)$ denotes aggregate final good sales from t to j, with $t, s \in \mathcal{S}$ and $j \in \mathcal{J}$. Final good flows are simply equal to the simplest GVC object

$$F\left(t, j\right) \equiv \mathcal{G}^1\left(t, j\right). \tag{11}$$

In sharp contrast, the full richness of GVC flows at more upstream stages of production are entirely compacted into bilateral intermediate input flows. The latter are given by

$$X\left(t, s\right) \equiv \sum_{N=2}^{\infty} \sum_{l^{N-2} \in \mathcal{S}} \cdots \sum_{l^1 \in \mathcal{S}} \sum_{j \in \mathcal{J}} \mathcal{G}^N\left(t, s, l^{N-2}, \ldots, l^1, j\right). \tag{12}$$

Sales between t and s occur between any stages $N > 1$ and $N-1$, and conditional on N there exist $\left|\mathcal{S}^{N-2} \times \mathcal{J}\right|$ possible downstream uses of these exchanges. Aggregate bilateral intermediate input flows equal the sum across all stages of production and all downstream uses.

Note that equations (11) and (12) describe the observable data in terms of the primitives $\mathcal{G}^N\left(\cdot\right)$. Contrary to the current empirical GVC literature which takes WIOT data as the core building blocks, I take the stand that the observable data is only a partial reflection of the true

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primitives of interest and that they only mirror a limited amount of information. Hence, I will argue that in order to do empirical GVC work one must take a stand on how to reverse-engineer the objects of interest $\mathcal{G}^N(\cdot)$ from the observable data. Before I describe this issue in further detail, note that equations (10), (11), and (12) readily imply that

$$\beta(s) = \frac{Y(s) - \sum_{t \in \mathcal{S}} X(t,s)}{Y(s)}, \quad \text{with } Y(s) = \sum_{t \in \mathcal{S}} X(s,t) + \sum_{j \in \mathcal{J}} F(s,j). \tag{13}$$

In other words, $\beta(s)$ is the value-added to gross-output ratio of s.

3.3 The Fundamental GVC Estimation Problem

The empirical GVC literature's key challenge is that we do not know how to deconstruct truncated GVCs into their upstream input purchases. That is, the following mapping is unknown

$$\mathcal{G}^N\left(s^N, \ldots, s^1, j\right) \xrightarrow{\;?\;} \mathcal{G}^{N+1}\left(t, s^N, \ldots, s^1, j\right). \tag{14}$$

Equation (10) imposes an aggregate flow constraint that makes sure that aggregate input sales to sequence $s^N \to \cdots \to s^1 \to j$ equal this sequence's aggregate input purchases. However, the input purchases from each specific supplier $t \in \mathcal{S}$ is unknown.

Most theories of intermediate input trade can be recast as providing a solution to the mapping in (14). In general, the resolution of the mapping is partially informed by the fact that aggregate bilateral intermediate input sales are observed but this is hardly sufficient. The main motivation of this paper is that up to now the literature has assumed that these flows *are* sufficient by invoking the tools of I-O analysis as I show below. In a nutshell, this paper is about trying to solve for the mapping in (14) in settings in which I-O analysis does not hold or, more colloquially, about disentangling GVCs.

Finally, note that determining this mapping is a high-dimensional problem. In the most general case the mapping between $\mathcal{G}^N(\cdot)$ and its upstream suppliers $\mathcal{G}^{N+1}(\cdot)$ depends on the whole sequence of $\mathcal{G}^N(\cdot)$. Since $\mathcal{G}^N(\cdot)$ can vary across up to $\left|\mathcal{S}^N \times \mathcal{J}\right|$ sequences and input shares need be determined across all $t \in \mathcal{S}$ this implies that the full mapping between $\mathcal{G}^N(\cdot)$ and $\mathcal{G}^{N+1}(\cdot)$ consists of up to $(|\mathcal{S}| - 1) \times \left|\mathcal{S}^N \times \mathcal{J}\right|$ input shares. Splitting GVCs across a further upstream input stage increases the number of input shares by a factor $|\mathcal{S}|$ so that the input shares to be determined increases exponentially with N and is thus impossible to solve unless further structure is imposed. This issue is even more salient when noting that Assumption 3.2 implies that GVCs can be decomposed for any N and thus need be determined when $N \to \infty$.

3.4 I-O Analysis: The Roundabout Solution

The theories of intermediate input trade featuring roundabout production are the one case in which bilateral trade data is sufficient for resolving the mapping in (14). More generally,

Leontief (1941) provided the key tool for constructing GVCs in this context by assuming a production process requiring a fixed proportion of inputs for every unit of output produced. Specifically, define $a\,(t|s)$ as the *technical coefficient* determining the expenditure on inputs from t of every dollar of s production. As before, the notation is defined so that it is explicit that input shares are conditional on the purchasing industry s. Output can be decomposed into intermediate input expenditures and value-added so that for all s

$$\sum_{t\in\mathcal{S}} a\,(t|s) + \beta\,(s) = 1.$$

These coefficients can be iterated so that $a\,(s''|s')\,a\,(s'|s)$ is the dollar expenditure on inputs from from s'' used by s production and purchased indirectly through s'. These relations can be extended to any number of upstream stages of production and fully characterize production processes (in equilibrium).

The empirical GVC literature has taken this insight at face value and assumed that WIOTs are defined such that every dollar of production requires the same input shares regardless of the stage of production and where output is sold to.[20] In particular, this theory imposes a simple solution on the mapping in (14).

Assumption 3.3. *Input-Output analysis assumes that for all* $N > 1$

$$\mathcal{G}^{N+1}\left(t, s^N, \ldots, s^1, j\right) = a\left(t|s^N\right) \mathcal{G}^N\left(s^N, \ldots, s^1, j\right). \tag{15}$$

This solves the dimensionality problem since GVCs are constructed recursively and require only final good flows $\mathcal{G}^1\left(s^1, j\right)$ and technical coefficients $a\,(t|s)$, which are all directly observed.

Note that I have still not defined precisely what the technical coefficients that solve the mapping in (14) are. The standard approach is to define them directly as an assumption. I now show that actually we need not do so, they are already implied by the previous assumptions.

Lemma 3.4. *The Input-Output analysis assumption in equation* (15) *implies that the aggregate input purchases from* t *of* s *equals*

$$X\,(t, s) = a\,(t|s)\,Y\,(s). \tag{16}$$

This follows from rearranging equation (12) and imposing the I-O analysis assumption in (15). The left-hand side equals aggregate input purchases from t by s while the right-hand side equals the product of the technical coefficient and gross output. Hence, I-O analysis requires

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[20]A key, but often overlooked, point is that Leontief (1941) defined I-O analysis in terms of quantities while modern GVC empirics implement it in terms of dollar-values. The technical coefficients that arise from each approach are only equivalent to the true technological technical coefficients if a fixed proportions production function is assumed when working with quantities while a Cobb-Douglas production function need be assumed when working with dollar-values (cf. Burress 1994). However, as Samuelson (1951) pointed out, at a given equilibrium I-O analysis is well defined as long as production features constant returns to scale. The stronger assumptions are needed in counterfactual analysis in order to keep the technical coefficients constant across equilibria.

$a(t|s)$ to be the term that relates aggregate output in s to its input purchases from t. This equation is crucial because it shows that bilateral trade data are sufficient for characterizing GVCs in the presence of roundabout production.

Corollary 3.5. *The **Input-Output analysis technical coefficients** are given by*

$$a(t|s) = \frac{X(t,s)}{Y(s)} = \frac{X(t,s)}{\sum_{t'\in\mathcal{S}} X(t',s)} (1 - \beta(s)). \tag{17}$$

The technical coefficients follow from rearranging (16) together with the definitions in (13).

Remember that Fact 2.1 claimed that roundabout structural models deliver the same GVC statistics as I-O analysis. This occurs because these models impose a mapping of output to intermediate inputs as in Assumption 3.3 and thus GVCs can be characterized with a Leontief matrix with elements as in (17). For example, look at the similarities with the toy roundabout model in (2) and (16), and in (6) and (17). Furthermore, remember that Fact 2.2 claimed that I-O analysis has no degrees of freedom. This is clear from (17) since the technical coefficients are fully characterized by the observable data.

Unfortunately, global trade is too complex to be studied solely through the lens of bilateral trade flows. Specifically, Figure 1.1 showed that Assumption 3.3 fails when using Mexican microdata while Section 2 described the perils of overlooking these issues in a pair of toy models. The rest of the paper focuses on solutions to the mapping in (14) that go beyond I-O analysis.

3.5 The Specialized Inputs Measurement Framework

I now generalize I-O analysis so that it can incorporate specialized inputs forces by imposing a similar but more relaxed version of Assumption 3.3.

Assumption 3.6. *Let $\mathcal{M} \in \mathbb{Z}^+$. The **specialized inputs measurement framework** with $\mathcal{M}-$proportional input shares assumes that for all $N > \mathcal{M}$*

$$\mathcal{G}^{N+1}\left(t, s^N, \dots, s^1, j\right) = a^{\mathcal{M}}\left(t \middle| s^N, \dots, s^{N-(\mathcal{M}-1)}\right) \mathcal{G}^N\left(s^N, \dots, s^1, j\right). \tag{18}$$

The technical coefficients are now conditional on the immediate $\mathcal{M}$ downstream stages of production. Whenever a GVC $\mathcal{G}^N(\cdot)$ is to be split into its direct upstream input purchases, the share flowing from a specific supplier $t \in \mathcal{S}$ may differ depending on the immediate sequence of length $\mathcal{M}$ through which these inputs will be further embedded into more downstream intermediate inputs. Note that $\mathcal{M} = 1$ corresponds to standard I-O analysis, as in (15), and that $\mathcal{M} > 1$ provides a more flexible framework since the technical coefficients have more degrees of freedom. An useful way of thinking about these generalized technical coefficients is that they represent an $\mathcal{M}-$th order Markov chain. I-O analysis is the simplest case in which input shares depend solely on the purchasing country-industry while $\mathcal{M} > 1$ implies that these input shares depend on the subsequent $\mathcal{M}$ nodes of production (see Solow 1952).

There are at least two reasons why Assumption 3.6 is desireable. First, these input shares can be made consistent with specialized inputs forces such as the input shares observed in the firm-level data in Figure 1.1, while the special case of I-O analysis cannot. That is, when $\mathcal{M} > 1$ input shares depend not only on the direct purchaser of these inputs but also on where this industry itself sells its own output to. Second, in Section 5 I develop a full-blown structural model with specialized inputs linkages that features input shares of this nature and justifies this approach. Thus, Assumption 3.6 can be rationalized by a class of models that feature rich intermediate input sourcing strategies at odds with I-O analysis.

I now derive the general form that $\mathfrak{a}^{\mathcal{M}}(\cdot)$ takes. It is useful to define a generalization of the gross bilateral input flows in (12) to gross input flows through longer sequences. Specifically, let $s^{\mathcal{M}} \to \cdots \to s^1$ be a specific sequence of $\mathcal{M}$ country-industry pairs. The gross input flow between t and this sequence equals the sum of these exchanges across all upstream stages of production and to be used through all further downstream sequences

$$\mathcal{X}^{\mathcal{M}}\left(t, s^{\mathcal{M}}, \ldots, s^1\right) \equiv \sum_{N=\mathcal{M}+1}^{\infty} \sum_{l^{N-\mathcal{M}-1} \in \mathcal{S}} \cdots \sum_{l^1 \in \mathcal{S}} \sum_{j \in \mathcal{J}} \mathcal{G}^N\left(t, s^{\mathcal{M}}, \ldots, s^1, l^{N-\mathcal{M}-1}, \ldots, l^1, j\right). \quad (19)$$

Country-industry t sells inputs directly to $s^{\mathcal{M}}$ to be further used through $s^{\mathcal{M}-1} \to \cdots \to s^1$ at the upstream stage $N = \mathcal{M} + 1$, and these flows are consumed in all $j \in \mathcal{J}$. However, t also sells inputs to be used through this specific sequence at upstream stage $N = \mathcal{M} + 2$ and in this case the flow is consumed in all $j \in \mathcal{J}$ but after flowing through any $l^1 \in \mathcal{S}$ at the last stage of production. Thus, input flows occur at all $N > \mathcal{M}$ and $\mathcal{X}^{\mathcal{M}}(\cdot)$ represents the dollar value that t sells to sequence $s^{\mathcal{M}} \to \cdots \to s^1$ across all stages of production and all further downstream uses. This definition is a simple generalization of (12) since $\mathcal{M} = 1$ equals the case in which input use is conditioned only on the industry of purchase so that this variable equals bilateral intermediate input trade $\mathcal{X}^1(t, s^1) = X(t, s^1)$. An example with $\mathcal{M} = 2$ could be $\mathcal{X}^2(t, s^2, s^1) = \mathcal{X}^2(\{\text{China,steel}\}, \{\text{U.S.,car parts}\}, \{\text{Mexico,cars}\})$, the dollar sales of Chinese steel sold as intermediate inputs to the U.S. car part industry to be used exclusively in the production of intermediate inputs that are sold directly to the Mexican car industry.

The notion of $\mathcal{M}-$proportionality is that we can obtain sufficient statistics that fully characterize GVC flows as long as these statistics condition on a sequence of $\mathcal{M}$ stages of production. Hence the following generalization of Lemma 3.4 applies.

Lemma 3.7. *The specialized inputs measurement framework with $\mathcal{M}-$proportional input shares as in (18) implies that the aggregate input purchases from t of a sequence $s^{\mathcal{M}}, \ldots, s^1$ equals*

$$\mathcal{X}^{\mathcal{M}}\left(t, s^{\mathcal{M}}, \ldots, s^1\right) = \mathfrak{a}^{\mathcal{M}}\left(t \,\middle|\, s^{\mathcal{M}}, \ldots, s^1\right) \left[\sum_{l \in \mathcal{S}} \mathcal{X}^{\mathcal{M}}\left(s^{\mathcal{M}}, \ldots, s^1, l\right) + \sum_{j \in \mathcal{J}} \mathcal{G}^{\mathcal{M}}\left(s^{\mathcal{M}}, \ldots, s^1, j\right)\right]. \quad (20)$$

This follows from rearranging equation (19) and imposing the specialized inputs assump-

tion in (18). The left-hand side equals aggregate input purchases from t by the sequence $s^{\mathcal{M}} \to \cdots \to s^1$ while the term in square brackets equals aggregate output of $s^{\mathcal{M}}$ sold to $s^{\mathcal{M}-1} \to \cdots \to s^1$. In other words, sequence $s^{\mathcal{M}} \to \cdots \to s^1$ buys inputs, produces along this chain, and after s^1 output is used as further intermediate inputs and sold to all $l \in \mathcal{S}$ but also as final consumption and sold to all $j \in \mathcal{J}$. Hence, $a^{\mathcal{M}}\left(t \,\middle|\, s^{\mathcal{M}}, \ldots, s^1\right)$ has to be the term relating aggregate input purchases to aggregate input sales.

Corollary 3.8. *The **specialized inputs technical coefficients** are given by*

$$a^{\mathcal{M}}\left(t \,\middle|\, s^{\mathcal{M}}, \ldots, s^1\right) = \frac{\mathcal{X}^{\mathcal{M}}\left(t, s^{\mathcal{M}}, \ldots, s^1\right)}{\sum_{t' \in \mathcal{S}} \mathcal{X}^{\mathcal{M}}\left(t', s^{\mathcal{M}}, \ldots, s^1\right)} \left(1 - \beta\left(s^{\mathcal{M}}\right)\right). \tag{21}$$

The technical coefficients follow from rearranging (20) together with the definition of $\mathcal{X}^{\mathcal{M}}\left(\cdot\right)$ in (19) and the aggregate input flow constraint (10). The value of production flowing through $s^{\mathcal{M}} \to \cdots \to s^1$ is entirely attributed to its factors of production since[21]

$$\sum_{t \in \mathcal{S}} a^{\mathcal{M}}\left(t \,\middle|\, s^{\mathcal{M}}, \ldots, s^1\right) + \beta\left(s^{\mathcal{M}}\right) = 1.$$

In particular, the specialized inputs technical coefficients simply tell the share of inputs purchased from any source t to be used for production at $s^{\mathcal{M}}$ and to be embedded in goods sold through the downstream sequence $s^{\mathcal{M}-1} \to \cdots \to s^1$. Hence, just like GVCs based on I-O analysis are fully determined by bilateral trade data, GVCs based on specialized input analysis can be derived recursively conditional on knowledge of the $\mathcal{M}-$stage gross input flows $\mathcal{X}^{\mathcal{M}}\left(\cdot\right)$ and the baseline GVC $\mathcal{G}^{\mathcal{M}}\left(\cdot\right)$. In practice, it is easier to work directly with $\mathcal{X}^{\mathcal{M}}\left(\cdot\right)$ than with $a^{\mathcal{M}}\left(\cdot\right)$ since the former permits a deeper exploitation of the linearity embedded in specialized inputs analysis, though both approaches are equivalent because of Corollary 3.8.

3.5.1 Relation to Specialized Inputs Toy Model

To fix ideas, let me briefly relate this framework to the above specialized inputs toy model. To do this, first note that the model has a single sector so that $\mathcal{S} = \mathcal{J}$. Furthermore, input shares depend only on the immediate use of output and thus correspond to a world in which input shares can be characterized with primitives of length $\mathcal{M} = 2$ (but not $\mathcal{M} = 1$, since that would correspond to a roundabout world). The primitives are then

$$\mathcal{X}^2\left(j'', j', j\right) = \tilde{\pi}_{j''j',j} \sum_{i \in \mathcal{J}} \tilde{\pi}_{j'j,i} \hat{F}\left(j, i\right) + \tilde{\pi}_{j''j',j} \sum_{i' \in \mathcal{J}} \tilde{\pi}_{j'j,i'} \sum_{i \in \mathcal{J}} \tilde{\pi}_{ji',i} \hat{F}\left(i', i\right) + \ldots,$$
$$\mathcal{G}^2\left(j'', j', j\right) = \tilde{\pi}_{j''j',j} \alpha_{j'j} w_j. \tag{22}$$

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[21]A natural question is whether it is appropriate to keep value-added shares constant. That is, why not define these as $\beta\left(s^{\mathcal{M}}, \ldots, s^1\right)$? This is indeed an interesting question, but I leave it open to future research since this adds another layer of complexity in that these shares need also be estimated. See Nomaler and Verspagen (2014).

The overall flow of intermediate input sales from j'' to j' to be embedded in intermediates sold to j can be computed by summing up these flows across all upstream stages of production and for all downstream uses, while the overall flow of intermediate input sales from j'' to j' to be embedded in final good sales to j is simply the proportion of the latter flow that uses inputs from j''. The specialized inputs technical coefficients are given by equation (21) and equal

$$a^2\left(j''|j',j\right) = \tilde{\pi}_{j''j',j}. \tag{23}$$

I-O analysis is only correct in the roundabout model since $a^2\left(j''|j',j\right) = a^1\left(j''|j'\right) = \tilde{\pi}_{j''j'}$.

3.5.2 GVC Statistics with Specialized Inputs

At the end of the day, the GVC primitives $\mathcal{X}^{\mathcal{M}}\left(\cdot\right)$ and $\mathcal{G}^{\mathcal{M}}\left(\cdot\right)$ are interesting in their own right but, more importantly, are a means to the end of computing GVC related statistics in a world of specialized inputs. Importantly, all empirical GVC measures based on I-O analysis can be extended to this framework and all that is missing is the mapping from these new objects into the old statistics. I illustrate this by deriving the decomposition of final good exports into value-added by source. Specifically, this decomposition can be defined in general, regardless of the theory of intermediate trade we impose, as follows.

Definition 3.9. The value-added from $t \in \mathcal{S}$ embedded in the final good exports of $s \in \mathcal{S}$ to consumers in country j equals

$$VA_t\left(s,j\right) = 1_{[t=s]}\beta\left(s\right)F\left(s,j\right) + \beta\left(t\right)\sum_{N=2}^{\infty}\sum_{l^{N-1}\in\mathcal{S}}\cdots\sum_{l^2\in\mathcal{S}}\mathcal{G}^N\left(t,l^{N-1},\ldots,l^2,s,j\right). \tag{24}$$

The first term is the value that t embedds directly into final goods and appears only when $t = s$, the second term includes the value produced in t at any upstream stage of production that is eventually used by s to produce final goods sold to consumers in j. This decomposition is useful for computing the dollar value of U.S. value-added that makes its way back to U.S. consumers through Mexican final good exports, a key measure of integration within the NAFTA region. That is, $\sum_{t\in\text{USA}\times\mathcal{K}}\sum_{s\in\text{MEX}\times\mathcal{K}}VA_t\left(s,\text{USA}\right)$ is the total value produced in all U.S. industries and exported in Mexican final goods of all industries to U.S. consumers.

If we are willing to assume that the specialized inputs assumption in equation (18) is an accurate theory of intermediate input trade, then this decomposition can be written with linear algebra in a similar way to the Leontief inverse in I-O analysis. Throughout I will stack up individual variables into vectors and the ordering is always done first along the first dimension, then along the second, so on and so forth (see Appendix Section B.1 for details). To simplify notation, I will refer to $\mathcal{S}$ and $\mathcal{J}$ as both the sets and the total number of elements contained therein. Let $\mathcal{G}\left(s,j\right) = \left[\mathcal{G}^{\mathcal{M}}\left(t^{\mathcal{M}},\ldots,t^2,s,j\right)\right]$ be a vector of size $\mathcal{S}^{\mathcal{M}-1}\times 1$ of the elements leading to final production in s that is exported to j, and let the overall matrix $\mathcal{G} = [\mathcal{G}\left(s,j\right)]$ of size

$\mathbb{S}^{\mathcal{M}-1} \times \mathbb{S}\mathcal{J}$ be the column-wise stacked up version. Likewise, let $\mathbf{a}^{\mathcal{M}}(t) = \left[a^{\mathcal{M}}\left(t \,\middle|\, s^{\mathcal{M}}, \ldots, s^1 \right) \right]$ be the vector of size $1 \times \mathbb{S}^{\mathcal{M}}$ of technical coefficients for t inputs, as defined in equation (21). I can now generalize the Leontief matrix by defining $\mathbf{a}^{\mathcal{M}} = \left[\mathbf{a}^{\mathcal{M}}(t) \right]$ as the matrix of size $\mathbb{S} \times \mathbb{S}^{\mathcal{M}}$ of technical coefficients such that the generalized Leontief matrix of size $\mathbb{S}^{\mathcal{M}} \times \mathbb{S}^{\mathcal{M}}$ is

$$\mathbf{A}^{\mathcal{M}} = \mathbf{a}^{\mathcal{M}} * \left(\mathbb{I}_{\mathbb{S}^{\mathcal{M}-1} \times \mathbb{S}^{\mathcal{M}-1}} \otimes \mathbf{1}_{1 \times \mathbb{S}} \right), \tag{25}$$

where $\otimes$ is the Kronecker product and $*$ is the column-wise Kronecker product (i.e. Khatri-Rao product). Finally, let $\boldsymbol{\beta} = [\beta(s)]$ be the vector of size $1 \times \mathbb{S}$ of value-added shares, define the vector of gross-output to intermediate input purchases as $\tilde{\boldsymbol{\beta}} = [1/(1 - \beta(s))]$, and let $\tilde{\boldsymbol{\beta}}^{\otimes n}$ denote the $n-$fold Kronecker product of $\tilde{\boldsymbol{\beta}}$ with itself.

Lemma 3.10. *The specialized inputs measurement framework implies that value-added in final good exports as in equation (24) can be decomposed as*

$$\mathbf{VA} = \mathrm{diag}\{\boldsymbol{\beta}\} \left[\begin{array}{c} \sum_{N=1}^{\mathcal{M}-1} \left(\mathbf{1}_{1 \times \mathbb{S}} \otimes \tilde{\boldsymbol{\beta}}^{\otimes(\mathcal{M}-N-1)} \right) \otimes \left(\mathrm{diag}\{\tilde{\boldsymbol{\beta}}\} \otimes \mathbf{1}_{1 \times \mathbb{S}^{N-1}} \right) \\ + \left(\mathbb{I}_{\mathbb{S} \times \mathbb{S}} \otimes \mathbf{1}_{1 \times \mathbb{S}^{\mathcal{M}-1}} \right) + \mathbf{a}^{\mathcal{M}} \left(\mathbb{I}_{\mathbb{S}^{\mathcal{M}} \times \mathbb{S}^{\mathcal{M}}} - \mathbf{A}^{\mathcal{M}} \right)^{-1} \end{array} \right] \left[\mathcal{G} * \left(\mathbb{I}_{\mathbb{S} \times \mathbb{S}} \otimes \mathbf{1}_{1 \times \mathcal{J}} \right) \right]. \tag{26}$$

In the special case in which I-O analysis holds, either $\mathcal{M} = 1$ or $\mathcal{M} > 1$ with primitives as in equation (35), this reduces to

$$\mathbf{VA} = \mathrm{diag}\{\boldsymbol{\beta}\} \left[\mathbb{I}_{\mathbb{S} \times \mathbb{S}} - \mathbf{A} \right]^{-1} \left[\mathbf{F} * \left(\mathbb{I}_{\mathbb{S} \times \mathbb{S}} \otimes \mathbf{1}_{1 \times \mathcal{J}} \right) \right]. \tag{27}$$

Thus $\mathbf{VA} = [VA_t(s, j)]$ is the matrix of size $\mathbb{S} \times \mathbb{S}\mathcal{J}$ with row elements indexing the source dimension t and columns indexing final good exports from s to market j. The terms in the big parenthesis in equation (26) depend now on three terms instead of the Leontief inverse matrix that arises in I-O analysis. The first two terms compute value-added directly observed through $\mathcal{G}^{\mathcal{M}}(\cdot)$ with the first tracing the value embedded in stages $N = 1, \ldots, \mathcal{M} - 1$ and the second tracing the value produced at stage $N = \mathcal{M}$. Meanwhile, the last term computes value-added at all stages $N > \mathcal{M}$ using a similar insight to the Leontief inverse matrix. That is, value-added at these upstream stages is computed with the recursion in equation (18) and is entirely summarized by the specialized inputs technical coefficients in $\mathbf{A}^{\mathcal{M}}$. Indeed I will call $\left(\mathbb{I}_{\mathbb{S}^{\mathcal{M}} \times \mathbb{S}^{\mathcal{M}}} - \mathbf{A}^{\mathcal{M}} \right)^{-1}$ the generalized Leontief inverse matrix since it summarizes all information contained in $\mathbf{a}^{\mathcal{M}}(\cdot)$. The intuition is that it is of size $\mathbb{S}^{\mathcal{M}} \times \mathbb{S}^{\mathcal{M}}$ since it embeds the knowledge of constant input shares across any sequence of $\mathcal{M}$ production stages.[22]

This decomposition relates to the existing literature in three ways. First, empirical GVC analyses have exclusively focused on the special case of I-O analysis which is nested within this framework. Specifically, when I-O analysis holds the decomposition (27) yields the famil-

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[22]The invertibility of this matrix can be shown with arguments similar to those in Hawkins and Simon (1949). In the words of Solow (1952), the necessary condition is that no group of industries be "self-exhausting".

iar value-added formula as defined in Johnson and Noguera (2012). Second, in Section 3.4 I claimed that the use of I-O analysis avoids the need of explicitly using GVC notation. This is revealed in (27) since it depends solely on bilateral trade data while the broader notions of proportionality in (26) require the use of $a^{\mathcal{M}}(\cdot)$ and $\mathcal{G}^{\mathcal{M}}(\cdot)$, and which cannot be written solely in terms of WIOT data unless I-O analysis is assumed. Third, though I have focused on the decomposition for final good exports it should be stressed that all GVC statistics that rely on the Leontief inverse can be generalized similarly. Though the derivation of these formulas is tedious and the notation cumbersome, they are easy to compute in standard computers.[23]

3.5.3   Implementing the Specialized Inputs Measurement Framework

More generally, the key challenge for implementing this framework is that its primitives, $\mathcal{X}^{\mathcal{M}}(\cdot)$ and $\mathcal{G}^{\mathcal{M}}(\cdot)$, are not observed and have to be estimated. Though more flexible than I-O analysis, it has the downside that it cannot be fully characterized by bilateral trade data since the latter are an aggregation of these richer primitives and a lot of information is lost through this aggregation. In consequence, there are many possible sets of primitives that are consistent with the same observable data and it is not obvious how to recuperate the true ones.

The standard approach would be to use a structural model, such as the toy specialized inputs model, but in which the input shares $\pi_{j''j',j}$ are microfounded. Indeed I show how one can do this in Section 5, but there are two big drawbacks. First, a full-blown model depends on a large number of parameters that may be very hard to discipline in practice. Second, even if this were possible it is likely that the parameterized model does not fit the data perfectly and thus whatever GVC estimates it delivers will be subject to the lack-of-fit error.

Instead, I argue in Section 4 that there is a more natural route in which this framework can be implemented directly if we have at least some knowledge of the supply chain data underlying bilateral trade flows. Before delving into these issues, I discuss the benefits and costs of modeling proportionality more flexibly (i.e. the number $\mathcal{M}$ takes).

3.5.4   How General Should the Measurement Framework Be?

In principle, as general as possible. That is, the space of GVCs that can be estimated from bilateral trade data will always be bigger the higher $\mathcal{M}$ is. Conditioning input shares on longer sequences of input use is desirable since more complex supply chains can be accommodated. To see this let $\mathcal{M}, \mathcal{M}' \in \mathbb{Z}^{+}$ with $\mathcal{M} > \mathcal{M}'$ and suppose the true GVCs involve technical coefficients of length $\mathcal{M}'$. Working with $\mathcal{M}$ is not restrictive since technical coefficients can be defined as

$$a^{\mathcal{M}}\left(t \,\middle|\, s^{\mathcal{M}}, \ldots, s^{1}\right) = a^{\mathcal{M}'}\left(t \,\middle|\, s^{\mathcal{M}}, \ldots, s^{\mathcal{M}-(\mathcal{M}'-1)}\right), \tag{28}$$

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[23]In particular, the value-added deficit as defined in Johnson and Noguera (2012) can be computed with this same formula. The value-added trade balance between, say, China and the U.S. is simply the difference between Chinese consumption of U.S. value-added and U.S. consumption of Chinese value-added: $\sum_{t \in US \times \mathcal{K}} \sum_{s \in \mathcal{J} \times \mathcal{K}} VA_t(s, China) - \sum_{t \in China \times \mathcal{K}} \sum_{s \in \mathcal{J} \times \mathcal{K}} VA_t(s, US)$.

and the true GVCs can be correctly backed out. Crucially, the converse is not true. If the true GVCs require technical coefficients with $\mathcal{M}$ but $\mathcal{M}' < \mathcal{M}$ is used then the GVCs that arise will be necessarily biased since $\mathcal{M}'$ is imposing too stringent assumptions. For example, when the roundabout model is true then both $\mathcal{M} = 1$ or $\mathcal{M} = 2$ can back out the true GVCs, but when the specialized inputs model is true the estimates can only be recovered with $\mathcal{M} = 2$.

The tradeoff is that generality comes at a cost in dimensionality. Working with $\mathcal{M} \in \mathbb{Z}^+$ requires knowledge of $\mathcal{X}^{\mathcal{M}}(\cdot)$ and $\mathcal{G}^{\mathcal{M}}(\cdot)$ and the estimation framework will thus be of dimensionality $|\mathcal{S}|^{\mathcal{M}}(|\mathcal{S}| + |\mathcal{J}|)$. I-O analysis is based on $\mathcal{M} = 1$ and the dimensionality $|\mathcal{S}|(|\mathcal{S}| + |\mathcal{J}|)$ corresponds to the size of bilateral trade flows, and that is where the sufficiency comes from, while $\mathcal{M} > 1$ requires more complex primitives and thus involves higher dimensionality.

In sum, roundabout production or I-O analysis exactly and uniquely correspond to $\mathcal{M} = 1$ but are also nested in the specialized inputs framework for all $\mathcal{M} \in \mathbb{Z}^+$. When $\mathcal{M} > 1$ the I-O technical coefficients may arise, but this is far from necessary. The Mexican microdata clearly rejects $\mathcal{M} = 1$ but is not rich enough to tell whether GVCs should be constructed with $\mathcal{M} = 2$ or an even higher $\mathcal{M} > 2$ is required. In practice, the dimensionality problem curtails the magnitude of $\mathcal{M}$ and so in the empirics of the paper I will simply use $\mathcal{M} = 2$.

4 Implementation: The QP Framework

I now propose a numerical framework for obtaining estimates of the primitives $\mathcal{X}^{\mathcal{M}}(\cdot)$ and $\mathcal{G}^{\mathcal{M}}(\cdot)$. Conceptually, this exercise implements a class of structural specialized inputs models with primitives that are observationally equivalent in equilibrium and where the specific underlying microstructure is irrelevant. This is exactly analogous to the connection between I-O analysis and roundabout models, while there are many microfoundations for the latter the only thing that matters for GVC estimation is the numbers the trade shares take in equilibrium. The new challenge is that there are many primitives consistent with the same bilateral data.

I exploit the linearity embedded in the specialized inputs measurement framework in order to use the tools of quadratic programming to tackle the high-dimensionality associated with this estimation. In order to illustrate the computational burden, note that at the end of this section I implement this approach empirically with 24 countries, 17 sectors and the simplest generalization with $\mathcal{M} = 2$. In this case the primitives have 72 million degrees of freedom, given by $|\mathcal{S}|^{\mathcal{M}}(|\mathcal{S}| + |\mathcal{J}|)$, while there are only 176 thousand datapoints, given by $|\mathcal{S}|(|\mathcal{S}| + |\mathcal{J}|)$.

I eliminate the degrees of freedom in three steps: (i) I constrain the primitives to be consistent with the observable WIOT data, (ii) I constrain the primitives to represent an internally consistent system, and (iii) I eliminate the remaining degrees of freedom by allowing the researcher to incorporate additional information. Appendix Section A contains a graphical description of the numerical method and is an useful companion to the main text.

4.1 The Linear Constraints

The first step is to determine a pair of relations that ensure that the primitives are consistent with bilateral WIOT data. Since $\mathcal{X}^{\mathcal{M}}\left(t, s^{\mathcal{M}}, \ldots, s^1\right)$ is the aggregate input flow between t and sequence $s^{\mathcal{M}} \to \cdots \to s^1$, note that $\sum_{t \in \mathcal{S}} \mathcal{X}^{\mathcal{M}}\left(t, s^{\mathcal{M}}, \ldots, s^1\right)$ is the aggregate input purchases of this sequence. Furthermore multiplying this sum by $1/\left(1 - \beta\left(s^{\mathcal{M}}\right)\right)$, the dollar output per dollar of intermediate input purchases, imputes the value that is added at $s^{\mathcal{M}}$ and thus provides the aggregate input sales of $s^{\mathcal{M}}$ to sequence $s^{\mathcal{M}-1} \to \cdots \to s^1$. This logic can be repeated until obtaining the aggregate input sales from s^2 to s^1, which is an observable data point in WIOTs. Thus, $\mathcal{X}^{\mathcal{M}}\left(\cdot\right)$ is consistent with bilateral intermediate input trade if the following holds

$$X\left(t, s\right) = \sum_{l^{\mathcal{M}+1} \in \mathcal{S}} \cdots \sum_{l^3 \in \mathcal{S}} \frac{\mathcal{X}^{\mathcal{M}}\left(l^{\mathcal{M}+1}, \ldots, l^3, t, s\right)}{\prod_{m=3}^{\mathcal{M}}\left(1 - \beta\left(l^m\right)\right) \times \left(1 - \beta\left(t\right)\right)}. \tag{29}$$

Likewise, $\mathcal{G}^{\mathcal{M}}\left(\cdot\right)$ is consistent with bilateral final good trade if the following holds

$$F\left(t, j\right) = \sum_{l^{\mathcal{M}+1} \in \mathcal{S}} \cdots \sum_{l^3 \in \mathcal{S}} \frac{\mathcal{G}^{\mathcal{M}}\left(l^{\mathcal{M}+1}, \ldots, l^3, t, j\right)}{\prod_{m=3}^{\mathcal{M}}\left(1 - \beta\left(l^m\right)\right) \times \left(1 - \beta\left(t\right)\right)}. \tag{30}$$

The aggregate constraint on $\mathcal{X}^{\mathcal{M}}\left(\cdot\right)$ in (29) is a direct implication of the definitions (12) and (19) and the mapping in (10) while the aggregate constraint on $\mathcal{G}^{\mathcal{M}}\left(\cdot\right)$ in (30) is a direct implication of the definition (11) and mapping in (10). Equations (11) and (12) defined WIOT data as functions of the underlying (unobserved) GVCs $\mathcal{G}^{N}\left(\cdot\right)$, but the observable data contains the former and not the latter. Hence, the idea is to make assumptions over $\mathcal{G}^{N}\left(\cdot\right)$, such as the $\mathcal{M}-$proportional input shares in (18), and reverse-engineer the primitives. These constraints ensure that the reverse-engineered primitives are consistent with the observed WIOT data.[24]

The second step is to ensure that the primitives determine an internally consistent system. Summing across $t \in \mathcal{S}$ in (20) and substituting $1 - \beta\left(s^{\mathcal{M}}\right) = \sum_{t \in \mathcal{S}} a^{\mathcal{M}}\left(t \mid s^{\mathcal{M}}, \ldots, s^1\right)$ delivers the following relation that forces the inflows and outflows of sequence $s^{\mathcal{M}} \to \cdots \to s^1$ to match

$$\frac{1}{1 - \beta\left(s^{\mathcal{M}}\right)} \sum_{t \in \mathcal{S}} \mathcal{X}^{\mathcal{M}}\left(t, s^{\mathcal{M}}, \ldots, s^1\right) = \sum_{l \in \mathcal{S}} \mathcal{X}^{\mathcal{M}}\left(s^{\mathcal{M}}, \ldots, s^1, l\right) + \sum_{j \in \mathcal{J}} \mathcal{G}^{\mathcal{M}}\left(s^{\mathcal{M}}, \ldots, s^1, j\right). \tag{31}$$

The sum on the left-hand side is aggregate input purchases of sequence $s^{\mathcal{M}} \to \cdots \to s^1$ with the scalar $1/\left(1 - \beta\left(s^{\mathcal{M}}\right)\right)$ imputing the value of production at $s^{\mathcal{M}}$. Hence, the left-hand side is aggregate input purchases of sequence $s^{\mathcal{M}-1} \to \cdots \to s^1$ from $s^{\mathcal{M}}$. Meanwhile, the right-hand side equals aggregate input sales from $s^{\mathcal{M}}$ to sequence $s^{\mathcal{M}-1} \to \cdots \to s^1$ when summing across all its further downstream uses as intermediates or final goods. Thus (31) ensures that trade flows across sequences of length $\mathcal{M}$ are consistent in levels or, to put it more bluntly, that everything that comes in must come out.

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[24]To be fully clear, $\mathcal{M}$ is fixed throughout at some positive integer and the goal is to estimate the primitives $\mathcal{X}^{\mathcal{M}}\left(\cdot\right)$ and $\mathcal{G}^{\mathcal{M}}\left(\cdot\right)$ so that the $\mathcal{M}-$proportional recursion in (18) can be used to build $\mathcal{G}^{N}\left(\cdot\right)$ for all $N \in \mathbb{Z}_{+}$.

To further provide intuition focus on the relation imposed in I-O analysis, i.e. when $\mathcal{M} = 1$

$$\frac{1}{1 - \beta\left(s^1\right)} \sum_{t \in \mathcal{S}} \mathsf{X}\left(t, s^1\right) = \sum_{l \in \mathcal{S}} \mathsf{X}\left(s^1, l\right) + \sum_{j \in \mathcal{J}} \mathsf{F}\left(s^1, j\right). \tag{32}$$

The left-hand side equals aggregate input purchases by s^1 with the scalar $1/\left(1 - \beta\left(s^1\right)\right)$ imputing the value of production of s^1. The right-hand side equals aggregate sales of s^1 and thus both sides equal aggregate output of s^1. This equation should be familiar since it is a key identity in national accounting. Indeed, rearranging this equation yields the definition of $\beta\left(s^1\right)$ in (13) which is simply the GDP to gross output ratio. That (31) is a generalization of (32) should not be surprising as they have been derived similarly but with the latter imposing $\mathcal{M} = 1$.

Though the whole point of this numerical framework is to avoid explicit structural modeling, it is important to note that these constraints not only make sense from a purely logical standpoint but also characterize a class of specialized inputs structural models. In order to fix ideas and relay more intuition, focus on the specialized inputs toy model where $\mathcal{S} = \mathcal{J}$, $\mathcal{M} = 2$, and with primitives as in equation (22). Substituting into the first set of constraints delivers

$$\sum_{l \in \mathcal{J}} \frac{\mathcal{X}^2\left(l, j', j\right)}{\left(1 - \beta\left(j'\right)\right)} = \hat{\mathsf{X}}\left(j', j\right), \quad \sum_{l \in \mathcal{J}} \frac{\mathcal{G}^2\left(l, j', j\right)}{\left(1 - \beta\left(j'\right)\right)} = \hat{\mathsf{F}}\left(j', j\right),$$

where the hat variables correspond to the simulated WIOT of the structural model. But since the toy model is parameterized to match the data, these primitives naturally satisfy the linear constraints in equations (29) and (30). Likewise, plugging in the structural primitives into each side of the internal consistency constraint in equation (31) delivers

$$\frac{1}{1 - \beta\left(j'\right)} \sum_{t \in \mathcal{J}} \mathcal{X}^2\left(t, j', j\right) = \hat{\mathsf{X}}\left(j', j\right), \quad \sum_{l \in \mathcal{J}} \mathcal{X}^2\left(j', j, l\right) + \sum_{l \in \mathcal{J}} \mathcal{G}^2\left(j', j, l\right) = \hat{\mathsf{X}}\left(j', j\right),$$

so that flows are indeed consistent along chains of length $\mathcal{M} = 2$.

GVCs in the specialized inputs measurement framework are thus fully characterized by $\mathcal{X}^{\mathcal{M}}\left(\cdot\right)$ and $\mathcal{G}^{\mathcal{M}}\left(\cdot\right)$ and equations (29), (30), and (31) ensure that they deliver the same bilateral trade flows as a specific WIOT database and that they represent an internally consistent system. The remaining issue is that many $\mathcal{X}^{\mathcal{M}}\left(\cdot\right)$ and $\mathcal{G}^{\mathcal{M}}\left(\cdot\right)$ satisfy these relations since these linear constraints only eliminate up to $|\mathcal{S}|\left(|\mathcal{S}| + |\mathcal{J}|\right) + |\mathcal{S}|^{\mathcal{M}}$ degrees of freedom.

4.2 Exploiting The Degrees of Freedom

As a third and final step, I propose a minimization problem that permits a researcher to discipline the estimates of $\mathcal{X}^{\mathcal{M}}\left(\cdot\right)$ and $\mathcal{G}^{\mathcal{M}}\left(\cdot\right)$ through her own priors or through other data sources while satisfying the above constraints. In particular, the advantage of the constraints on this system is that they are all linear. Let $\mathcal{X}$ and $\mathcal{G}$ be the stacked up vectors of $\mathcal{X}^{\mathcal{M}}\left(\cdot\right)$ and $\mathcal{G}^{\mathcal{M}}\left(\cdot\right)$

and define the quadratic programming (QP) approach to estimating GVC flows as

$$\min_{\{\mathcal{X},\mathcal{G}\}} \Xi(\mathcal{X},\mathcal{G}) = \frac{1}{2} \begin{bmatrix} \mathcal{X} - \mathbf{c}_{\mathcal{X}} \\ \mathcal{G} - \mathbf{c}_{\mathcal{G}} \end{bmatrix}^{\top} \mathbf{Q} \begin{bmatrix} \mathcal{X} - \mathbf{c}_{\mathcal{X}} \\ \mathcal{G} - \mathbf{c}_{\mathcal{G}} \end{bmatrix}$$

$$\text{s.t. } \mathcal{X} \text{ and } \mathcal{G} \text{ satisfy the bilateral data constraints (29) and (30)} \tag{33}$$
$$\mathcal{X} \text{ and } \mathcal{G} \text{ satisfy the consistency constraints (31)}$$
$$\mathcal{X}, \mathcal{G} \geqslant 0$$

Appendix section [B.1](#) shows how to write the constraints with linear algebra. This problem has a unique global minimum as long as $\mathbf{Q}$ is positive semi-definite.

The constraints on the QP framework depend solely on $\mathcal{X}$ and $\mathcal{G}$ and observable data, while the remaining degrees of freedom are eliminated in the objective function through the weighting matrix $\mathbf{Q}$, and the targets $\mathbf{c}_{\mathcal{X}}$ and $\mathbf{c}_{\mathcal{G}}$. Throughout this paper I will focus on the special case in which the weighting matrix has diagonal form so that the objective function of the QP problem (33) becomes a weighted sum of squared differences

$$\Xi(\mathcal{X},\mathcal{G}) = \frac{1}{2} \sum \mathbf{q}_{\mathcal{X}} \left(s^{\mathcal{M}}, \ldots, s^1, s^0 \right) \left(\mathcal{X}^{\mathcal{M}} \left(s^{\mathcal{M}}, \ldots, s^1, s^0 \right) - \mathbf{c}_{\mathcal{X}} \left(s^{\mathcal{M}}, \ldots, s^1, s^0 \right) \right)^2$$
$$+ \frac{1}{2} \sum \mathbf{q}_{\mathcal{G}} \left(s^{\mathcal{M}}, \ldots, s^1, j^0 \right) \left(\mathcal{G}^{\mathcal{M}} \left(s^{\mathcal{M}}, \ldots, s^1, j^0 \right) - \mathbf{c}_{\mathcal{G}} \left(s^{\mathcal{M}}, \ldots, s^1, j^0 \right) \right)^2 . \tag{34}$$

The intuition for this objective function is that $\mathbf{c}_{\mathcal{X}}$ and $\mathbf{c}_{\mathcal{G}}$ act as targets that shape the estimates and this is where a researcher can incorporate her priors over the underlying data generating process. The weights $\mathbf{q}_{\mathcal{X}}$ and $\mathbf{q}_{\mathcal{G}}$ rank the importance of matching each specific target. Defining this as an optimization problem is necessary in order to impose the linear constraints and is a standard tool for finding solutions in underdetermined linear systems. The special feature of this particular quadratic program is that its linear constraints implement a class of specialized inputs models. However, note that this is program is stronger in that it does not including bilateral data as moments to be fitted but rather imposes that they hold exactly. This is feasible in practice because of the flexibility of working with the primitives directly, whereas the microstructure is often too rigid in specific models for this to be possible. Finally, linearly constrained quadratic programming is exceptionally suitable for this high-dimensional problem because it has linear first order conditions whereas imposing a more sophisticated nonlinear objective function is infeasible in practice.

The following lemma proves useful.

Lemma 4.1. *Suppose $\mathcal{X}^*$ and $\mathcal{G}^*$ satisfy the linear constraints (29), (30), (31) and are non-negative. Let the terms in the QP objective function be $\mathbf{c}_{\mathcal{X}} = \mathcal{X}^*$ and $\mathbf{c}_{\mathcal{G}} = \mathcal{G}^*$. Then the solution to the QP framework for any positive semi-definite $\mathbf{Q}$ is $\mathcal{X}_{\text{QP}} = \mathcal{X}^*$ and $\mathcal{G}_{\text{QP}} = \mathcal{G}^*$.*

That is, the QP framework nests any solution that satisfies the constraints on (33). The proof of this result is trivial since $\Xi(\mathcal{X}^*,\mathcal{G}^*) = 0$ and the solution is feasible. This is an existence result

which assures us that the QP framework samples from the totality of the class of specialized inputs models consistent with a specific WIOT database.

A trivial application of Lemma 4.1 proves that I-O analysis is nested in the QP framework for any $\mathcal{M} \in \mathbb{Z}^+$. Specifically, note that the primitives associated with I-O analysis are

$$
\begin{aligned}
\mathcal{X}_{\text{I-O}}^{\mathcal{M}}\left(s^{\mathcal{M}}, \ldots, s^1, s^0\right) &= a\left(s^{\mathcal{M}} \mid s^{\mathcal{M}-1}\right) \cdots a\left(s^2 \mid s^1\right) X\left(s^1, s^0\right), \\
\mathcal{G}_{\text{I-O}}^{\mathcal{M}}\left(s^{\mathcal{M}}, \ldots, s^1, j^0\right) &= a\left(s^{\mathcal{M}} \mid s^{\mathcal{M}-1}\right) \cdots a\left(s^2 \mid s^1\right) F\left(s^1, j^0\right),
\end{aligned}
\tag{35}
$$

with $a\left(\cdot\right)$ the I-O technical coefficients in (17). To fix ideas, remember that the specialized inputs model with primitives in equation (22) nests the roundabout model. That is, $\mathcal{X}_{\text{I-O}}^2\left(j'', j', j\right) = \tilde{\pi}_{j''j} X\left(j', j\right)$ and $\mathcal{G}_{\text{I-O}}^2\left(j'', j', j\right) = \tilde{\pi}_{j''j} F\left(j', j\right)$, with $a\left(j'' \mid j'\right) = \tilde{\pi}_{j''j'}$ as shown in equation (6).

Corollary 4.2. *For any $\mathcal{M} \in \mathbb{Z}^+$, if $c_{\mathcal{X}} = \mathcal{X}_{\text{I-O}}$ and $c_{\mathcal{G}} = \mathcal{G}_{\text{I-O}}$, then $\mathcal{X}_{\text{QP}} = \mathcal{X}_{\text{I-O}}$ and $\mathcal{G}_{\text{QP}} = \mathcal{G}_{\text{I-O}}$.*

This result is the QP parallel of equation (28) since technical coefficients with $\mathcal{M}' < \mathcal{M}$ are also consistent with $\mathcal{M}$. The I-O GVCs are the unique solution when $\mathcal{M} = 1$ and this is the formal proof of Fact 2.2. In this case the QP framework simply estimates a roundabout model that matches bilateral trade data, abstracts away from the specific microstructure underlying the roundabout primitives, and backs out the unique GVCs that could arise in this class of models. When $\mathcal{M} > 1$ the exercise is analogous but within the class of specialized inputs models, the key difference being that now there is a continuum of primitives that could fit the same observable data and the targets $c_{\mathcal{X}}$ and $c_{\mathcal{G}}$ discipline which is chosen.

4.3 Incorporating Additional Sources of Information

It is easiest to explain how to incorporate new data through specific examples. The input shares for Mexican vehicle exports in Figure 1.1 provide a glimpse of how the supply chains that cross through Mexico work and can be used to discipline the primitives for Mexican production. For example, the dollar flow of German car parts purchased by the Mexican car industry to produce exports for German consumers, i.e. $\mathcal{G}^2\left(\{\text{DEU}, \text{car parts}\}, \{\text{MEX}, \text{cars}\}, \text{DEU}\right)$, is actually observed. Defining the corresponding target $c_{\mathcal{G}}\left(\cdot\right)$ as this number and with the corresponding weight $q_{\mathcal{G}}\left(\cdot\right)$ as a large positive number ensures that the QP framework will take care to target this value. To name a more extreme example, suppose that an insider at FedEx or UPS tells us that there is a specific supply chain which never occurs. In such a case we could place the corresponding value of $c_{\mathcal{G}}\left(\cdot\right) = 0$ and $q_{\mathcal{G}}\left(\cdot\right)$ to a large positive number. Researchers with access to other snapshots of the supply chains underlying bilateral trade data can use that information to discipline their own GVC estimates.

4.3.1 Incorporating Mexican Microdata

I now implement the QP framework using the WIOD for 2014 aggregated to the world's 24 largest economies and 17 industries per country, 15 in manufacturing plus agriculture and

services aggregates. I also incorporate, as additional information, (confidential) Mexican firm-level data containing the universe of manufacturing import and export shipments. The data has three important drawbacks. First, domestic transactions are not observed. This poses a problem because it is then impossible to know whether imported intermediate inputs are embedded in goods sold on the domestic market or whether exported goods require domestic intermediate inputs. Second, services trade is not observed and so it is impossible to know the linkages with manufacturing. Third, the data on intermediate input exports contains information on the destination market but not to which industry.

These issues imply that the data cannot be readily used for the purposes of this paper without additional assumptions. I make the restrictive assumption that manufacturing imports are only used to produce exports. This basically implies that Mexican manufacturing trade corresponds exclusively to processing trade, an obviously strong assumption and not fully accurate but not too far-fetched for Mexico since previous research has shown that Mexico is one of the two countries in the world for which processing trade is most important (the other is China); see Koopman et al. (2010) and De La Cruz et al. (2011).[25] Nonetheless, this is an useful starting point since the goal of this paper is to show how its tools can be used. A researcher that disagrees with these assumptions can make her own but keep the numerical procedure. Thus we can begin a conversation about GVC estimation that cannot be done with I-O analysis.

The firm-level data then delivers the following primitives (written for $\mathcal{M} = 2$)

$$\sum_{k \in \mathcal{K}_{MAN}} \mathfrak{X}\left(\{j'', k''\}, \{\mathbf{MEX}, k'\}, \{j, k\}\right), \quad \text{for } j'', j \in \mathcal{J} \backslash \{\mathbf{MEX}\}, k'', k' \in \mathcal{K}_{MAN},$$

$$\mathcal{G}\left(\{j'', k''\}, \{\mathbf{MEX}, k'\}, j\right), \quad \text{for } j'', j \in \mathcal{J} \backslash \{\mathbf{MEX}\}, k'', k' \in \mathcal{K}_{MAN},$$

where $\mathcal{K}_{MAN}$ is the set of 15 manufacturing sectors. The first set of datapoints are aggregate manufacturing intermediate input imports used by the Mexican manufacturing industries to produce intermediate input exports. The second set of datapoints are aggregate manufacturing intermediate input imports used by the Mexican manufacturing industries to produce final good exports. These datapoints are targeted in the QP framework by setting the corresponding targets $c_{\mathfrak{X}}(\cdot)$ and $c_{\mathcal{G}}(\cdot)$ to these values.[26] Since I do not have microdata to discipline all other targets, I set these to the values given by the benchmark I-O primitives in equation (35).

4.3.2 Foreign and U.S. Content in Mexican Exports to the U.S.

Figure 4.1 shows how pervasive Mexico-U.S. supply chain integration is once GVC estimates incorporate these forces. Specifically, the left panel depicts the dollar value of final good exports

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[25]This applies to intermediate input imports; Mexican consumers do purchase final goods from abroad.

[26]In order to obtain the use of intermediate input exports I assume that this is determined by bilateral shares

$$c_{\mathfrak{X}}\left(\{j'', k''\}, \{\mathbf{MEX}, k'\}, \{j, k\}\right) = \sum_{t \in \mathcal{K}_{MAN}} \mathfrak{X}\left(\{j'', k''\}, \{\mathbf{MEX}, k'\}, \{j, t\}\right) \times \frac{X\left(\{\mathbf{MEX}, k'\}, \{j, k\}\right)}{\sum_{t \in \mathcal{K}_{MAN}} X\left(\{\mathbf{MEX}, k'\}, \{j, t\}\right)}.$$
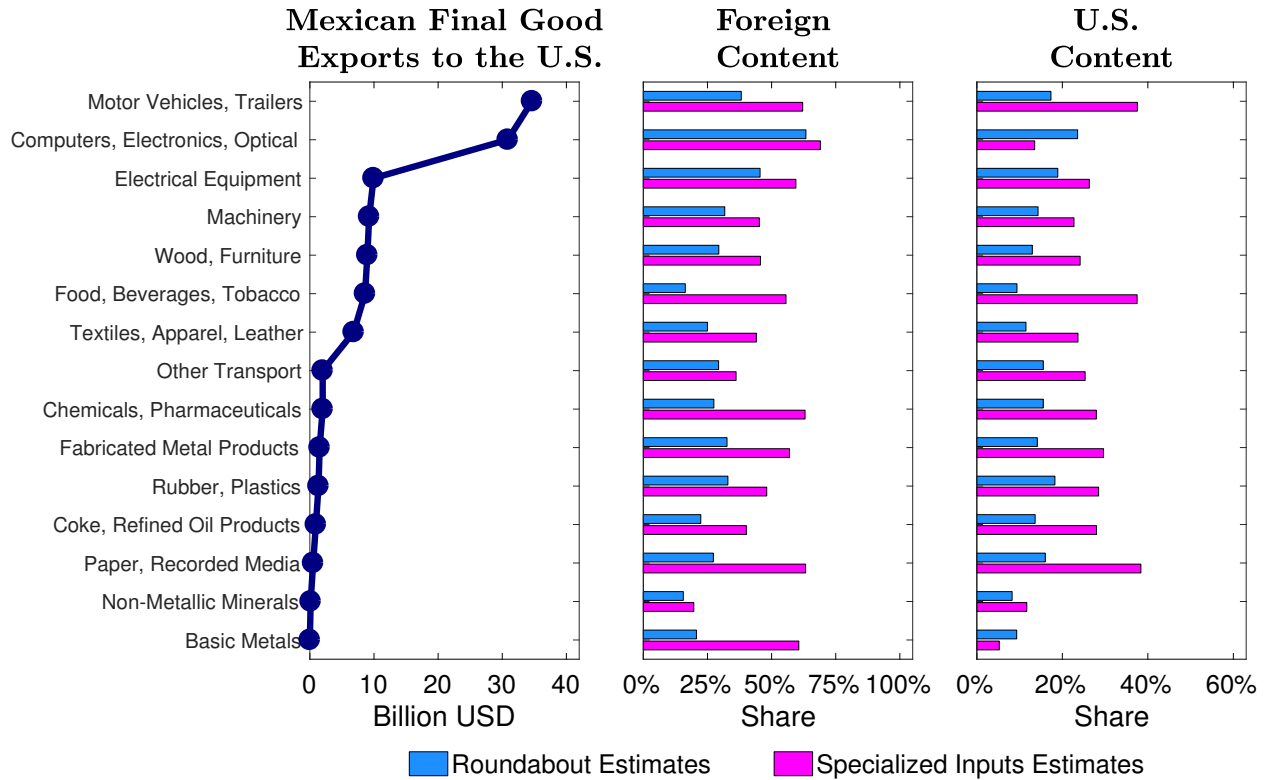
Figure 4.1: Foreign and U.S. Content in Mexican Manufacturing Exports of Final Goods to the U.S. The roundabout estimates are computed with I-O analysis while the specialized inputs estimates are computed with the QP framework.

in 2014 for each of the manufacturing sectors labeled on the y-axis. The middle panel depicts the shares of foreign content in these exports while the right panel plots the shares of U.S. content. In the last two panels, the dashed line depicts the shares that would arise in a roundabout model, and computed with I-O analysis as given by the decomposition in equation (27), while the dotted line presents QP framework shares when disciplining the GVC estimates with the Mexican microdata and computing these shares with equation (26).[27]

Mexico is much more integrated with both the world and U.S. economies than what our current estimates imply. For example, I obtain that 38 cents of every dollar of Mexican vehicle exports to U.S. consumers corresponds to value-added created in the U.S and a full 62 cents are re-exports of value-added created abroad. Ignoring the specialized inputs channel predicts much lower shares at 17 and 38 cents per dollar, respectively. The same is true for overall manufacturing final good exports with the specialized inputs estimates standing at 58 and 27 cents per dollar, while the roundabout estimates are 41 and 17 cents per dollar.

This occurs for two reasons. First, the extreme but not too far-off assumption of processing trade increases the share of foreign content in Mexican exports as seen in the middle panel. Second, and more importantly, Mexico utilizes a disproportionate share of U.S. inputs to pro-

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[27]Solving the QP problem numerically with $|\mathcal{J}| = 24$ and $|\mathcal{K}| = 17$ takes about 5 days and requires around 100-150 GBs of memory when using Gurobi, the fastest solver for mathematical programming.

duce exports to the U.S. and this drives up the U.S. content relatively more than it pushes up overall foreign content in the majority of industries, as can be seen in the right panel. In particular, while the roundabout estimates imply that the U.S. accounts for 45% of all foreign content in Mexican vehicle exports to U.S. consumers this statistic increases to 60% in the specialized inputs estimates. Furthermore, while the roundabout estimates are independent of the use and destination of exports, this is not true for the specialized inputs estimates. The main difference is that most of the U.S. content in Mexican exports appears to be embedded in final good exports (see further results in Appendix Section D.1).

In sum, the small picture takeaway is that accounting for specialized inputs yields a much more integrated view of Mexico-U.S. trade and this confirms the concerns regarding a potential increase in trade barriers within the NAFTA region. The big picture take is that new data can be incorporated in order to ensure that GVC estimates take the newly revealed empirical regularities into account. I have posted the code that implements the quadratic programming framework permanently in estimategvcs.com so that anyone can apply it immediately.

4.3.3 Local Information Has Local Effects

Though the overall structure of GVC flows is interdependent, I now show evidence that local data has only local effects. That is, remember that the benchmark QP estimation targets the primitives given by firm-level data for Mexico but targets the primitives for all other countries using the I-O analysis values. The results in Figure 4.1 clearly show that incorporating the Mexican targets has very important implications for GVC statistics related to Mexico. However, this has little effects on GVC statistics that are not directly linked to Mexico. For example, the left panel of Figure 4.2 shows that deviation with respect to I-O analysis in the share of U.S. value-added in Canadian exports to the U.S. is quantitatively insignificant.

The local effects of local data is a positive feature since it implies that researchers can focus on obtaining a limited amount of additional data in order to study a specific statistic. First, note that this result is not generally true across any network structure. Rather, global trade networks are so concentrated that indirect linkages often have second or third order effects and so while Mexican microdata is key for understanding Mexico-U.S. trade it is almost irrelevant for understanding Canada-U.S. trade even though Canada is the third member of NAFTA. Second, this also suggests that the new estimates of Mexico-U.S. integration are accurate in the sense that including additional Canadian, Chinese or Japanese microdata would change them little. The exception is U.S. microdata which would likely have a substantial effect.

4.3.4 Robustness

I present two robustness tests in Figure 4.1. First, statistical offices often struggle trying to determine whether production is used as an intermediate or final good. Since it is rarely possible to know what output is being used for (i.e. by destination industry or consumers),
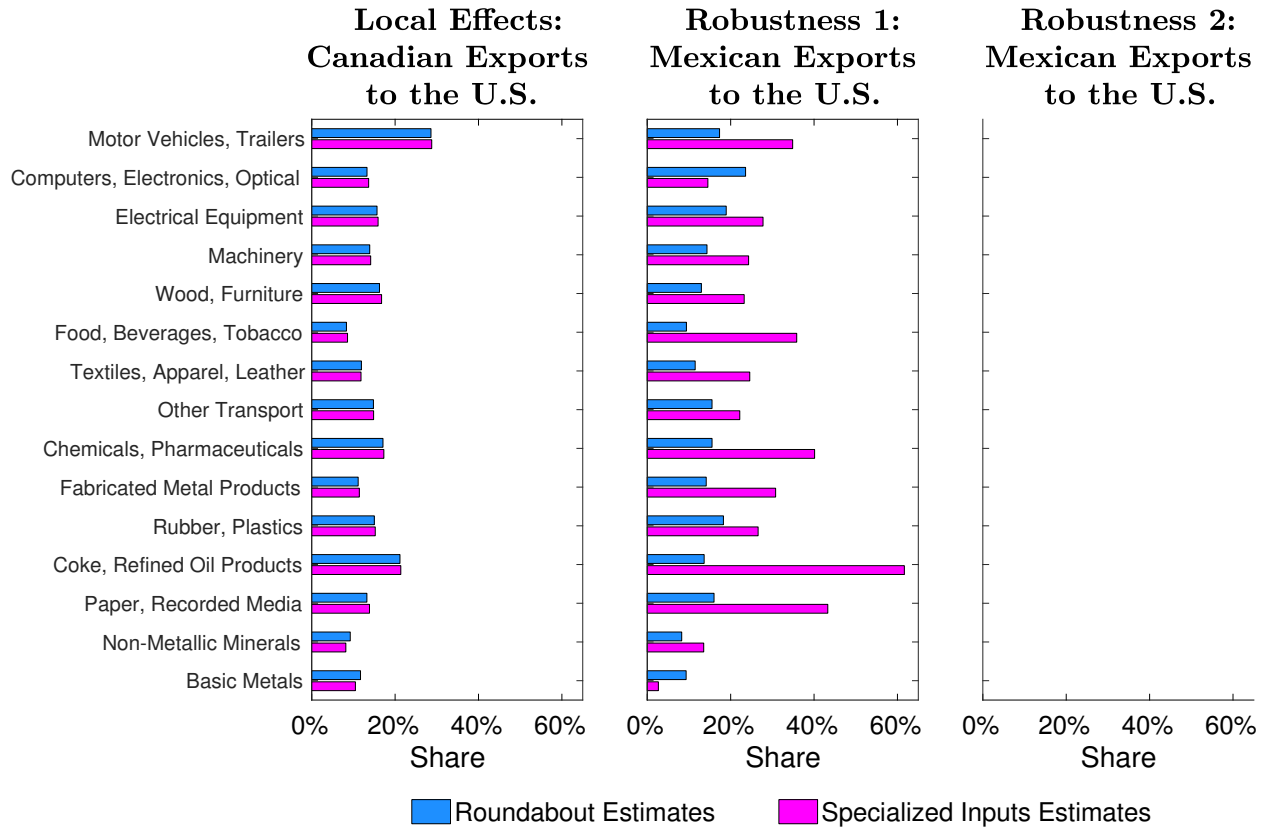
Figure 4.2: U.S. Content in Manufacturing Exports of Final Goods to the U.S.: The left panel presents the shares for Canadian exports using the benchmark QP results. The last two panels present the shares for Mexican exports. The middle panel uses estimates from the QP results using common input shares for both types of targets. The right panel lowers the processing trade assumption to 75% of foreign inputs in exports and 25% of foreign inputs in domestic sales.

they regularly infer the use by the type of product according to well-established classifications. For example, it is fairly certain that a car will be used as final consumption while it is also fairly certain the iron ore will be used as an intermediate. The middle panel in Figure 4.1 shows the results for the case in which intermediates and final goods cannot be distinguished an in which the relative use of inputs from s'' by s' is common across $\mathcal{X}(s'', s', s)$ and $\mathcal{G}(s'', s', j)$ regardless of what it is used for (i.e. $\mathcal{X}(s'', s', s) / \mathcal{G}(s'', s', j) = \mathsf{X}(s', s) / \mathsf{F}(s', j)$). The results are mostly similar.

Second, though processing trade is very important in Mexico one might wonder how much it influences the GVC estimates. The right panel of Figure 4.1 presents the results when lowering the use of foreign inputs in exports to only 75% and increasing the use of foreign inputs in domestic sales to 25%. TBD.

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5 Counterfactuals: Multi-Sector, Specialized Inputs Ricardo

This section provides a solution to the mapping in equation (14) through the lens of a fully-fledged Ricardian specialized inputs model featuring complex spider-snake supply chains with cross-sector and cross-stage of production input-output linkages. Specifically, producing a finished good requires several stages of production and input linkages are specialized in that a stage m good requires an input from the upstream stage $m + 1$.

The main contribution to GVC estimation is that the model's GVCs can be built recursively as in the specialized inputs measurement framework, with the specialized inputs technical coefficients being defined structurally in terms of deep parameters and general equilibrium variables. The model thus provides a microfounded rationale for the previous measurement framework, much like structural roundabout models justify the use of I-O analysis.

The main contribution to structural modeling is that I provide the first tractable multi-sector specialized inputs model. Specifically, the I extend Antràs and de Gortari (2017) to a multi-sector setting that features spider-snake input sourcing patterns since firms producing at stage m require stage $m + 1$ inputs from various sectors. Hence, input sourcing decisions branch out as more upstream stages are included.

5.1 Primitives

Let $\mathcal{J}$ be the set of countries and $\mathcal{K}$ be the set of industrial sectors. Each country $j \in \mathcal{J}$ supplies L_j units of effective labor at wage w_j and aggregate income is $w_j L_j$. Preferences are such that each country j spends a share α_j^k of its income on final consumption goods from sector $k \in \mathcal{K}$. Market structure is perfect competition.

I adopt a double taxonomy to refer to goods. First, inputs are specialized in that producing a *finished* good requires $\mathcal{M} \in \mathbb{Z}^+$ stages of sequential production, with $\mathcal{M}$ also indexing the initial stage, so that production of the stage $m < \mathcal{M}$ *unfinished* good requires inputs from the upstream stage $m + 1$. I refer to a good produced in stage $m = 1$ as a finished good. Second, a finished good can be used either for *final consumption* or as additional *intermediate inputs* by firms producing at any stage. Thus firms at stage $m = \mathcal{M}$ produce with labor and finished goods while firms at stage $m < \mathcal{M}$ produce with labor, unfinished goods from stage $m + 1$, and finished goods. Finally, firms purchase both types of intermediate inputs from all sectors.

I now define the notation that summarizes firm sourcing decisions of unfinished good inputs; Figure 5.1 depicts an example of overall sourcing decisions when $\mathcal{M} = 3$ and is intended to aid the reader. Focus on a firm producing at stage m in country j. When $m = \mathcal{M}$, no unfinished inputs are required and thus there are no sourcing decisions to make. When $m = \mathcal{M} - 1$, firms need to decide where to source the stage $\mathcal{M}$ unfinished input from and the sourcing decisions are summarized by the following set

$$\boldsymbol{\ell}_j(\mathcal{M}) \in \left\{ \ell(\mathcal{M}, k) \in \mathcal{J}, \forall k \in \mathcal{K} \right\}.$$

Location $\ell(\mathcal{M}, k)$ is the source of the sector k input and so the set $\boldsymbol{\ell}_j(\mathcal{M})$ has $|\mathcal{K}|$ elements. Sourcing decisions are unrestricted so that $\boldsymbol{\ell}_j(\mathcal{M}) \in \mathcal{J}^{|\mathcal{K}|}$ and in principle a stage $m = \mathcal{M} - 1$ firm can source from $|\mathcal{J}|^{|\mathcal{K}|}$ possible combinations of countries. Each element in both of the top brackets in Figure 5.1 corresponds to some location $\ell(\mathcal{M}, k)$ while the full brackets correspond to some set $\boldsymbol{\ell}_j(\mathcal{M})$.

The sourcing decisions of firms producing further downstream at $m < \mathcal{M} - 1$ are more complex since it is not sufficient to simply choose the locations from which to source the stage $m + 1$ inputs from. The upstream firms supplying the stage $m + 1$ inputs in a specific location may be able to offer different unit prices for their output depending on where they purchase their own stage $m+2$ inputs from. Hence, stage $m < \mathcal{M}-1$ firms actually need to choose a whole path of upstream sourcing decisions up to the initial stage $\mathcal{M}$. The complexity of these sourcing decisions derives from each input supplier having its own $|\mathcal{K}|$ upstream input suppliers and thus overall sourcing decisions branch out as further upstream stages are incorporated.

I define the sourcing decisions of stage $m < \mathcal{M} - 1$ firms in country j recursively as follows

$$\boldsymbol{\ell}_j(m+1) \in \left\{ \left\{ \boldsymbol{\ell}_{\ell(m+1,k)}(m+2), \ell(m+1,k) \right\} \in \prod_{\mu=2}^{\mathcal{M}-m} \mathcal{J}^{|\mathcal{K}|^\mu} \times \mathcal{J}, \forall k \in \mathcal{K} \right\}. \tag{36}$$

Location $\ell(m+1, k)$ is where the immediate supplier of the sector k unfinished good of stage $m + 1$ is located. In addition, $\boldsymbol{\ell}_{\ell(m+1,k)}(m+2)$ fully specifies the upstream sourcing decisions of this supplier and the upstream sourcing decisions of its own suppliers. I defined $\boldsymbol{\ell}_j(m+1)$ recursively but I will often refer to it as a set of $|\mathcal{K}|^{\mathcal{M}-m}$ chains of the form

$$\ell\left(\mathcal{M}, k^{\mathcal{M}}\right) \to \cdots \to \ell\left(m+2, k^{m+2}\right) \to \ell\left(m+1, k^{m+1}\right) \to j.$$

The superscript on k^m is meant only to reference the stage for which this sector is relevant. This chain indicates that $\ell(m+1, k^{m+1})$ is the source of the sector k^{m+1} input from stage $m + 1$, that the firm producing this input sources its stage $m + 2$ input from sector k^{m+2} from $\ell(m+2, k^{m+2})$, and so on. Hence, $\boldsymbol{\ell}_j(m+1) \in \prod_{\mu=1}^{\mathcal{M}-m} \mathcal{J}^{|\mathcal{K}|^\mu}$ and can take up to $|\mathcal{J}|^{\sum_{\mu=1}^{\mathcal{M}-1} |\mathcal{K}|^\mu}$ different combinations. In the example of Figure 5.1, each element of the middle bracket corresponds to a supplier of the $m = \mathcal{M}-1$ stage, $\ell(\mathcal{M}-1, k)$, and each has its own associated set of input suppliers given by $\boldsymbol{\ell}_{\ell(\mathcal{M}-1,k)}(\mathcal{M})$. Likewise the full set of stage $\mathcal{M}$ and $\mathcal{M}-1$ suppliers in the first two levels are summarized in $\boldsymbol{\ell}_j(\mathcal{M}-1)$.

Finally, I define the notation for sourcing decisions of finished goods as

$$\ell \in \left\{ \left\{ \boldsymbol{\ell}_{\ell(1)}(2), \ell(1) \right\} \in \prod_{\mu=1}^{\mathcal{M}-1} \mathcal{J}^{|\mathcal{K}|^\mu} \times \mathcal{J} \right\}. \tag{37}$$

In contrast to $\boldsymbol{\ell}_j(m+1)$, ℓ characterizes a single location for the last production stage. Location $\ell(1)$ indicates the location of assembly while $\boldsymbol{\ell}_{\ell(1)}(2)$ summarizes the whole upstream path
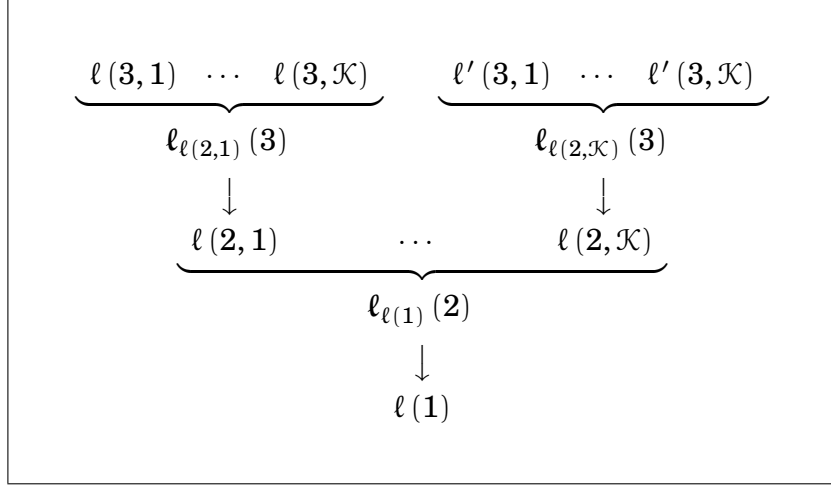
Figure 5.1: Unfinished Good Input Sourcing Decisions When $\mathcal{M} = 3$.

of input sourcing decisions. Overall sourcing strategies of finished goods can thus take up to $|\mathcal{J}|^{\sum_{\mu=0}^{\mathcal{M}-1} |\mathcal{K}|^\mu}$ different combinations.[28] Hence, the full path of inputs in Figure 5.1 is summarized by ℓ with $\ell(1)$ the assembly location and $\boldsymbol{\ell}_{\ell(1)}(2)$ the set of all its upstream suppliers.

Having defined notation I now introduce the main technological assumption.

Assumption 5.1. *Production features constant returns to scale and the market structure is perfect competition. Specifically, every dollar of production of a firm producing in country $j \in \mathcal{J}$, at some stage $m = 1, \ldots, \mathcal{M}$, in sector $k \in \mathcal{K}$, and sourcing inputs from $\boldsymbol{\ell}_j(m+1)$ can be split across its factors of production with shares*

(i) $\beta_j^{m,k}$ *on labor,*

(ii) $\xi_j^{m,k,k'}\left(\boldsymbol{\ell}_j(m+1)\right)$ *on unfinished goods from sector k',*

(iii) $\gamma_j^{m,k,k'}\left(\boldsymbol{\ell}_j(m+1)\right)$ *on finished goods from sector k',*

with

$$\beta_j^{m,k} + \sum_{k' \in \mathcal{K}} \xi_j^{m,k,k'}\left(\boldsymbol{\ell}_j(m+1)\right) + \sum_{k' \in \mathcal{K}} \gamma_j^{m,k,k'}\left(\boldsymbol{\ell}_j(m+1)\right) = 1.$$

Part *(i)* of this assumption is strongest since value-added shares are country-stage-industry specific but do not depend on upstream sourcing decisions. This ensures that the model is in line with Assumption 3.2 and that the aggregate relation in equation (10) applies (see footnote 21). The use of intermediate inputs is more flexible since firms can substitute across unfinished and finished goods and across sectors depending on the specific sequence of production. Note that at stage $m = \mathcal{M}$, $\xi_j^{\mathcal{M},k,k'}(\varnothing) = 0$ always holds. Also note that with a general constant returns to scale production function these expenditure shares may depend on general equilibrium variables such as wages. I omit this explicit dependence to save on notation.

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[28]Note how this contrasts with a single sector world, i.e. $|\mathcal{K}| = 1$, where unfinished input sourcing decisions occur in pure snake form: $\ell(\mathcal{M}) \to \cdots \to \ell(2) \to \ell(1)$.

Finally, I will denote the distribution of country j's consumption of sector k finished goods with $\pi_j^k(\ell)$ such that $\sum_{\ell \in \mathcal{J}^{\sum_{\mu=0}^{\mathcal{M}-1} |\mathcal{K}|^\mu}} \pi_j^k(\ell) = 1$. This distribution can also be interpreted as the share of expenditure on finished goods produced through ℓ. In order to focus on the specialized inputs linkages I leave the specific structure of $\pi_j^k(\ell)$ unspecified for the time being. The reader can think of this distribution as a function of deep parameters and general equilibrium variables or, more simply, as a set of numbers determined by nature. Furthermore, I have so far described the input sourcing decisions as if there are independent firms producing at each stage of production. I will return later to discuss a specific microfoundation for $\pi_j^k(\ell)$ and the required assumptions for working with either a decentralized equilibrium or one in which a global firm organizes the whole supply chain.

Both the distribution π and the expenditure shares ξ and γ are sequence-specific but have different implications. Since firms are perfectly competitive they take general equilibrium variables such as wages as given and thus input shares act as technological constraints imposing input expenditure requirements along the production of ℓ. Meanwhile, π determines the distribution of firms sourcing inputs across different strategies ℓ. These three variables jointly determine the aggregate distribution of input expenditures at the country-industry-stage level.

5.2 $\mathcal{M}-$**Proportional GVCs**

I now show that this model is consistent with the measurement framework of Section 3. In a nutshell, it takes $\mathcal{M}$ stages of sequential production to transform an unfinished good into a finished good and further upstream input linkages cycle over through finished goods inputs (governed by γ). Hence, GVCs can be constructed recursively by conditioning input expenditures on the immediate $\mathcal{M}$ downstream locations through which these inputs flow. In what follows I further restrict the assumptions on technology by imposing Cobb-Douglas production and by assuming that technology is independent of the upstream sourcing decisions. Thus, $\beta_j^{m,k}$, $\xi_j^{m,k,k'}$, and $\gamma_j^{m,k,k'}$ are parameters and the latter two no longer depend on $\ell_j(m+1)$. All of the results in this section carry through to the general technology in Assumption 5.1 and are discussed in Appendix Section B.4.

Input linkages occur through unfinished and finished goods. In order to derive input shares, remember the notation of previous sections: (i) let $\mathcal{S} = \mathcal{J} \times \mathcal{K} \times \mathcal{M}$ be the set of country-industry-stages and define an element $s^n \in \mathcal{S}$ as a triple $s^n = \{j^n, k^n, m^n\}$ where n is used only to indicate the relation to a specific triple, (ii) a sequence of production of length $N \in \mathbb{Z}^+$ is given by $s^N \to \cdots \to s^1$. I now derive the share of inputs from $\bar{s} = \{\bar{j}, \bar{k}, \bar{m}\}$ purchased by an arbitrary sequence of production $s^N \to \cdots \to s^1$ that produces goods to be consumed in location j^0.

I. Unfinished Goods Input Shares. These input sales occur only if the following three conditions are satisfied: (i) $\bar{m} > 1$ so that sales are of unfinished goods; (ii) $m^N = \bar{m} - 1, m^{N-1} = \bar{m} - 2, \ldots, m^{N-(\bar{m}-2)} = 1$ so that the use of these inputs is through the appropriate sequence of stages; and (iii) $N \geqslant \bar{m} - 1$ so that the sequence is long enough to fully specify the use of the unfinished input through the assembly stage. When these conditions are satisfied

the share of inputs from $\bar{s}$ flowing through sequence $s^N \to \cdots \to s^1 \to j^0$ of length N equals

$$a^N\left(\bar{s}\,\middle|\,s^N,\ldots,s^1,j^0\right) = \underbrace{\frac{\sum_{\ell \in \mathcal{L}^{\bar{m},\bar{k}}\left(\bar{j}|s^N,\ldots,s^1\right)} \pi_{j^{N-(\bar{m}-1)}}^{k^{N-(\bar{m}-2)}}(\ell)}{\sum_{\ell \in \bigcup_{\bar{j}' \in \mathcal{J}} \mathcal{L}^{\bar{m},\bar{k}}\left(\bar{j}'|s^N,\ldots,s^1\right)} \pi_{j^{N-(\bar{m}-1)}}^{k^{N-(\bar{m}-2)}}(\ell)}}_{\substack{\text{Probability of sourcing unfinished}\\ \text{good input from location } \bar{j}}} \times \underbrace{\xi_{j^N}^{m^N,k^N,\bar{k}}}_{\substack{\text{Input share of } s^N \text{ on sector } \bar{k}\\ \text{unfinished good inputs}}}, \qquad (38)$$

with

$$\mathcal{L}^{\bar{m},\bar{k}}\left(\bar{j}\,\middle|\,s^N,\ldots,s^1\right) = \left\{ \ell \in \mathcal{J}^{\sum_{\mu=0}^{\mathcal{M}-1}|\mathcal{K}|^\mu} : \begin{array}{c} \ell\left(\bar{m},\bar{k}\right) \to \ell\left(m^N,k^N\right) \to \cdots \to \ell\left(2,k^{N-(\bar{m}-3)}\right) \to \ell\left(1\right) \\ \text{is given by } \bar{j} \to j^N \to \cdots \to j^{N-(\bar{m}-3)} \to j^{N-(\bar{m}-2)} \end{array} \right\}.$$

The input share consists of two terms. First, $\xi_{j^N}^{m^N,k^N,\bar{k}}$ is the overall expenditure of s^N on unfinished inputs from sector $\bar{k}$. Second, the distribution of sourcing strategies needs to be taken into account in order to compute the share of this expenditure spent on unfinished goods from a specific location $\bar{j}$. More specifically, when $\bar{m} < \mathcal{M}$ there are multiple sourcing sequences ℓ that are consistent with the production sequence in $\bar{s} \to s^N \to \cdots \to s^{N-(\bar{m}-2)}$ since further upstream sources are not specified and the set $\mathcal{L}^{\bar{m},\bar{k}}\left(\bar{j}|s^N,\ldots,s^1\right)$ is defined such that it includes all of these chains. The ratio of probabilities in (38) is thus the conditional probability of sourcing the $\bar{k}$ sector unfinished good from a specific $\bar{j}$ given that the input will flow downstream through $s^N \to \cdots \to s^{N-(\bar{m}-2)}$.

II. Finished Goods Input Shares. In contrast to the above, finished goods are sourced by all stages of production and thus the input shares are defined whenever $\bar{m} = 1$ as

$$a^N\left(\bar{s}\,\middle|\,s^N,\ldots,s^1,j^0\right) = \underbrace{\sum_{\ell \in \mathcal{L}(\bar{j})} \pi_{j^N}^{\bar{k}}(\ell)}_{\substack{\text{Probability of sourcing finished}\\ \text{good input from location } \bar{j}}} \times \underbrace{\gamma_{j^N}^{m^N,k^N,\bar{k}}}_{\substack{\text{Input share of } s^N \text{ on sector } \bar{k}\\ \text{finished good inputs}}}, \qquad (39)$$

with

$$\mathcal{L}\left(\bar{j}\right) = \left\{ \ell \in \mathcal{J}^{\sum_{\mu=0}^{\mathcal{M}-1}|\mathcal{K}|^\mu} : \ell\left(1\right) = \bar{j} \right\}.$$

As before, the input share consists of two terms. The overall expenditure of s^N on sector $\bar{k}$ inputs is similar to the previous case except that now this expenditure is on finished goods. The probability term is simpler since assembly takes place in $\bar{s}$ and thus the first term in (39) is the sum of chains that assemble finished goods in $\bar{j}$, with the set of these given by $\mathcal{L}\left(\bar{j}\right)$.

There are four observations about the input shares that are important to keep in mind. First, whenever $\bar{s} \to s^N \to \cdots \to s^1 \to j^0$ does not satisfy the conditions of the unfinished or finished goods input flows then $a^N\left(\cdot\right) = 0$. Second, it is easy (but tedious) to see that the input shares are defined appropriately in the sense that the value of production of s^N to be used

through sequence $s^{N-1} \to \cdots \to s^1 \to j^0$ is fully accounted for by the expenditure on its factors of production. That is, the following always holds

$$\beta_{j^N}^{m^N, k^N} + \sum_{\bar{s} \in \mathcal{S}} a^N \left(\bar{s} \, \middle| \, s^N, \ldots, s^1, j^0 \right) = 1.$$

Third, note that specifying the sequence through which s^N production is used is only relevant for the linkages through unfinished goods. That is, while the input shares of unfinished goods (38) may depend on the sequence $s^{N-1} \to \cdots \to s^1 \to j^0$ those of finished goods (39) do not. This distinction is at the heart of this paper because it is precisely this type of specialized input linkages, in which firms make their input sourcing decisions conditional on where they sell their own output to, that makes these models inconsistent with roundabout production. Fourth, it will never be necessary to condition input shares on a sequence of more than $\mathcal{M}$ stages. That is, there will never be any need to compute the input shares in (38) for $N > \mathcal{M}$ since sourcing strategies in ℓ are defined only up to $\mathcal{M}$ stages of production and further input linkages only occur through finished goods. Furthermore, when $N = \mathcal{M}$ there is no need to specify the final location j^0 since sequential inputs flow at the most upstream between an input producer in $\bar{m} = \mathcal{M}$ and the location of purchase of the finished good being produced will at most be at the downstream stage s^1. The implication is that when $N = \mathcal{M}$, input shares can be conditioned as $a^{\mathcal{M}} \left(\bar{s} \, \middle| \, s^{\mathcal{M}}, \ldots, s^1 \right)$.

The terms $a^N \left(\cdot \right)$ fully characterize GVC flows $\mathcal{G}^N \left(s^N, \ldots, s^1, j \right)$ for any $N \in \mathbb{Z}^+$. To see this, begin by computing final consumption flows and remember that $\alpha_j^k w_j L_j$ is country j's aggregate consumption of sector k goods. Purchases from $s^1 \in \mathcal{S}$ occur only if $m^1 = 1$ and are given by

$$\mathcal{G}^1 \left(s^1, j \right) = \left[\sum_{\ell \in \mathcal{L}(j^1)} \pi_j^{k^1} \left(\ell \right) \right] \times \alpha_j^{k^1} w_j L_j. \tag{40}$$

The share of final good flows purchased from the location in s^1 is equal to the total probability of sourcing shares ℓ that assemble the finished good of sector k^1 in location j^1. Longer GVC flows can be computed recursively for $N = 1, \ldots, \mathcal{M} - 1$ as

$$\mathcal{G}^{N+1} \left(s^{N+1}, \ldots, s^1, j \right) = a^N \left(s^{N+1} \, \middle| \, s^N, \ldots, s^1, j \right) \mathcal{G}^N \left(s^N, \ldots, s^1, j \right).$$

Proposition 5.2. *GVCs in the specialized inputs model can be constructed for any $N \geqslant \mathcal{M}$ as*

$$\mathcal{G}^{N+1} \left(s^{N+1}, s^N, \ldots, s^1, j \right) = a^{\mathcal{M}} \left(s^{N+1} \, \middle| \, s^N, \ldots, s^{N+(\mathcal{M}-1)} \right) \mathcal{G}^N \left(s^N, \ldots, s^1, j \right) \tag{41}$$

Hence, this structural model provides an answer to the fundamental problem of GVC estimation in that it delivers a precise way for solving the mapping in equation (14). Furthermore, the model delivers the same answer as the specialized inputs measurement framework as defined in (18) and thus provides a microfounded justification for its use just as roundabout

production does for I-O analysis.

5.2.1 General Equilibrium

Computing the equilibrium wages is immediate now that the model has been mapped into its GVCs since these can be used to trace value across all stages of production. Specifically, an implication of Proposition 5.2 is that that the decomposition of value-added in final good consumption can be done with the formula in (26). Since labor is the only factor of production, wages are pinned down in general equilibrium by equating labor income to value-added

$$w_j L_j = \sum_{t \in j \times \mathcal{K} \times \mathcal{M}} \sum_{s \in j \times \mathcal{K} \times \mathcal{M}} VA_t(s, j).$$

The term on the right depends on wages through final good consumption and through the variables defining the input shares $a^{\mathcal{M}}(\cdot)$, so that this equation presents an useful fixed point.[29]

This ends the description of the model which holds regardless of how $\pi_j^k(\ell)$ is determined. Though there are many different stories behind what could potentially determine the GVC distribution it will always be the case that with constant returns to scale as in Assumption 5.1 the mapping in (14) can be resolved with (41).

5.3 Welfare Analysis: A Ricardian Microfoundation

The specialized inputs measurement framework estimates GVCs directly with primitives since it assimilates the fact that all that matters for GVC computation is the values that these take in equilibrium. Computing counterfactuals is a very different matter because now we need a theory for constructing unobserved, in the sense that they do not exist, equilibria. In order to do so, I now impose additional structure and parametric characterization of $\pi_j^k(\ell)$. I discuss this microfoundation in terms of there being a lead firm that decides the overall sourcing strategy ℓ, and will discuss at the end how this can be decentralized into a world where there are independent firms producing at each stage of production.

Assume that each sector produces a continuum of measure one of differentiated varieties indexed by ω. As before, each variety requires $\mathcal{M} \in \mathbb{Z}^+$ stages of sequential production. An mth stage variety ω requires variety-specific inputs from stage $m + 1$. That is, the unfinished inputs needed for producing the mth stage variety ω of sector k are stage $m + 1$ varieties ω from every sector t. Finished varieties of stage $m = 1$ are consumed by final consumers and also used by firms as intermediate inputs through constant-elasticity-of-substitution composite bundles of the continuum of varieties of each sector. Specifically, let $\sigma_j^k > 1$ be the elasticity-of-substitution and P_j^k the unit price of one unit of the sector k composite bundle in country j. Finally, all international trade flows are subject to iceberg trade costs with τ_{ji}^k indicating

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[29]Trade imbalances can easily be incorporated as in Dekle et al. (2007) by including an exogenous trade deficit $D_j < 0$ in overall final demand in (40) as $w_j L_j - D_j$.

how many units from sector k need to be sent from j for one unit to arrive in i. The following restrictions apply: $\tau_{ji}^k = \tau_{ij}^k$, $\tau_{jj}^k = 1$, and $\tau_{ji}^k \leqslant \tau_{jh}^k \tau_{hi}^k$ for all $i, j, h \in \mathcal{J}$ and for all sectors $k \in \mathcal{K}$.

The environment is perfect competition and since technology is Cobb-Douglas I work directly with prices (the dual). Throughout it will be useful to write the following auxiliary variable related to the unit cost of labor and composite inputs

$$c_j^{m,k} = \left(w_j\right)^{\beta_j^{m,k}} \prod_{k' \in \mathcal{K}} \left(P_j^t\right)^{\gamma_j^{m,k,k'}}.$$

The stage $m = \mathcal{M}$ variety ω of sector k requires no unfinished good inputs so that the cost of labor and composite inputs in each location j fully characterize its price up to a Ricardian productivity shifter $z_j^{\mathcal{M},k}(\omega)$. That is

$$p_j^{\mathcal{M},k}(\omega) = z_j^{\mathcal{M},k}(\omega)\, c_j^{\mathcal{M},k}. \tag{42}$$

Computing the prices of stage $m < \mathcal{M}$ varieties is slightly more complicated since they depend on the whole upstream sequence through which the unfinished inputs are sourced from. The Cobb-Douglas assumption implies that these can be computed recursively as

$$p_j^{m,k}\left(\omega\,\middle|\,\ell_j\left(m+1\right)\right) = z_j^{m,k}(\omega)\, c_j^{m,k} \prod_{k' \in \mathcal{K}} \left(p_{\ell(m+1,k')}^{m+1,k'}\left(\omega\,\middle|\,\ell_{\ell(m+1,k')}\left(m+2\right)\right) \tau_{\ell(m+1,k')j}^{k'} \right)^{\xi_j^{m,k,k'}}.$$

$$\tag{43}$$

The price depends on a variety-specific Ricardian productivity shifter $z_j^{m,k}(\omega)$, the local price of labor and composites through $c_j^{m,k}$, and also on the set of prices that its direct input suppliers of stage $m+1$ from each sector k' located at $\ell\left(m+1,k'\right)$ command when they source their own inputs through the chains specified by $\ell_{\ell(m+1,k')}\left(m+2\right)$. Note that geography plays a role since input prices are shifted upwards by the trade cost $\tau_{\ell(m+1,k')j}^{k'}$.

After $\mathcal{M}$ stages of sequential production each variety becomes a finished good and can be purchased in location j at a price that varies depending on the sequence of locations ℓ through which the upstream variety-specific inputs were sourced from. Specifically, the price of a sector k variety ω produced through ℓ (with $\ell\left(1\right)$ the finished good assembly stage) in country j equals

$$p_j^{F,k}\left(\omega\,\middle|\,\ell\right) = p_{\ell(1)}^{1,k}\left(\omega\,\middle|\,\ell_{\ell(1)}\left(2\right)\right) \tau_{\ell(1)j}^k, \tag{44}$$

$$= \prod_\ell \prod_{m=2}^{\mathcal{M}} \left[z_{\ell(m,k^m)}^{m,k^m}(\omega)\, c_{\ell(m,k^m)}^{m,k^m} \tau_{\ell(m,k^m)\ell(m-1,k^{m-1})}^{k^m} \right]^{\prod_{\mu=1}^{m-1} \xi_{\ell(\mu,k^\mu)}^{\mu,k^\mu,k^{\mu+1}}} \times z_{\ell(1)}^{1,k}(\omega)\, c_{\ell(1)}^{1,k} \tau_{\ell(1)j}^k.$$

Notation is such that the product over ℓ denotes the multiplication of the term in square parenthesis for each of the $\sum_{m=1}^{\mathcal{M}-1} |\mathcal{K}|^m$ elements of the $|\mathcal{K}|^{\mathcal{M}-1}$ chains in ℓ that fully describe the input sourcing decisions used to produced a finished variety.[30]

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[30]For example, following the case of $\mathcal{M} = 3$ in Figure 5.1, this product includes a term for each of the stage $m = 2$ suppliers $\ell\left(2,1\right), \ldots, \ell\left(2,\mathcal{K}\right)$; and a term for each of the stage $m = 3$ suppliers $\ell\left(3,1\right), \ldots, \ell\left(3,\mathcal{K}\right)$ from which each

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The key challenge for deriving the distribution $\pi_j^k(\ell)$ is that that the literature normally assumes that $1/z_{\ell(m,k^m)}^{m,k^m}(\omega)$ is distributed as a Fréchet random variable. This poses a challenge because if there exist individual firms producing at each stage of production and purchasing inputs from the cheapest upstream suppliers then this problem becomes intractable since the overall distribution of sourcing decisions is given by the product of Fréchet random variables, as in (44), which has no closed form probability distribution. Alternatively, as initially shown in Antràs and de Gortari (2017), this obstacle can be surmounted by assuming that there is a *lead firm* which organizes the overall supply chain and faces a random productivity shock that is specific to the overall input sourcing decision ℓ. Assume that the productivity distribution of lead firms selling to market j and producing through ℓ is given by

$$1 \Big/ \prod_\ell \prod_{m=2}^{\mathcal{M}} \left[z_{\ell(m,k^m)}^{m,k^m}(\omega) \right]^{\prod_{\mu=1}^{m-1} \xi_{\ell(\mu,k^\mu)}^{\mu,k^\mu,k^{\mu+1}}} \times z_{\ell(1)}^{m,k}(\omega) \sim \text{Fréchet}\left(T_j^k(\ell), \theta \right). \qquad (45)$$

That is, the random variable is given by the product of productivities across all input sourcing chains in ℓ weighted by the input expenditure share relative to the final output value on each upstream input as denoted by the exponent $\prod_{\mu=1}^{m-1} \xi_{\ell(\mu,k^\mu)}^{\mu,k^\mu,k^{\mu+1}}$.

The lead firm assumption in (45) renders the problem tractable since final good prices follow an extreme value distribution and the optimal sourcing strategies for each variety can thus be easily characterized using the standard techniques introduced in Eaton and Kortum (2002). In the most general case, lead firms face different overall productivities depending on the market at which they sell their finished good in (and determined by the scale parameter $T_j^k(\ell)$).

There are various reasons why compatibility may drive variation in this parameter across consumer markets. For example, physical compatibility may require that machines have similar voltages while regulatory compatibility may arise because emission or quality standards vary across countries. Furthermore, this parameter can also be used to capture multinational activity by proxying the offshoring practices of global firms. Finally, a very similar model can be written in which trade costs are sequence-specific, i.e. $\tau_{ij}^k(\ell)$ and can reflect trade barriers that are related to content such as rules of origin. For example, Mexico can export a car to the U.S. at a lower tariff whenever its upstream inputs were purchased from the U.S. itself.

The share of varieties that country j sources through a particular sequence equals

$$\pi_j^k(\ell) = \Pr\left(\ell = \underset{\ell' \in \mathcal{J}^{\sum_{\mu=0}^{\mathcal{M}-1} |\mathcal{K}|^\mu}}{\arg\min}\; p_j^{F,k}(\omega|\ell') \right), \qquad (46)$$

$$= \frac{1}{\Theta_j^k} T_j^k(\ell) \prod_\ell \prod_{m=2}^{\mathcal{M}} \left(c_{\ell(m,k^m)}^{m,k^m} \tau_{\ell(m,k^m)\ell(m-1,k^{m-1})}^{k^m} \right)^{-\theta \prod_{\mu=1}^{m-1} \xi_{\ell(\mu,k^\mu)}^{\mu,k^\mu,k^{\mu+1}}} \times \left(c_{\ell(1)}^{1,k} \tau_{\ell(1)j}^k \right)^{-\theta}.$$

stage $m=2$ supplier $\ell(2,k)$ sources its inputs from. Thus, there are $|\mathcal{K}|^{3-1} = |\mathcal{K}|^2$ chains leading to assembly in $\ell(1)$ from some firm at $m = \mathcal{M}$ and $|\mathcal{K}|^1 + |\mathcal{K}|^2$ elements in the product.

with the proportionality constant Θ_j^k being the sum of the numerator across all possible input sourcing strategies $\ell \in \mathcal{J}^{\sum_{\mu=0}^{\mathcal{M}-1} |\mathcal{K}|^\mu}$. Finally, composite prices are given by

$$P_j^k = \left[\Theta_j^k\right]^{-\frac{1}{\theta}} \Gamma\left(1 + \frac{1-\sigma_j^k}{\theta}\right)^{\frac{1}{1-\sigma_j^k}}. \tag{47}$$

A key property in equation (46) is that the trade elasticity of shipping goods through $\ell(m, k^m) \to \ell(m-1, k^{m-1})$ and to be used subsequently through $\ell(m-2, k^{m-2}) \to \cdots \to \ell(2, k^2) \to \ell(1)$ is given by $-\theta \xi_{\ell(1)}^{1,k,k^2} \prod_{\mu=2}^{m-1} \xi_{\ell(\mu,k^\mu)}^{\mu,k^\mu,k^{\mu+1}}$ and thus increases as goods move downstream. This result echoes that found early on in Yi (2010) and more recently in Antràs and de Gortari (2017). Trade costs are proportional to gross output and as unfinished varieties flow down the supply chain more and more value is added so that cost of shipping becomes more and more important. However, a crucial difference with the previous literature is that the supply chain linkages do not occur solely in pure snake form. That is, the trade elasticity is always greater at more downstream stages within a specific chain $\ell(\mathcal{M}, k^{\mathcal{M}}) \to \ldots \to \ell(2, k^2) \to \ell(1)$ but it is entirely possible that the trade elasticity of upstream production stages in another chain, even within the same ℓ, is higher than the downstream trade elasticities of the former. The reason for this is that the trade elasticities are attenuated by input expenditure shares so that chains that deliver very few inputs to assembly will have low trade elasticities in downstream stages while chains that deliver a lot of inputs to assembly may have high trade elasticities even in upstream stages of production.

5.3.1 Gains from Trade

Implementing counterfactuals with specialized inputs models is a daunting task given the number and complexity of the parameters on which they depend. In particular, the compatibility parameters $\mathsf{T}_j^k(\ell)$ determine the average productivity across sequences of production and are thus constitute major force in shaping supply chain patterns. An immediate implication of this that they can only be calibrated with supply chain data, or at the very least more dissagregate moments than those contained in bilateral trade flows, and are thus impossible to parameterize given current data limitations. Furthermore, a similar critique applies to the intermediate input expenditure since it bilateral flows do not provide information on how to disentangle expenditures across intermediates and on composites γ and variety-specific inputs ξ. Hence the direct use of this model appears to be limited.

Fortunately, as noted by Antràs and de Gortari (2017), the broad insight from Arkolakis et al. (2012) carries through to this class of Ricardian supply chain models and the welfare gains from trade can be stated in terms of a few sufficient statistics and key elasticities. Specif-

ically, the change in real income in country j equals

$$\hat{W}_j = \prod_{k^1 \in \mathcal{K}} \left(\left[\hat{\pi}_j^{k^1}\left(\ell_j^*\right) \right]^{-\frac{1}{\theta} \frac{1}{\sum\limits_{k^{\mathcal{M}},\ldots,k^2} \sum\limits_{m=1}^{\mathcal{M}} \beta_j^{m,k^m} \prod\limits_{\mu=1}^{m-1} \xi_j^{\mu,k^\mu,k^{\mu+1}}} } \prod_{t \in \mathcal{K}} \left[\frac{\hat{P}_j^t}{\hat{P}_j^{k^1}} \right]^{-\frac{\sum\limits_{k^{\mathcal{M}},\ldots,k^2} \sum\limits_{m=1}^{\mathcal{M}} \gamma_j^{m,k^m,t} \prod\limits_{\mu=1}^{m-1} \xi_j^{\mu,k^\mu,k^{\mu+1}}}{\sum\limits_{k^{\mathcal{M}},\ldots,k^2} \sum\limits_{m=1}^{\mathcal{M}} \beta_j^{m,k^m} \prod\limits_{\mu=1}^{m-1} \xi_j^{\mu,k^\mu,k^{\mu+1}}}} \right)^{\alpha_j^{k^1}}, \tag{48}$$

where ℓ_j^* is the input sourcing strategy that sources every single input from domestic firms. In terms of sufficient statistics it depends on the probability of purely domestic sourcing chains $\hat{\pi}_j^{k^1}\left(\ell_j^*\right)$ and changes in relative prices while he key elasticity is given by the trade elasticity attenuated by the sum of the value-added shares relative to the assembly stage across all upstream stages of production.

In order to better understand how multi-sector production and specialized inputs linkages jointly shape this equation, I first discuss its relation to special cases found in earlier work.

I. Eaton and Kortum (2002) developed the first tractable multi-country Ricardian model with a single sector and no specialized inputs linkages: $|\mathcal{K}| = 1$ and $\mathcal{M} = 1$. The welfare gains depend simply on the share of expenditure on domestic goods, the trade elasticity, and the value-added share

$$\hat{W}_j = \left[\hat{\pi}_{jj} \right]^{-\frac{1}{\theta \beta_j}}.$$

In the absence of intermediate inputs, $\beta_j = 1$, goods cross borders a single time so that the gains from trade are given by the change in domestic expenditures and amplified by the degree of comparative advantage $1/\theta$. The latter occurs because, for a given change in domestic expenditure, stronger comparative advantage implies that a country benefits more from sourcing foreign goods. However, when intermediates are present, $\beta_j < 1$, aggregate purchases equal a share $1 + \left(1 - \beta_j\right)/\beta_j = 1/\beta_j$ of income and the amplification is greater since changes in trade costs have ripple effects across all stages of production.

II. Caliendo and Parro (2015) generalized the model to multiple sectors and included input-output linkages, defined as the linkages through finished (composite) goods in this paper, but no specialized inputs: $|\mathcal{K}| > 1$ and $\mathcal{M} = 1$. The welfare gains from trade now also depend on the sectorial input expenditure shares, the consumer's sectorial final good expenditure shares, and the changes in relative prices

$$\hat{W}_j = \prod_{k \in \mathcal{K}} \left(\left[\hat{\pi}_{jj}^k \right]^{-\frac{1}{\theta \beta_j^k}} \prod_{k' \in \mathcal{K}} \left[\frac{\hat{P}_j^{k'}}{\hat{P}_j^k} \right]^{-\frac{\gamma_j^{k,k'}}{\beta_j^k}} \right)^{\alpha_j^k}.$$

The new terms reflect the input-output linkages through which increases (decreases) in sector-level prices of composite inputs relative to output decrease (increase) labor productivity. This

translates directly into welfare up to the weight $\gamma_j^{k,k'}/\beta_j^k$, which proxies the importance of the sector k' intermediates relative to value-added in sector k.

III. Antràs and de Gortari (2017) incorporated specialized inputs into a one-sector model: $|\mathcal{K}| = 1$ and $\mathcal{M} > 1$. Specifically, specialized input linkages are occur as pure snakes in the sense that the chain $\ell(\mathcal{M}) \to \cdots \to \ell(1)$ fully characterizes input sourcing decisions since each production stage sources a single upstream input and these chains do not 'branch-out' as in the multi-sector case. The welfare gains equal

$$\hat{W}_j = \left[\hat{\pi}_j\left(\ell_j^*\right)\right]^{-\frac{1}{\theta \sum\limits_{m=1}^{\mathcal{M}} \beta_j^m \prod\limits_{\mu=1}^{m-1} \xi_j^\mu}}.$$

Now the sufficient statistic is given by the expenditure on goods produced through purely domestic chains, $\ell_j^* = \{j \to \cdots \to j\}$, and the amplification is given by the weighted trade elasticity across all stages of production $\beta_j^1 + \beta_j^2 \xi_j^1 + \beta_j^3 \xi_j^2 \xi_j^1 + \dots$. The latter proxies the fact that trade costs are leveraged over gross output and this increases as specialized inputs flow down the chain of production. Note that, in principle, the overall amplification effect might not be higher than in the Eaton and Kortum (2002) since θ means different things across models and while empirical estimates of the trade elasticity map directly into θ in a structural gravity world this is not so in a world with specialized inputs.

The key reason why bilateral trade data is not sufficient for characterizing welfare in specialized inputs models is that changes in geography have asymmetric effects over intermediate input purchases. That is, structural gravity implies that when Mexico sells one more car to the U.S. this has the exact same effects on its purchases of car parts than when it sells one more car to Germany. In a world of specialized inputs this is not so since Mexico uses different inputs to produce cars sold to different markets. More formally, this can be stated as third country trade costs having an asymmetric effect on the elasticity of relative imports from two sources. That is, the macro-level restriction that Arkolakis et al. (2012) call "the import demand system is CES" fails and I show this formally in Appendix Section B.6.

The general formula presented in equation (48) contains elements from all three papers. First, the gains depend on a domestic share variable and the trade elasticity as in Eaton and Kortum (2002). Second, the gains also depend on the shifts in relative prices as in Caliendo and Parro (2015). Third, the trade elasticity is amplified through intermediate input linkages as in Eaton and Kortum (2002) but this amplification also depends on the specialized inputs channel as in Antràs and de Gortari (2017). However, now the amplification is even richer since specialized inputs occur as spiders and is given by the sum of value-added shares across the total $|\mathcal{K}|^{\mathcal{M}-1}$ domestic chains in ℓ_j^* as noted by the summatory $\sum_{k^{\mathcal{M}},\dots,k^2}$.

Current data limitations also preclude the use of the sufficient statistics approach for the time being. While knowing the deep parameters governing compatibility $T_j^k(\ell)$ are no longer needed, the (currently unavailable) equilibrium supply chain flows are required in order to compute domestic expenditure shares. This is further compounded by the fact that the spe-

cialized input expenditure elasticities $\xi_j^{k,k'}$ are also unknown and that shifts in relative prices are hard to obtain in both general counterfactual exercises and real world trade liberalizations.

6 Conclusions

I have developed a broad GVC framework consistent with the specialized inputs linkages that permeate today's global trade arena. The small picture takeaway is that this channel yields a much more integrated view of Mexico-U.S. trade and this confirms the worries arising from a potential increase in trade barriers following the current renegotiation of NAFTA.

The big picture take is that additional sources of information can be used in a piecemeal basis to obtain more accurate GVC estimates. I have focused on the GVCs that cross through Mexico and studied statistics heavily influenced by these variables given my access to the Mexican firm-level data. Other researchers can readily incorporate their own data into this estimation framework in order to study whichever questions are pertinent. Moreover, multiple sources of data, say U.S. and Chinese customs data, can be jointly incorporated to study relevant statistics such as U.S.-China value-added trade imbalances. I have posted the quadratic programming code permanently in estimategvcs.com so that anyone can immediately use it.

Ultimately, measuring regional integration properly matters since deep integration is associated with potentially more costly supply chain disruption. I provided a model that satisfies the maxim of developing theory in order to guide the recollection of data since, though of limited current use, it provides statistical offices with a map of which data to collect and report. In particular, it calls for obtaining better domestic transaction data and reporting measures of cross-industry domestic supply chain expenditures. While countries with value-added taxes often collect this data, it is much less prevalent in those without such as the U.S.

The most pressing need facing the theoretical GVC field involves addressing a major issue that I have disregarded entirely: Fixed costs of production. There is ample anecdotal evidence suggesting that this is a key concern regarding supply chain disruption since it is very costly, in terms of both time and money, for manufacturers to transfer production facilities across borders when trade breaks down. I have ignored this margin not out of choice but out of necessity since combining specialized inputs linkages with fixed costs is extremely challenging given that one quickly runs into multiple equilibria. Specifically, fixed costs imply that upstream marginal costs depend on the downstream use of output and so general equilibrium cannot be computed through a recursive characterization of firm supply chain decisions. Developing a specialized inputs model that nests Melitz (2003) as a knife-edge case would be a great step towards better understanding the welfare losses from supply chain disruption.

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A   Graphical Intuition

Concretely, the paper's message is simple and the intuition can be conveyed through the graphical representation in Figure A.1. Let $\mathcal{G}$ summarize all GVC flows leading to final consumption. In other words, let $\mathcal{G}$ proxy a specific GVC data generating process that is consistent with some aggregate WIOT data. The large cloud in Figure A.1a represents the highly-dimensional space of all data generating processes that are consistent with some aggregate WIOT data, with the yellow star representing the
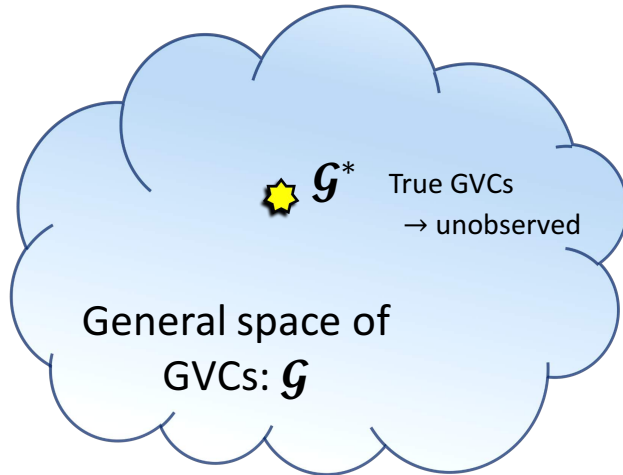
(a) GVCs live in a high dimensional space of which the true GVCs are a single point.
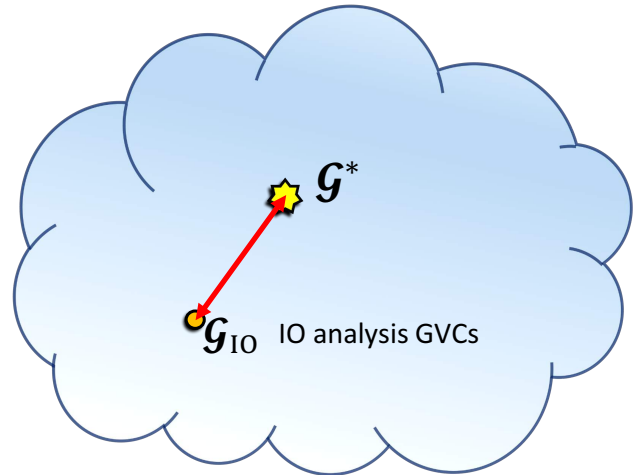
(b) The roundabout or I-O analysis GVCs are only one of many possible data generating processes. The distance between two points proxies how similar they are.
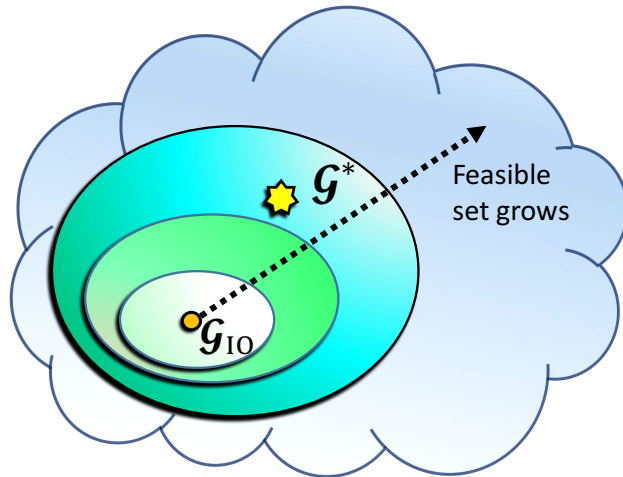
(c) Relaxing the proportionality assumptions as in the specialized inputs measurement framework increases the size of the set of GVCs that can be estimated. The QP framework can back out any flows consistent with a given degree of proportionality.
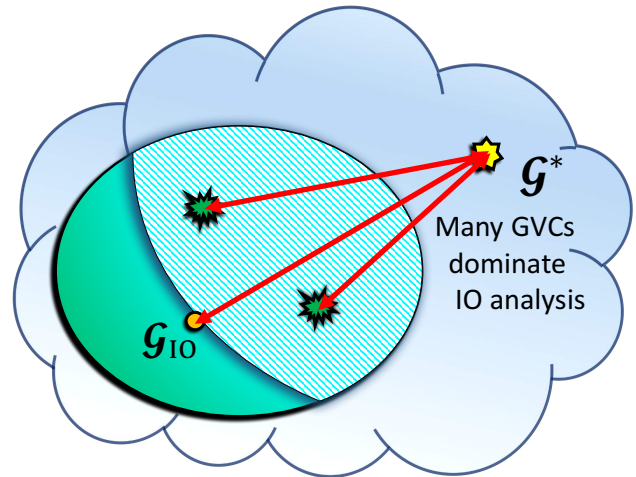
(d) Even if we cannot work with the correct degree of proportionality, there exist a whole set of solutions to the QP framework that dominate the I-O analysis GVCs.

Figure A.1: The Specialized Inputs Measurement Framework and QP Intuition.

true (unobserved) one.

Figure A.1b shows that the I-O analysis GVCs are mismeasured and let the distance between two dots proxy how close they are (for example, as given by the Euclidean distance). The set of GVCs consistent with WIOT data that can be measured expands as the I-O analysis input shares in Assumption 3.3 are relaxed to hold only across longer sequences of production as with specialized inputs in Assumption 3.6. This can be seen in Figure A.1c with the larger circles representing the set of GVCs consistent with weaker assumptions (i.e. larger $\mathcal{M}$). Relaxing the proportionality assumption is desirable since the QP framework can potentially back out the correct ones once the the assumptions are weak enough and the true GVCs fall in this set.

More generally, it may occur that the true GVCs do not satisfy any notion of fixed input shares or satisfy one that is beyond our computing power. In such a case, Figure A.1d shows that the QP framework can still improve upon I-O analysis as the GVC estimates in the shaded region, though still imperfect, are more accurate. The key identification assumption can be described as shifting the GVC estimates into the shaded region. A researcher can use additional data or her own priors over the data generating process to discipline the GVC estimates as she considers reasonably. If done correctly, this improves the GVC estimates relative to I-O analysis.

B Mathematical Derivations

B.1 Linear Algebra for the QP Framework

All of the vectors throughout the paper are stacked in the same way with the sorting done first along the first dimension, then along the second, so on and so forth. Formally, let $v\left(s^{\mathcal{M}}, \ldots, s^1, s^0\right)$ be a variable of $\mathcal{M} + 1$ dimensions where each has the range of elements in $\mathcal{S}$ and to make notation clener assume that the set notation also denote the number of elements contained therein. Hence, $\mathcal{S}$ is the set of all country-industry pairs and index a pair $s \in \mathcal{S}$ consisting of an industry $k \in \mathcal{K}$ in country $j \in \mathcal{J}$ as $s = (j-1)\mathcal{K} + k$ so that these can be referred to with $s = 1, \ldots, \mathcal{S}$. I define the stacking recursively. Define $v^0\left(s^{\mathcal{M}}, \ldots, s^0\right) = v\left(s^{\mathcal{M}}, \ldots, s^0\right)$ as the initial vector of size 1×1. Each of the $\mathcal{M} + 1$ dimensions are stacked up as

$$\underset{\mathcal{S}^n \times 1}{\boldsymbol{v}^n(s^{\mathcal{M}}, \ldots, s^n)} = \begin{bmatrix} \boldsymbol{v}^{n-1}(s^{\mathcal{M}}, \ldots, s^n, 1) \\ \vdots \\ \boldsymbol{v}^{n-1}(s^{\mathcal{M}}, \ldots, s^n, \mathcal{S}) \end{bmatrix},$$

with $n = 1, \ldots, \mathcal{M}$. Finally, $\boldsymbol{v}^{\mathcal{M}+1}$ is the stacked vector of size $\mathcal{S}^{\mathcal{M}+1} \times 1$ of vectors $\boldsymbol{v}^{\mathcal{M}}(s)$.

I now define the matrices for the QP framework (33) for an arbitrary $\mathcal{M} \in \mathbb{Z}^+$. Stack up $\boldsymbol{c}_{\mathcal{X}}$, $\boldsymbol{q}_{\mathcal{X}}$, and $\mathcal{X}$ as before so that they are vectors of size $\mathcal{S}^{\mathcal{M}+1} \times 1$ and stack up $\boldsymbol{c}_{\mathcal{G}}$, $\boldsymbol{q}_{\mathcal{G}}$ and $\mathcal{G}$ analogously but with the initial stacking across $\mathcal{J}$ destinations only so that these are vectors of size $\mathcal{S}^{\mathcal{M}}\mathcal{J} \times 1$. The WIOT data is stacked up similarly but only up to $n = 1$ so that X is a $\mathcal{S}^2 \times 1$ vector and F is a $\mathcal{S}\mathcal{J} \times 1$ vector. Finally,

let $\tilde{\beta} = \left[\frac{1}{1-\beta(s)}\right]$ be an auxiliary vector of size $1 \times \mathcal{S}$. Define the auxiliary matrices

$$\underset{\mathcal{S}^2 \times \mathcal{S}^{\mathcal{M}+1}}{\mathbf{B}_{\mathcal{X}}} = \left[\left(\mathbf{1}_{1\times\mathcal{S}} \otimes \tilde{\beta}^{\otimes(\mathcal{M}-2)}\right) \otimes \mathrm{diag}\left\{\tilde{\beta} \otimes \mathbf{1}_{1\times\mathcal{S}}\right\}\right],$$

$$\underset{\mathcal{S}\mathcal{J} \times \mathcal{S}^{\mathcal{M}}\mathcal{J}}{\mathbf{B}_{\mathrm{F}}} = \left[\left(\mathbf{1}_{1\times\mathcal{S}} \otimes \tilde{\beta}^{\otimes(\mathcal{M}-2)}\right) \otimes \mathrm{diag}\left\{\tilde{\beta} \otimes \mathbf{1}_{1\times\mathcal{J}}\right\}\right],$$

$$\underset{\mathcal{S}^{\mathcal{M}} \times \mathcal{S}^{\mathcal{M}+1}}{\mathbf{D}_{\beta\mathcal{X}}} = \left[\mathbf{1}_{1\times\mathcal{S}} \otimes \mathrm{diag}\left\{\tilde{\beta} \otimes \mathbf{1}_{1\times\mathcal{S}^{\mu-1}}\right\}\right],$$

$$\underset{\mathcal{S}^{\mathcal{M}} \times \mathcal{S}^{\mathcal{M}+1}}{\mathbf{D}_{\mathcal{X}}} = \left[\mathbb{I}_{\mathcal{S}^{\mathcal{M}} \times \mathcal{S}^{\mathcal{M}}} \otimes \mathbf{1}_{1\times\mathcal{S}}\right],$$

$$\underset{\mathcal{S}^{\mathcal{M}} \times \mathcal{S}^{\mathcal{M}}\mathcal{J}}{\mathbf{D}_{\mathcal{G}}} = \left[\mathbb{I}_{\mathcal{S}^{\mathcal{M}} \times \mathcal{S}^{\mathcal{M}}} \otimes \mathbf{1}_{1\times\mathcal{J}}\right],$$

$$\underset{\mathcal{S}^{\mathcal{M}}(\mathcal{S}+\mathcal{J}) \times \mathcal{S}^{\mathcal{M}}(\mathcal{S}+\mathcal{J})}{\mathbf{Q}} = \mathrm{diag}\left\{\mathbf{q}_{\mathcal{X}}, \mathbf{q}_{\mathcal{G}}\right\}.$$

with $\otimes$ the Kronecker product, and where $\tilde{\beta}^{\otimes(\mathcal{M}-2)}$ is the $(\mathcal{M}-2)-$fold Kronecker product of $\tilde{\beta}$ with itself and of size $1 \times \mathcal{S}^{\mathcal{M}-2}$. The QP framework is

$$\min \begin{bmatrix} \mathcal{X} - \mathbf{c}_{\mathcal{X}} \\ \mathcal{G} - \mathbf{c}_{\mathcal{G}} \end{bmatrix}^{\mathsf{T}} \mathbf{Q} \begin{bmatrix} \mathcal{X} - \mathbf{c}_{\mathcal{X}} \\ \mathcal{G} - \mathbf{c}_{\mathcal{G}} \end{bmatrix}$$

$$\text{s.t.} \quad \begin{bmatrix} \mathbf{B}_{\mathcal{X}} & \mathbf{0}_{\mathcal{S}^2 \times \mathcal{S}^{\mathcal{M}}\mathcal{J}} \\ \mathbf{0}_{\mathcal{S}\mathcal{J} \times \mathcal{S}^{\mathcal{M}+1}} & \mathbf{B}_{\mathrm{F}} \\ \mathbf{D}_{\beta\mathcal{X}} - \mathbf{D}_{\mathcal{X}} & -\mathbf{D}_{\mathcal{G}} \end{bmatrix} \begin{bmatrix} \mathcal{X} \\ \mathcal{G} \end{bmatrix} = \begin{bmatrix} \mathbf{X} \\ \mathbf{F} \\ \mathbf{0}_{\mathcal{S}^{\mathcal{M}} \times 1} \end{bmatrix}$$

$$\mathbb{I}_{\mathcal{S}^{\mathcal{M}}(\mathcal{S}+\mathcal{J}) \times \mathcal{S}^{\mathcal{M}}(\mathcal{S}+\mathcal{J})} \begin{bmatrix} \mathcal{X} \\ \mathcal{G} \end{bmatrix} \geqslant \mathbf{0}_{\mathcal{S}^{\mathcal{M}}(\mathcal{S}+\mathcal{J}) \times 1}$$

The first two sets of linear constraints represent the bilateral data constraints (29) and (30) while the last set of linear constraints represent the consistency constraint (31).

B.2 Proof of Lemma 3.7

Proof. From the definition of $\mathcal{X}^{\mathcal{M}}(\cdot)$ in (19) and the specialized inputs assumption (18)

$$\mathcal{X}^{\mathcal{M}}\left(t, s^{\mathcal{M}}, \ldots, s^1\right),$$

$$= \sum_{N=\mathcal{M}+1}^{\infty} \sum_{l^{N-\mathcal{M}-1}\in\mathcal{S}} \cdots \sum_{l^1\in\mathcal{S}} \sum_{j\in\mathcal{J}} \mathcal{G}^N\left(t, s^{\mathcal{M}}, \ldots, s^1, l^{N-\mathcal{M}-1}, \ldots, l^1, j\right),$$

$$= \sum_{N=\mathcal{M}+1}^{\infty} \sum_{l^{N-\mathcal{M}-1}\in\mathcal{S}} \cdots \sum_{l^1\in\mathcal{S}} \sum_{j\in\mathcal{J}} a^{\mathcal{M}}\left(t\,|\,s^{\mathcal{M}}, \ldots, s^1\right) \mathcal{G}^{N-1}\left(s^{\mathcal{M}}, \ldots, s^1, l^{N-\mathcal{M}-1}, \ldots, l^1, j\right),$$

$$= a^{\mathcal{M}}\left(t\,|\,s^{\mathcal{M}}, \ldots, s^1\right)\left[\sum_{l\in\mathcal{S}} \sum_{N=\mathcal{M}+1}^{\infty} \sum_{l^{N-\mathcal{M}-1}\in\mathcal{S}} \cdots \sum_{l^1\in\mathcal{S}} \sum_{j\in\mathcal{J}} \mathcal{G}^N\left(s^{\mathcal{M}}, \ldots, s^1, l, l^{N-\mathcal{M}-1}, \ldots, l^1, j\right) + \sum_{j\in\mathcal{J}} \mathcal{G}^{\mathcal{M}}\left(s^{\mathcal{M}}, \ldots, s^1, j\right)\right],$$

$$= a^{\mathcal{M}}\left(t\,|\,s^{\mathcal{M}}, \ldots, s^1\right)\left[\sum_{l\in\mathcal{S}} \mathcal{X}^{\mathcal{M}}\left(s^{\mathcal{M}}, \ldots, s^1, l\right) + \sum_{j\in\mathcal{J}} \mathcal{G}^{\mathcal{M}}\left(s^{\mathcal{M}}, \ldots, s^1, j\right)\right].$$

$\square$

B.3 Corollary 3.8

Proof. From equation (10) we have

$$\mathcal{G}^N\left(s^{\mathcal{M}}, \ldots, s^1, l, l^{N-\mathcal{M}-1}, \ldots, l^1, j\right) = \frac{1}{(1 - \beta(s^{\mathcal{M}}))} \sum_{t' \in \mathcal{S}} \mathcal{G}^{N+1}\left(t', s^{\mathcal{M}}, \ldots, s^1, l, l^{N-\mathcal{M}-1}, \ldots, l^1, j\right)$$

and

$$\mathcal{G}^{\mathcal{M}}\left(s^{\mathcal{M}}, \ldots, s^1, j\right) = \frac{1}{(1 - \beta(s^{\mathcal{M}}))} \sum_{t' \in \mathcal{S}} \mathcal{G}^{\mathcal{M}+1}\left(t', s^{\mathcal{M}}, \ldots, s^1, j\right).$$

Substituting these terms into the fourth line of the previous proof we obtain that

$$
\begin{aligned}
\mathcal{X}^{\mathcal{M}}\left(t, s^{\mathcal{M}}, \ldots, s^1\right) &= \frac{a^{\mathcal{M}}\left(t \mid s^{\mathcal{M}}, \ldots, s^1\right)}{(1 - \beta(s^{\mathcal{M}}))} \sum_{t' \in \mathcal{S}} \left[\sum_{N=\mathcal{M}+1}^{\infty} \sum_{l^{N-\mathcal{M}-1} \in \mathcal{S}} \cdots \sum_{l^1 \in \mathcal{S}} \sum_{j \in \mathcal{J}} \mathcal{G}^N\left(t', s^{\mathcal{M}}, \ldots, s^1, l^{N-\mathcal{M}-1}, \ldots, l^1, j\right) \right], \\
&= \frac{a^{\mathcal{M}}\left(t \mid s^{\mathcal{M}}, \ldots, s^1\right)}{(1 - \beta(s^{\mathcal{M}}))} \sum_{t' \in \mathcal{S}} \mathcal{X}^{\mathcal{M}}\left(t', s^{\mathcal{M}}, \ldots, s^1\right).
\end{aligned}
$$

□

B.4 $\mathcal{M}$-Proportional GVCs for Constant Returns to Scale

TBD

B.5 Decentralization

TBD

B.6 Import Demand Systems in a Specialized Inputs World Are Not CES

I illustrate how the presence of specialized inputs breaks the key macro-level restriction in Arkolakis et al. (2012) that relative imports be unaffected, in partial equilibrium, by changes in trade costs with third countries. To make this as simple as possible I assume a one-sector model with a single link of sequential inputs, i.e. $|\mathcal{K}| = 1$ and $\mathcal{M} = 1$. This is equivalent to the model in Antràs and de Gortari (2017). I make two additional assumptions in order to focus on a special case, which is easy to study analytically, but all the results carry through more generally. Specifically, I assume that (i) the compatibility parameters are separable across stages of production, weighted by the share of stage output relative to the finished good output, and independent of the consumption location and given by $T_j(\ell) = \left(T_{\ell(2)}\right)^{\xi_{\ell(1)}} T_{\ell(1)}$; (ii) the value-added relative to composite expenditure is constant across both stages of production so that $\beta_j^1/\gamma_j^1 = \beta_j^2/\gamma_j^2$ and $\beta_j^1 + \gamma_j^1 + \xi_j = \beta_j^2 + \gamma_j^2 = 1$. Finally, I focus on a more general notion of geography such that trade costs are stage specific and not necessarily symmetric (i.e. $\tau_{ij}^m \neq \tau_{ji}^m$); the importance of specialized inputs is even greater when trade costs are common across stages and symmetric.

The distribution over supply chains $h \to i \to j$, from which j sources finished varieties, is given by

$$\pi_{hi,j} = \frac{1}{\Theta_j} \left(T_h \left(c_h^2 \tau_{hi}^2\right)^{-\theta}\right)^{\xi_i} \times T_i \left(c_i^1 \tau_{ij}^1\right)^{-\theta},$$

and can be used to build bilateral trade shares. The share of final good flows that country j purchases from country i equals

$$\pi_{ij}^{F} = \frac{1}{\Theta_j^{F}} \sum_{h \in \mathcal{J}} \left(T_h \left(c_h^2 \tau_{hi}^2 \right)^{-\theta} \right)^{\xi_i} \times T_i \left(c_i^1 \tau_{ij}^1 \right)^{-\theta},$$

while the share of bilateral intermediate input purchases equals

$$\pi_{ij}^{X} = \frac{1}{\Theta_j^{X}} \left[\pi_{ij}^{F} \times \frac{\gamma_j^1}{\beta_j^1} GDP_j + \sum_{h \in \mathcal{J}} \frac{1}{\Theta_h} \left(\left(T_i \left(c_i^2 \tau_{ij}^2 \right)^{-\theta} \right)^{\xi_h} \times T_j \left(c_j^1 \tau_{jh}^1 \right)^{-\theta} \right) \times \xi_h \left(1 + \frac{\gamma_h^1}{\beta_h^1} \right) GDP_h \right].$$

The proportionality constants Θ_j, Θ_j^{F} and Θ_j^{X} are such that $\sum_{h,i \in \mathcal{J} \times \mathcal{J}} \pi_{hi,j} = \sum_{i \in \mathcal{J}} \pi_{ij}^{F} = \sum_{i \in \mathcal{J}} \pi_{ij}^{X} = 1$.

B.6.1   CES Import Demand System

Arkolakis et al. (2012) define their third macro-level restriction as follows: The import demand system is such that for any importer j and any pair of exporters $i \neq j$ and $i' \neq j$, the partial elasticity of imports from i relative to domestic expenditures with respect to trade costs

$$\frac{\partial \ln \left(Z_{ij} / Z_{jj} \right)}{\partial \ln \tau_{i'j}} = \begin{cases} -\theta & \text{if } i' = i, \\ 0 & \text{if } i' \neq i, \end{cases}$$

where Z_{ij} is overall imports from i in country j, τ_{ij} is variable trade costs between i and j, and θ is the trade elasticity. I will derive these elasticities for both intermediate and final good imports.

B.6.2   Import Demand System for Final Goods

Note that $F_{ij}/F_{jj} = \pi_{ij}^{F}/\pi_{jj}^{F}$ so that the partial elasticity of final good imports with respect to stage 1 trade costs equals

$$\frac{\partial \ln \left(\pi_{ij}^{F}/\pi_{jj}^{F} \right)}{\partial \ln \tau_{i'j}^1} = \begin{cases} -\theta & \text{if } i' = i, \\ 0 & \text{if } i' \neq i, \end{cases}$$

while the same elasticity but with respect to stage 2 trade costs equals

$$\frac{\partial \ln \left(\pi_{ij}^{F}/\pi_{jj}^{F} \right)}{\partial \ln \tau_{i'j}^2} = \theta \xi_j \frac{\left(T_{i'} \left(c_{i'}^2 \tau_{i'j}^2 \right)^{-\theta} \right)^{\xi_j}}{\sum_{h \in \mathcal{J}} \left(T_h \left(c_h^2 \tau_{hj}^2 \right)^{-\theta} \right)^{\xi_j}}.$$

When trade costs are common across stages of production then the overall partial elasticity equals

$$\frac{\partial \ln \left(\pi_{ij}^{F}/\pi_{jj}^{F} \right)}{\partial \ln \tau_{i'j}} = -\theta \left[\mathbf{1}_{[i=i']} - \xi_j \frac{\left(T_{i'} \left(c_{i'}^2 \tau_{i'j}^2 \right)^{-\theta} \right)^{\xi_j}}{\sum_{h \in \mathcal{J}} \left(T_h \left(c_h^2 \tau_{hj}^2 \right)^{-\theta} \right)^{\xi_j}} \right].$$

Clearly, the import demand system is not CES.

B.6.3 Import Demand System for Intermediate Goods

The partial elasticity for intermediate good imports with respect to stage 1 trade costs equals

$$
\frac{\partial \ln\left(\pi_{ij}^X/\pi_{jj}^X\right)}{\partial \ln \tau_{i'j}^1} = -\theta \frac{\mathrm{GDP}_j}{\Theta_j^X}\left[\frac{\gamma_j^1}{\beta_j^1}\left(\frac{\pi_{jj}^F \pi_{i'j}^F}{\pi_{jj}^X} - \frac{\pi_{ij}^F\left(\pi_{i'j}^F - 1_{[i=i']}\right)}{\pi_{ij}^X}\right) + \frac{\xi_j\left(1-\xi_j\right)}{\beta_j^1}\pi_{i'j}^F\left(\frac{\pi_{jj,j}}{\pi_{jj}^X} - \frac{\pi_{ij,j}}{\pi_{ij}^X}\right)\right],
$$

while the same elasticity but with respect to stage 2 trade costs equals

$$
\frac{\partial \ln\left(\pi_{ij}^X/\pi_{jj}^X\right)}{\partial \ln \tau_{i'j}^2} = -\theta \xi_j \frac{\mathrm{GDP}_j}{\Theta_j^X}
$$
$$
\times \left[\frac{\gamma_j^1}{\beta_j^1}\pi_{i'j,j}\left(\frac{\pi_{jj}^F - 1}{\pi_{jj}^X} - \frac{\pi_{ij}^F}{\pi_{ij}^X}\right) + \sum_{h\in\mathcal{J}}\xi_j\pi_{i'j,h}\left(\frac{\pi_{jj,h}}{\pi_{jj}^X} - \frac{\pi_{ij,h} - 1_{[i=i']}}{\pi_{ij}^X}\right)\times\frac{\frac{1-\xi_h}{\beta_h}\mathrm{GDP}_h}{\mathrm{GDP}_j}\right].
$$

When trade costs are common across stages of production then the overall partial elasticity equals

$$
\frac{\partial \ln\left(\pi_{ij}^X/\pi_{jj}^X\right)}{\partial \ln \tau_{i'j}} = \frac{\partial \ln\left(\pi_{ij}^X/\pi_{jj}^X\right)}{\partial \ln \tau_{i'j}^2} + \frac{\partial \ln\left(\pi_{ij}^X/\pi_{jj}^X\right)}{\partial \ln \tau_{i'j}^1}.
$$

Once again, the import demand system is not CES.

B.6.4 The Special Case of Roundabout Production

Suppose that there are no specialized inputs, that is $\alpha_1 = 1$, so that the roundabout assumption holds. It is then easy to see that

$$
\frac{\partial \ln\left(\pi_{ij}^F/\pi_{jj}^F\right)}{\partial \ln \tau_{i'j}} = \frac{\partial \ln\left(\pi_{ij}^X/\pi_{jj}^X\right)}{\partial \ln \tau_{i'j}} = -1_{[i=i']}\theta,
$$

so that it is only in this knife-edge case in which the conditions of Arkolakis et al. (2012) hold. The intuition is very simple, trade costs between third countries matter in a world of specialized inputs because these linkages move input demand asymmetrically and a fall in trade barriers will increase relative imports from those locations and those countries that are situated along the supply chains through which trade costs fell. In the special case of roundabout production, there are no specialized inputs linkages and changes in trade costs shift input demand symmetrically.

C Industry Aggregation Bias

I review in both theory and practice the perils of not taking into account the potential heterogeneity in input shares across industrial sectors.

C.1 Industry Aggregation Bias in Theory

For simplicity, assume that there is a single country and that $\mathcal{K}$ is the set of sectors of production. Let $X(t,k)$ be aggregate intermediate input sales from sector t to sector k and let $Y(k)$ be total sales of sector k. Finally, assume that each sector in $\mathcal{K}$ produces a single homogenous good and that IO analysis holds

in the sense that every dollar of production in $k \in \mathcal{K}$ uses the same amount of inputs from every other $t \in \mathcal{K}$. The technical coefficients determining the IO analysis input shares are

$$a(t|k) \equiv \frac{X(t,k)}{Y(k)},$$

where I have written $a(t|k)$ with the conditional sign to be explicit that input shares are specific to each industry $k \in \mathcal{K}$.

Suppose that our statistical office only reports data at the level of $\mathcal{K}^{AGG}$ sectors. In particular, assume that for each $k' \in \mathcal{K}^{AGG}$ the set $\kappa(k') \subset \mathcal{K}$ represents the sectors that are aggregated into k' (i.e. $\kappa(\cdot)$ is a partition of $\mathcal{K}$). The technical coefficients associated with $t', k' \in \mathcal{K}^{AGG}$ are then

$$a^{AGG}(t'|k') \equiv \frac{X^{AGG}(t',k')}{Y^{AGG}(k')} = \frac{\sum_{t \in \kappa(t')} \sum_{k \in \kappa(k')} X(t,k)}{\sum_{k \in \kappa(k')} Y(k)}.$$

The industry aggregation bias arises from the misspecification of the technical coefficients of the aggregate data and this occurs unless the aggregation is only done across sets of industries that share the same input mix. That is, aggregation is consistent when

$$a^{AGG}(t'|k') \text{ is unbiased } \Leftrightarrow \text{ For every } t \in \kappa(t') \text{ we have } a(t|k) = a(t|l) \, \forall k, l \in \kappa(k') \qquad (49)$$

When this condition fails the calculation of upstream input requirements are biased. To see why imagine we wish to compute the amount of inputs purchased from t' through k' for the production of l'. In general the true number does not equal that implied by the aggregate technical coefficients:

$$\sum_{t \in \kappa(t')} \sum_{k \in \kappa(k')} \sum_{l \in \kappa(l')} a(t|k) X(k,l) \neq a^{AGG}(t'|k') X^{AGG}(k',l').$$

On the left $X(k,l)$ is the inputs each l purchases directly from each k while a share $a(t|k)$ of that is spent on further upstream inputs from t. However, because each k has different input requirements from each t this implies that the the average input purchases from the industries in $\kappa(k')$ of the industries $\kappa(t')$, as indicated by $a^{AGG}(t'|k')$, may be poor approximation of the actual value of these input flows. The latter is only accurate in the special case in which $a(t|k)$ is constant across all the elements of $\kappa(k')$.[31] In sum, IO analysis estimates may be biased even if it is correct at a level of disaggregation $\mathcal{K}$ when the data is only observable at $\mathcal{K}^{AGG}$.

C.2 Industry Aggregation Bias in Practice

The IO tables on which GVCs are estimated typically contain around 50 or less industries per country which suggests that the industry aggregation bias is probably substantial.[32] I will now show that, at least for the U.S., this issue is indeed present. Specifically, the most disaggregate IO data for the U.S. is

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[31]In this case

$$\sum_{t \in \kappa(t')} \sum_{k \in \kappa(k')} \sum_{l \in \kappa(l')} a(t,k) X(k,l) = \sum_{t \in \kappa(t')} \frac{\sum_{k \in \kappa(k')} X(t,k)}{\sum_{k \in \kappa(k')} Y(k)} \left[\sum_{k \in \kappa(k')} \sum_{l \in \kappa(l')} X(k,l) \right] = a^{AGG}(t',k') X^{AGG}(k',l').$$

[32]In the words of Hatanaka (1952) and McManus (1956b), regarding the condition in (49) "There is very little chance that they will be fulfilled by any model".

available for 379 sectors for the year 2007 but, for simplicity, I will concentrate on manufactures which encompasses 237 sectors.

The exercise I run is the following, let $\mathcal{K}$ be the set of 6−digit NAICS sectors (237) and let $\mathcal{K}^{AGG}$ be the set of 3−digit NAICS sectors (19). Is it true that all of the 6−digit sectors within each 3−digit share the same input shares? No.

First, I focus on a specific 3−digit sector 'Computer and electronic products', which is composed of 20 6−digit sectors. The left panel of figure C.1 plots the implied input shares of its five largest 6−digit codes. That is, the five industries labeled on the y−axis account for 50% of the output of 'Computer and electronic products'. Meanwhile, the input shares are shown for the five most important input suppliers. In this case input expenditures on 'Other electronic components' accounts for 3.6% of the aggregate output value of the 3−digit sector and using the 3−digit data implies assuming that these are also the input expenditure shares of each specific 6−digit sector. However, the panel on the right shows the true input shares for each 6-digit sector. The differences are substantial. For example, the 6−digit code 'Electronic computers' spends 11% of its output value on 'Computer storage devices' but the other four 6−digit codes spend almost zero on these inputs. The aggregation bias is manifest in that the 3−digit sourcing shares in the left panel assume that each 6−digit sector actually spends a share of 2.7% on these inputs.
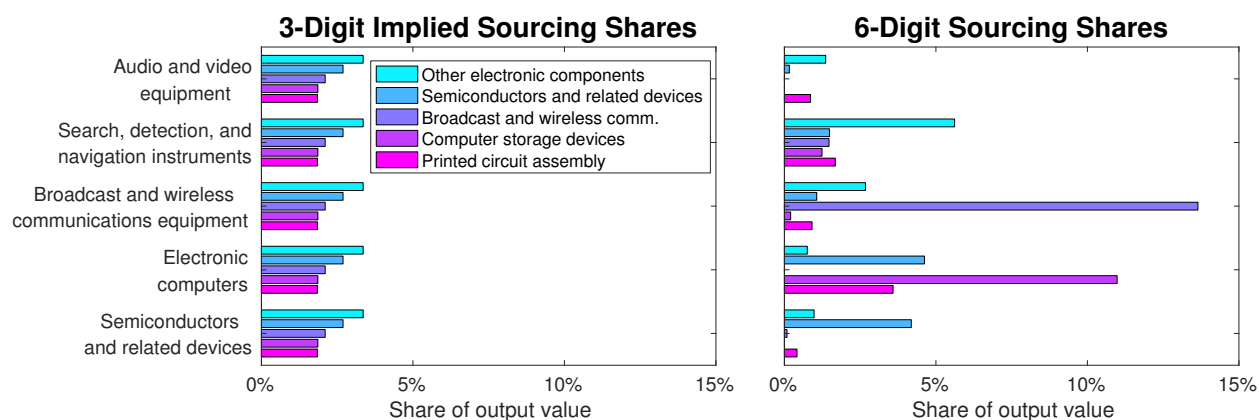

Figure C.1: Input sourcing shares of the five largest 6−digit sectors in 'Computer and electronic products'.

Second, to look at the overall picture I compute the coefficient of variation −standard deviation relative to mean− of input shares from each source within each 3−digit code. Specifically, for each 3−digit k' I compute the coefficient of variation of $a(t|k)$ for each $t \in \mathcal{K}$ and across $k \in \kappa(k')$. IO analysis at the 3−digit level is only correct under the assumptions that IO analysis is correct at the 6−digit level and that all 6−digit sectors $\kappa(k')$ in each 3−digit k' have the same sourcing shares from each industry. In such a case the coefficient of variation will be zero. When the aggregation is done across sectors with very different sourcing shares then this statistic will be large and positive.

Figure C.2 shows the coefficients of variation across all 3−digit sectors in manufactures and for each input supplier. Each circle represents the coefficient of variation of input shares from a specific source across all of the 6−digit codes contained in each 3−digit code; the size of each circle is proportional to the share of inputs purchased by the latter. There is one takeaway: There is substantial variation in input shares within each 3−digit sector. For example, for 'Computers and electronics' the five biggest circles are those corresponding to input shares from the sources in figure C.1 and the values are 1.0, 1.3,
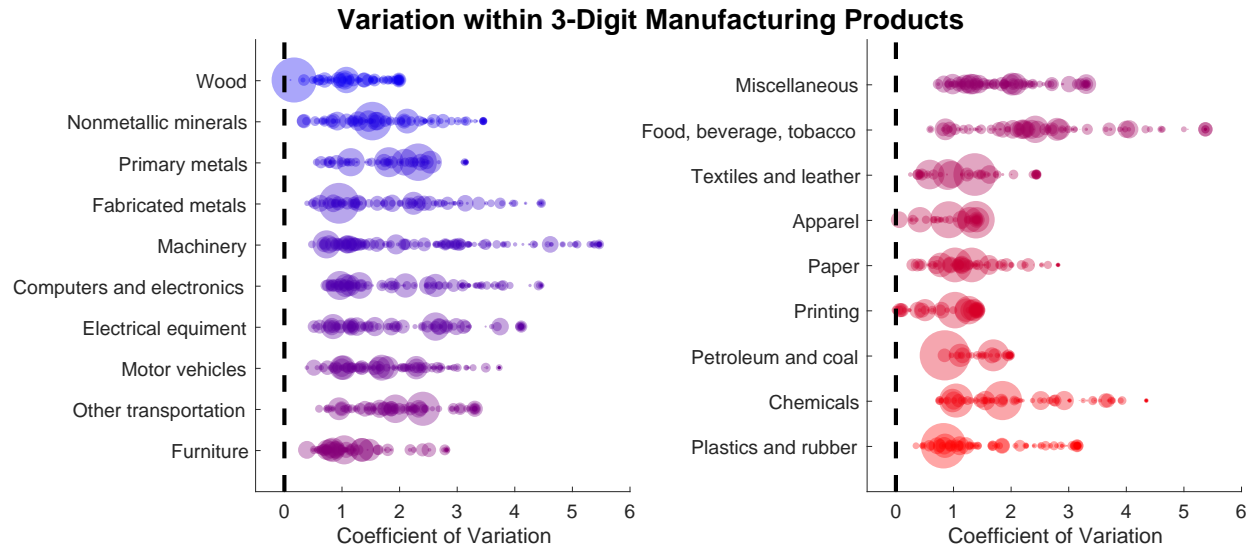
Figure C.2: Variation in 6−digit sourcing shares for each 3−digit sector in manufactures. Each circle corresponds to the coefficient of variation of the input sourcing shares from a specific source across all 6−digit sectors within each 3-digit sector. Circle size is proportional to the share of aggregate input purchases from each source. Data corresponds to 2007 U.S. IO tables.

2.1, 2.6, and 1.1. In this case, the largest input source is 'Other electronic components' and as figure C.1 shows there is relatively little variation in input shares and so the coefficient of variation is 1.0. In contrast, figure C.1 shows high variation for 'Broadcast and wireless communications equipments' and for 'Computer storage devices' and these appear with values 2.1 and 2.6.

D Additional Results

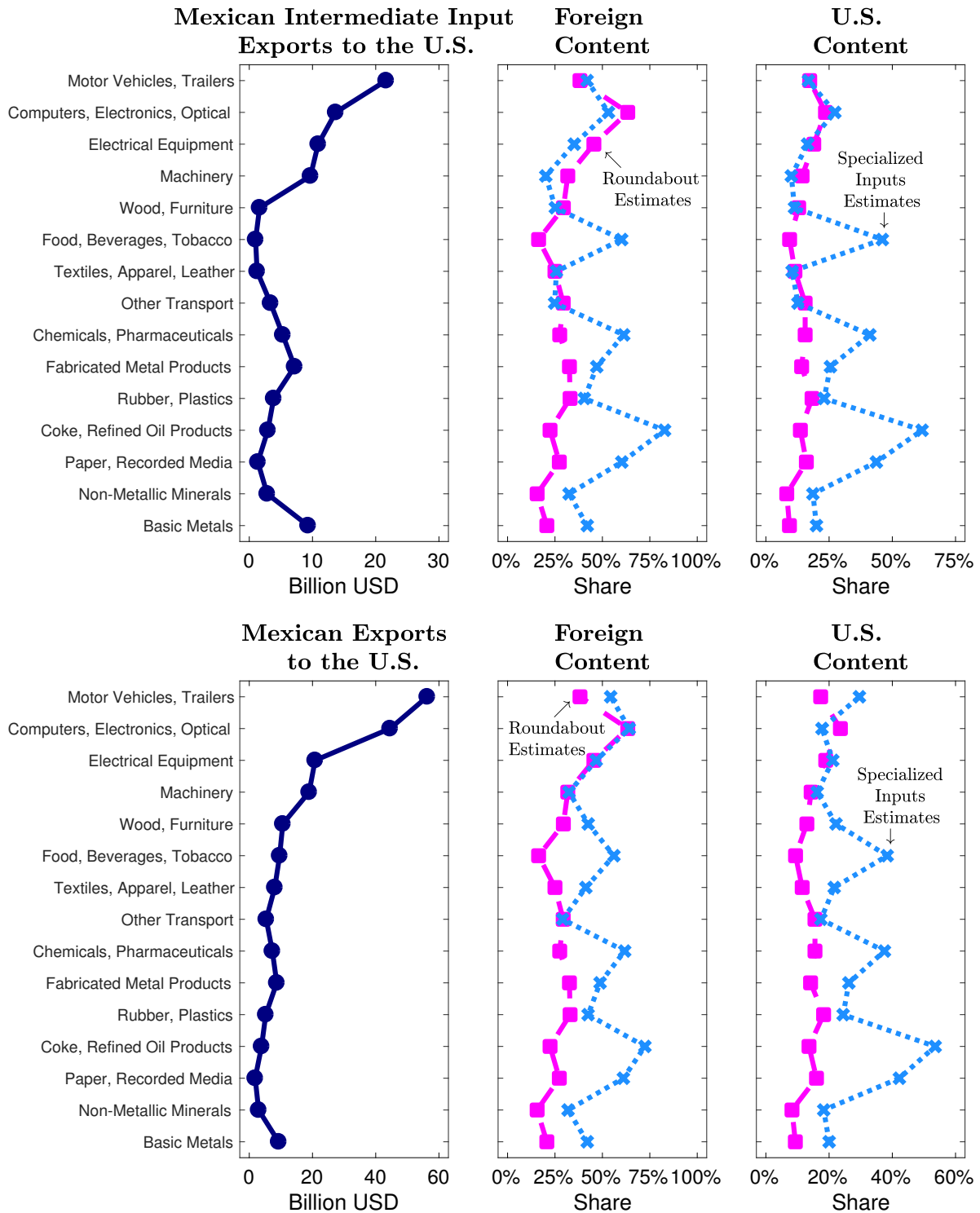## D.1 Foreign and U.S. Content in Mexican Exports

Figure D.1: Foreign and U.S. Content in Mexican Manufacturing Exports to U.S: Top panel corresponds to intermediate input exports while bottom panel to overall exports.